The BIG SIGMAA News

Volume 9, Number 2 **November 2013**

Business, Industry, and Government Special Interest Group of the Mathematical Association of America

BIG Events in Baltimore in January

The Joint Mathematics Meetings return to the East Coast in January. They will be held at the Baltimore Convention Center, January 15 – 18 (Wednesday – Saturday), 2014. JMM claims to be the largest mathematics meeting in the world. Over 6,600 people attended the 2013 meetings in San Diego, and in Baltimore there will be over 2,500 individual presentations. The complete program is on the meeting's website (http://jointmathematicsmeetings.org/jmm), including a paragraph about each of the invited speakers.

The BIG SIGMAA will have a BIG presence at the meetings. On Friday morning, the BIG SIGMAA will be jointly sponsoring (with the MAA Committee on Undergraduate Student Activities and Chapters) a panel discussion on Nonacademic Career Paths for Mathematicians. Several of the scheduled participants are BIG SIGMAA members. On Friday afternoon, the first of two contributed paper sessions will be held. Ten speakers will talk about their experiences using mathematics in business, industry, and government. The full program, with links to the abstracts of the talks, can be found at the JMM website.

On Friday evening, we are jointly sponsoring (with the History of Mathematics SIGMAA) a guest lecture by William Noel. Currently at the University of Pennsylvania, Dr. Noel was for many years a curator at the Walters Art Museum in Baltimore, where he directed an international program to conserve, image, and study the Archimedes Palimpsest. In that endeavor and others, Dr. Noel has done groundbreaking work in the application of digital technology to the study of ancient manuscripts. The title of Dr. Noel's talk at JMM is Eureka! The Archimedes Palimpsest. Here is a portion of Dr. Noel's description of his talk:

> This presentation centers on a manuscript of extraordinary importance to the history of science, the Archimedes Palimpsest. This thirteenth century prayer book contains erased texts that were written several centuries earlier still. These erased texts include two treatises by Archimedes that can be found nowhere else, The Method and Stomachion. The manuscript sold at auction to a private collector on the 29th October 1998. The owner

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Editor's note:

Thanks to Aaron Luttman for his article (beginning on page 2) comparing industrial mathematical research with academic research. Thanks also to Greg Cosxon for his helpful suggestions and to Collin Carbno for the interesting artwork.

If you would like to contribute an article, a poem, a puzzle, or anything else to future issues of the newsletter, please let me know.

Jim Fife
Vice Chair for Services
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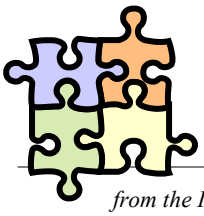
Inside this issue:



The Puzzle Corner

from the Internet

A mathematician is having coffee with two of her friends at a mathematics conference. "I am thinking of two integers greater than 1," she said. "What are they?" To help her friends out, she gives each of them a hint. She tells one friend the product of the two integers and she tells the other friend the sum. The friends then have this conversation:

First friend: I don't know the sum.
Second friend: I knew that. The sum is less than 14.
First friend: I knew that. But now I know both integers.
Second friend: So do I.

What are the two integers?

Solution later in this newsletter.

Real Problems, Real Data, and Real Impacts: Deciding Between Industrial and Academic Research

Aaron Luttman

In my previous life as a university professor – and now as a mathematician in the U. S. Department of Energy (DOE) research complex – I have had many opportunities to present to students on research and careers involving research. Since my move to industry, the question I am most commonly asked is "Do you like industry more than academia?" Naturally, there is no simple "yes" or "no" answer to that question, so my response usually focuses on differences between the two. While there are many logistical differences – daily schedules and salaries, etc. – the most important differences for me relate to the natures of the research I did in academia and now do in industry. The three primary differences for me are that the problems are real, the data are real, and the impacts are real. In their own ways, each of these ideas make industrial research both more and less interesting and more and less challenging than academic research, and understanding these differences can help students make informed decisions about whether they will be happier and more successful as researchers in industrial or academic settings.

Real Problems

As a professor, I had the luxury of choosing the problems I would work on, but, more importantly, I could choose the nature of the problems. This usually meant that I would start from a basic set of assumptions and develop a mathematical formulation of the problem from those assumptions. For example, collaborators and I had worked to develop methods for deblurring images contaminated with noise. Rather than allow for any kind of noise, we assumed the noise followed a Poisson distribution and designed a corresponding theory. The theory is elegant, because the Poisson distribution is well studied and has some very nice properties.

In my work now, I develop methods for analyzing X-ray radiographs – which are images created by a process similar to medical X-rays – and my job is to design techniques for analyzing the image data. Rather than assuming the data have certain properties – like being contaminated with Poisson noise – the data itself drives the formulation, no matter how messy the corresponding theory is. We have to solve the real problem that is presented to us, not the abstract problem we wish had been presented.

The fact that I don't define the problems on which I work has its pros and its cons. It is generally more interesting to work on the problems we define ourselves, because, tautologically, we can define the problems to be interesting. The problems are also not limited by reality, so we can design research problems with whatever properties we want. Developing one's own problems can also be more challenging than having them given to you, since nearly every abstract problem can be extended ad infinitum (or ad nauseum), always making it harder and harder. These facts make academic research great. On the other hand, when research problems are defined externally, as they are in industry, we can be introduced to new problems that we didn't even know we were interested in. (It never occurred to me that I would find shock physics so interesting, until my bosses convinced me to get involved in gas gun experiments where we shoot glass bullets at 6 km/s.) As far as challenges go, I am yet to ever hear of someone complaining about industrial research being too easy. Real and invented problems – industrial or academic – are both interesting and challenging, just in different ways, and we help students find what's right for them in their careers by explaining these differences and exposing them to the different kinds of research, for example through REU's or internships.

Real Data

The second major distinction I found between research as an academic and research as an industrial scientist is that the data are real and often fail to satisfy the assumptions we make in applied mathematics. For example, in controlled deuterium fusion reactions, neutrons are generated and then counted by neutron detectors. The detectors send a signal to a digital oscilloscope and the voltages on the scope are the measured data. The signals have a Gaussian noise due to heat on the scope, but they also have a signal-dependent noise component that is difficult to characterize, except to say that it does not follow any of the standard noise distributions. As an academic I would have assumed a form of the data and a mathematical model for how the data is captured, then tried to analyze the physics of the fusion reaction under those assumptions. Instead, after two years of research, we still haven't begun to really analyze the physics of the reactions, because we're still working to develop a mathematical model for the data capture that would result in the noise that we actually see. Instead of focusing on a fundamental question of the physics of controlled fusion, the data itself drives the research.

Working with synthetic data can be quite rewarding for a mathematician, because we can use it to demonstrate that our algorithms do what we claim they do. That means we can spend our time on the algorithms themselves, where many of our interests lie, rather than on studying the nature of the data. Real data must be studied carefully and the mathematical theory and computational methods we develop for analyzing the data must account for its nature, introducing a whole new set of problems that are both interesting and challenging. Again, both basic research as is done in academia and applied research as done in industry are interesting and challenging, just in subtly different ways.

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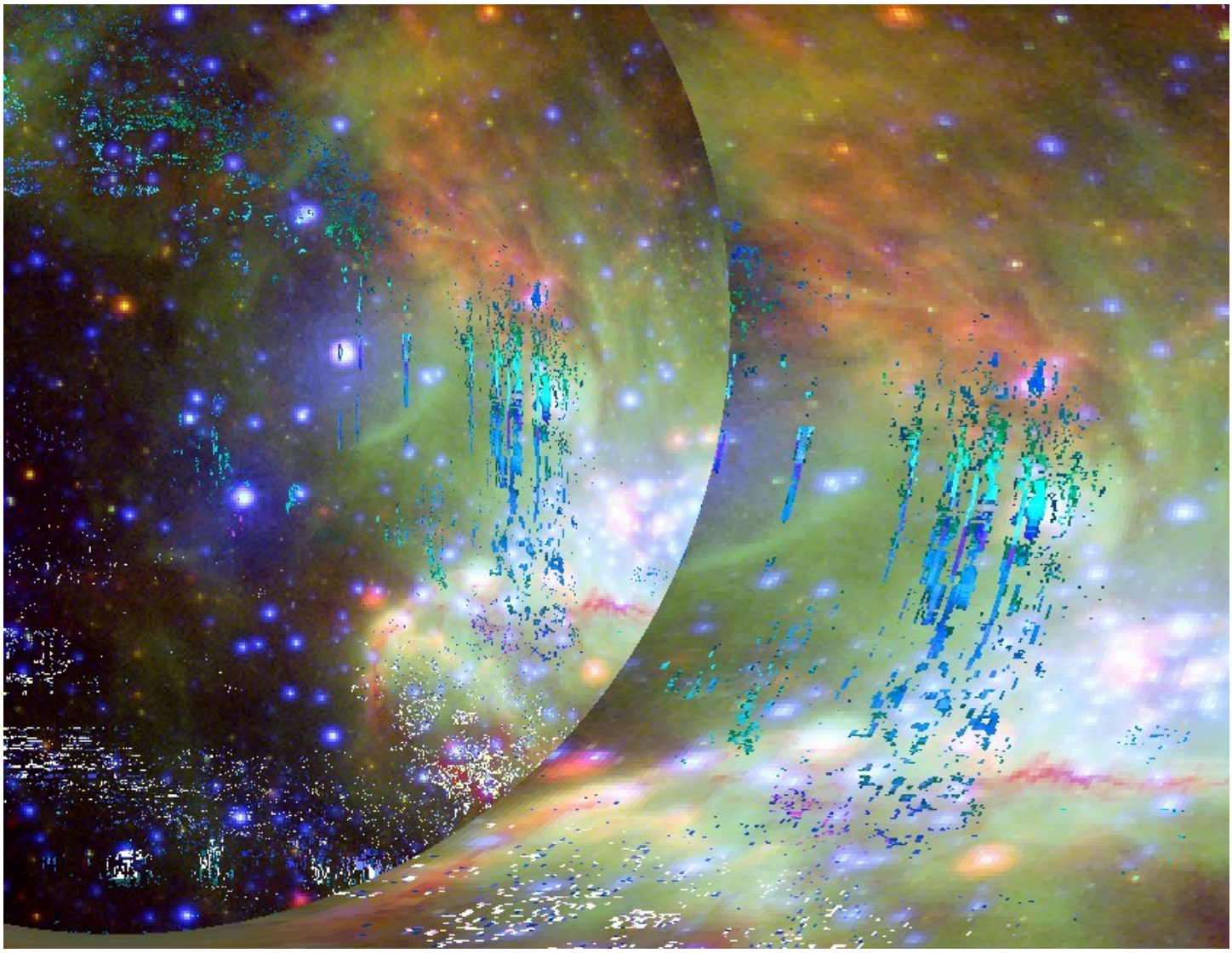

Super Bubble **Carlin Carbano, artist**

from *Mathematics: The Man-Made Universe*

In 1963, Sherman Stein published Mathematics: The Man -Made Universe, a text book for the math-for-liberal-arts crowd. The book is still in print. Here is an excerpt:

The universe of mathematics
grows out of the world about us like dreams
out of the events of the day.

The world serves as a cue, a point
of departure, suggesting questions
that lure the mathematician.

Out of these questions, or their answers, grow
more questions. As long as questions demand answers,
mathematics expands.

Euclid alone has looked on Beauty bare.

Edna St.Vincent Millay

Euclid alone has looked on Beauty bare.
Let all who prate of Beauty hold their peace,
And lay them prone upon the earth and cease
To ponder on themselves, the while they stare
At nothing, intricately drawn nowhere
In shapes of shifting lineage; let geese
Gabble and hiss, but heroes seek release
From dusty bondage into luminous air.
O blinding hour, O holy, terrible day,
When first the shaft into his vision shone
Of light anatomized! Euclid alone
Has looked on Beauty bare. Fortunate they
Who, though once only and then but far away,
Have heard her massive sandal set on stone.

A Brief History of the BIG SIGMAA

Phil Gustafson, Chair, BIG SIGMAA

The Business, Industry and Government Special Interest Group of the MAA, or BIG SIGMAA, was formed in 2001. In this article, we take a brief look at how the BIG SIGMAA came about, provide a partial listing of the officers over the years, and describe some of its activities during the formative years.

The MAA has worked to reach out to BIG mathematicians for many years. We begin our history in 1987, when the MAA Committee on Industrial and Government Mathematicians (CIGM) was formed. Its charge was to encourage a greater role in MAA activities for mathematicians outside of academia. The chair of the CIGM in 1998 was Michael Monticino of the University of North Texas. Michael was also a member of the newly formed MAA Special Interest Group Task Force, established by recommendation of the MAA Board of Governors and the MAA's strategic planning process. The task force developed guidelines for the establishment of Special Interest Groups within the MAA, and resolved to form "SIGMAAs", as they became known. By December 1999, the new SIGMAAs program was officially announced in FOCUS.

In 2000, Michael Monticino wrote the charter for the BIG SIGMAA, and in January 2001 the BIG SIGMAA was approved by the MAA. The BIG SIGMAA was the third SIGMAA to be approved, with the first being SIGMAA RUME and the second SIGMAA STAT-ED, see http://www.maa.org/community/sigmaas/history-of-sigmaas. Michael also recruited the first board of officers for BIG SIGMAA in January 2001, which included:

Michael Monticino, Chair
Pete DeLong, Vice Chair of Services
Phil Gustafson, Vice Chair of Programs
Leon Seitelman, Secretary/Treasurer

A summary of the officers over the years is given below in the BIG SIGMAA Officers Summary Table. The names, years, omissions are a reflection of what I can remember and what I was able to check for this article. I accept credit for all errors and oversights. *Article continues on next page.*

BIG SIGMAA Officers Summary Table

Year	Chair	Vice Chair of Programs	Vice Chair of Services	Vice Chair of Membership	Secretary / Treasurer
2001	Michael Monticino	Phil Gustafson	Pete DeLong	Peter Stanek (?)	Leon Seitelman
2002	Michael Monticino	Phil Gustafson	Pete DeLong	Peter Stanek (?)	Leon Seitelman
2003	Michael Monticino	Phil Gustafson	Pete DeLong		Leon Seitelman
2004	Michael Monticino	Phil Gustafson	Greg Coxson		Leon Seitelman
2005	Michael Monticino	Phil Gustafson	Greg Coxson		Allen Butler
2006	Michael Monticino	Phil Gustafson	Greg Coxson		Allen Butler
2007	Michael Monticino	Phil Gustafson	Greg Coxson		Allen Butler
2008	Pete DeLong	Phil Gustafson	Greg Coxson		Allen Butler
2009	Pete DeLong	Phil Gustafson	Greg Coxson	Kurt TelKoste	Allen Butler
2010	Phil Gustafson	Carla Martin	Greg Coxson	Kurt TelKoste	Allen Butler
2011	Phil Gustafson	Carla Martin	Greg Coxson	Kurt TelKoste	Allen Butler
2012	Phil Gustafson	Carla Martin	Greg Coxson	Kurt TelKoste	Allen Butler
2013	Phil Gustafson	Carla Martin	Jim Fife	Greg Coxson	Thomas Hoft
2014	Allen Butler	Carla Martin	Jim Fife	Greg Coxson	Thomas Hoft

A Brief History of the BIG SIGMAA (continued)

BIG folks are likely to be familiar with the BIG SIGMAA in several ways: (1) the listserv, (2) the BIG SIGMAA newsletter, (3) the annual article in the FOCUS, and (4) the programs we offer. The listserv was put in place by the efforts of Pete DeLong, our first Vice Chair of Services. In 2004, Greg Coxson continued Pete's work of moderating and maintaining the listserv. Many members may remember Greg's interesting postings and queries on the listserv. Greg has been active in several other ways, including initiating the BIG SIGMAA newsletter; being an early advocate of the JMM BIG SIGMAA Guest Lecturer position; and continually having good suggestions for speakers. Back-issues of the newsletter can be found at the BIG SIGMAA website http://sigmaa.maa.org/big/, and are worth looking through again. BIG SIGMAA has also offered a variety of interesting programs.

Since our early years, we have hosted a catered reception for all JMM participants interested in BIG mathematics. We also ran panel sessions at the 2001 and 2002 JMM, and then a Contributed Paper Session from 2003 - 2013, all of which centered on the topic of "Mathematics Experiences in Business, Industry and Government." This paper session was designed as a forum for mathematicians to share experiences they have with a direct connection to a BIG project. This aspect was intended to distinguish our paper session from sessions that feature more general applied mathematics topics. The speakers over the years have been consistently outstanding, and the topics interesting. We have published a FOCUS article each year since 2003 covering the topics in the paper sessions. As with the presentations, these articles offer a fascinating glimpse into the mathematics of the BIG world. Our JMM BIG SIGMAA Guest Lectures have included the following topics and presenters:

Is God a mathematician?, Mario Livio, Space Telescope Science Institute, January 2013

Rational rationing in healthcare: Observations from organ allocation, Sommer Gentry, U.S. Naval Academy, January 2012.

How mathematics is changing Hollywood, Tony DeRose, Pixar Animation Studios, January 2011.

From Netflix to Gerrymanders: A Sample of BIG Applications of Mathematics, Barry Cipra, January 2010.

Calculus in Orbit, Dan Kalman, American University, January 2009.

From Flapping Birds to Space Telescopes: The Modern Science of Origami, Robert Lang, January 2008.

Carla Martin took over as VC of Programs in 2009, and she has continued to connect with BIG mathematicians in important ways for our events. She has a variety of BIG experience as a mathematician, and has written articles for FOCUS about her professional experiences in BIG.

BIG SIGMAA history continues to unfold as each year goes by, and we encourage you to participate in it. We have new officers this year, Jim Fife and Thomas Hoft, as well as a new Chair in 2014, Allen Butler. Allen has been with BIG SIGMAA for many years, serving as Secretary/Treasurer, and brings valuable perspectives and experience from his BIG background. The outlook is truly exciting for BIG SIGMAA in the years ahead.

Mathematical Quotations

I thought the following four [rules] would be enough, provided that I made a firm and constant resolution not to fail even once in the observance of them.

The first was never to accept anything as true if I had not evident knowledge of its being so; that is, carefully to avoid precipitancy and prejudice, and to embrace in my judgment only what presented itself to my mind so clearly and distinctly that I had no occasion to doubt it.

The second, to divide each problem I examined into as many parts as was feasible, and as was requisite for its better solution.

The third, to direct my thoughts in an orderly way; beginning with the simplest objects, those most apt to be known, and ascending little by little, in steps as it were, to the knowledge of the most complex; and establishing an order in thought even when the objects had no natural priority one to another.

And the last, to make throughout such complete enumerations and such general surveys that I might be sure of leaving nothing out.

These long chains of perfectly simple and easy reasonings by means of which geometers are accustomed to carry out their most difficult demonstrations had led me to fancy that everything that can fall under human knowledge forms a similar sequence; and that so long as we avoid accepting as true what is not so, and always preserve the right order of deduction of one thing from another, there can be nothing too remote to be reached in the end, or to well hidden to be discovered.

René Descartes, *Discours de la Méthode*

I wanted certainty in the kind of way in which people want religious faith. I thought that certainty is more likely to be found in mathematics than elsewhere. But I discovered that many mathematical demonstrations, which my teachers expected me to accept, were full of fallacies, and that, if certainty were indeed discoverable in mathematics, it would be in a new field of mathematics, with more solid foundations than those that had hitherto been thought secure. But as the work proceeded, I was continually reminded of the fable about the elephant and the tortoise. having constructed an elephant upon which the mathematical world could rest, I found the elephant tottering, and proceeded to construct a tortoise to keep the elephant from falling. But the tortoise was no more secure than the elephant, and after some twenty years of very arduous toil, I came to the conclusion that there was nothing more that I could do in the way of making mathematical knowledge indubitable.

Bertrand Russell, *Portraits from Memory*

Memory believes before knowing remembers. Believes longer than recollects, longer than knowing even wonders.

William Faulkner, *Light in August*

BIG Events in Baltimore in January (continued)

(Continued from page 1)

> deposited the manuscript at The Walters Art Museum in Baltimore, Maryland, a few months later. Since that date the manuscript has been the subject of conservation, imaging and scholarship, in order to better read the texts. The Archimedes Palimpsest project, as it is called, has shed new light on Archimedes and revealed new texts from the ancient world….

If you've never heard of the Archimedes Palimpsest, you'll find the history of this document fascinating. I saw the Palimpsest in 1999 when it was on display at the Walters, and I still remember it as being an awesome and amazing document.

There will be a reception after Dr. Noel's talk, followed by the BIG SIGMAA business meeting, to which all members are invited.

Finally, on Saturday morning there will be a second contributed paper session, with seven more presentations. Like the Friday afternoon talks, the talks on Saturday morning promise to be especially interesting.

We hope as many BIG SIGMAA members as possible will attend the meetings. If you cannot arrange to attend the entire conference, but you live near Baltimore, perhaps you can attend the conference for one day. Note that you can register for just one day at the conference; no advance registration is required.

We hope to see you in Baltimore!

Real Problems, Real Data, and Real Impacts (continued)

(Continued from page 2)

Real Impacts

For me, the biggest difference between industrial and academic research that I had not anticipated when I left academia was the immediate and tangible impacts of the work in industry. As a university professor, I am unaware of any major decisions being made or anyone's life changing (other than my own) due to my research. During my time in industry, this has not been the case. I am one of only three mathematicians among a large group of physicists and engineers. In our research complex, the physicists tend to design experiments, the engineers design and field diagnostics (machines used to take measurements), and both analyze the data afterward. As a mathematician, I develop methods for analyzing the data and for quantifying the uncertainties that arise from the experimental design and measurement process. The result is that I often suggest ways of changing the experiments or the diagnostics, based on mathematical modeling and algorithm development in my group. These changes are usually implemented, sometimes immediately, impacting multi-million dollar projects.

Having a tangible impact on projects is both a blessing and a curse. As an academic it was disappointing that my research didn't seem to matter in any obvious way. Nonetheless, it was reassuring that my mistakes had no real consequences. Now I enjoy the fact that my research is considered in large-scale (dollarwise) decision-making, but I also carry the blame if I lead my colleagues and bosses astray. Many students want to have careers where they can see the impacts of their work, and such students may find more gratification out of industrial research. Others would prefer not to be responsible for decisions based on their research, and an academic setting may be more enjoyable for them.

Deciding on Industrial and Academic Careers

This past summer I attended the Summer School on Uncertainty Quantification at the University of Southern California, a workshop sponsored by the U.S. Department of Energy (DOE) Office of Science. After I described my job as a DOE scientist to the director of a research center at one of the national laboratories, he half-jokingly responded "You work with real data? Ooh. That's too hard." We both laughed at the comment, because it was funny but also because it had a grain of truth. That saying has now become one of my staple "jokes" in my presentations to students, and it gives me the opportunity to discuss some of the differences between research in industry and research in academia, especially in regard to the nature of the problems, data, and impacts.

Aaron Luttman began his research career with Pointcloud Inc., in Plymouth, MN, doing research and development in computer vision. After getting his Ph.D. in mathematics from the University of Montana, he spent 6 years as an assistant professor at Bethany Lutheran College in Mankato, MN, and at Clarkson University in Potsdam, NY. He is currently a Senior Scientist at National Security Technologies, LLC, the U. S. Department of Energy contractor for the Nevada National Security Site, where his research focuses on developing theory and computational methods for analyzing diagnostics in large-scale physics experimentation.

Michael Vogelius named next Division Director for Mathematical Sciences

Michael Vogelius of Rutgers University has been named as the next Division Director for Mathematical Sciences (DMS), at the U.S. National Science Foundation, effective January 2014. Dr. Vogelius replaces Sastry Pantula who stepped down from the position in September. Fleming Crim, Associate Director for Math and Physical Sciences (MPS) at NSF, who oversees DMS, issued the following announcement on Monday, August 12:

> I am pleased to announce that Dr. Michael Vogelius will join MPS as the Division Director for the Division of Mathematical Sciences in January 2014. Dr. Vogelius is currently a Board of Governors Professor in the Department of Mathematics at Rutgers University, where he has been since 1989, and where he served as the Department Chair from 2009-2013. Dr. Vogelius was previously on the faculty of the University of Maryland and has held visiting appointments at Stanford University, Ecole Polytechnique Federale de Lausanne, and the University of Copenhagen. Dr. Vogelius earned his Ph.D. in Mathematics from the University of Maryland, College Park in 1980, and his research interests include mathematical analysis, partial differential equations, and numerical analysis. He has published over 90 articles and is a Fellow of the American Mathematical Society, a Foreign Member of the Royal Danish Academy of Sciences and Letters, and a recipient of a Sloan Research Fellowship. Dr. Vogelius is also an Associate Editor of the SIAM Journal on Mathematical Analysis and serves on the Editorial Boards of several other mathematical journals.

Limericks

An Englishman living in Bree
Can't solve an equation for *z*.
When asked why, he said
"I solve it for *z*.
That's how I do math," answered he.

Limerick in honor of Phil Gustafson,
who is stepping down as BIG SIGMAA Chair

A mathematician named Phil
Did climb to the top of the hill.
BIG SIGMAA he led;
At last then he said,
"I'm done. But oh my! What a thrill!"

So How Do You Design An Aeroplane?

Michael Bartholemew-Biggs

This poem was delivered at the Bridges conference in Enschede, the Netherlands, in July.

Look for precious little help
from birds except that silhouette
of teardrop body; tapered wings
outspread around its one-third length;
a tail that fans out aft for balance.
No way to replicate their hollow
bones or feathers' flexibil-
ity and be a breathing structure light
enough to flap itself to flight

So like a child you have to learn
from other people's work. There's much
the miller or the yachtsman knows
about the shape and trim of sails
to stop them breaking when they borrow
from the wind sufficient force
to force flat heavy stones to turn
or drive a boat across the waves
skipping like a spinning pebble.

Beneath its pencil-shaded skin
your paper aeroplane requires
a skeleton of ribs and spars
so ask a carpenter about
the properties of wood and how
to reconcile the yin and yang
of weight and strength. A shipwright knows
how hull and keel support the mast
when canvas catches air in curves.
You have to choose from aerofoils
catalogued by lift and drag
find the size for fin and rudder
fix an angle of attack
ensure the elevators' power
provides sufficient pitching moment
check the stress at spanwise stations
in the end it all boils down
to calculations calculations...

Business, Industry, and Government Special Interest Group of the Mathematical Association of America

Business • Industry • Government
Special Interest Group of the MAA

2013 BIG SIGMAA Elections

Voting for this year's BIG SIGMAA elections took place online between October 8 and October 21. The two open positions were Chair and Vice Chair of Programs. Congratulations go out to Allen Butler (Chair) and Carla Martin (Vice Chair of Programs), the winners of this year's elections. These positions are two-year terms, and begin in January 2014. Thanks to all of you who voted in this year's election, and for being a part of BIG SIGMAA.

Next year the positions of Vice Chair of Services, Vice Chair of Membership, and Secretary/Treasurer will be open for election, also two year positions. If you are interested in participating in the leadership of BIG SIGMAA, informational emails will be sent out on this listserv next year when election season comes around again. The primary requirements include being a BIG SIGMAA member, participating in email discussions regarding the activities of BIG SIGMAA, and being able to attend the annual BIG SIGMAA Business Meeting and the MAA SIGMAA Officer's Meeting at the Joint Mathematics Meetings of the AMS and MAA (JMM) each year in January.

Phil Gustafson

Chair, BIG SIGMAA

from the MAA Website:

BIG SIGMAA serves as a unifying link between business, industry, and government mathematicians, academic mathematicians, and mathematics students. The SIGMAA provides resources and a forum for MAA members who share an interest in mathematics used in business, industry, and government, aids in professional development, helps build partnerships between industry and academics, and increases awareness of opportunities for mathematicians in business, industry, and government.

The Puzzle Corner Solution

The friends' conversation provides us with four clues.

First friend: *I don't know the sum.*

The first friend cannot deduce the sum of the two integers from their product. It follows that the two integers cannot both be prime numbers.

Second friend: *I knew that.*

The second friend knows that the first friend cannot deduce the sum from the product, and therefore the second friend must know that the two integers cannot both be prime numbers. Since the second friend only knows the sum of the two integers, it follows that the sum is a number than cannot be written as the sum of two primes. For example, the sum cannot be 8, since $8 = 3 + 5$.

Second friend: *... The sum is less than 14.*

The sum must be at least 4 since each integer must be at least 2. The only integer between 4 and 13 (inclusive) that cannot be written as the sum of two primes is 11. Thus the sum of the two integers is 11. Therefore the two integers are either 2 and 9, 3 and 8, 4 and 7, or 5 and 6, and hence their product is either 18, 24, 28, or 30.

First friend: *I knew that.*

The first friend knew that the sum of the integers must be less than 14. Since the first friend only knows the product of the two integers, it follows that the product is a number that cannot be written as the product of two integers whose sum is greater than or equal to 14. For example, the product cannot be 24, since $24 = 2 \times 12$ and $2 + 12 = 14$. Similarly, the product cannot be 28 or 30. Therefore the product must be 18.

Thus the sum of the two integers is 11 and their product is 18. It follows that the two integers are 2 and 9.