VOLUME 6, NUMBER 1 APRIL 2010

The BIG Notebook

A Newsletter of the MAA Special Interest Group for Mathematics in Business, Industry & Government

One Canadian's Dream (Come True) of Applying Mathematics in the Southwest

Rick Chartrand,
Los Alamos National Laboratory

Like many mathematicians working in industry or government, for years I assumed that my career path would be in academia. For me this started as an undergraduate at the University of Manitoba. I was drawn to pure mathematics, its logical preciseness appealing to my mind's fondness of order. I also greatly enjoyed being a teaching assistant, so much so that by my junior year I was teaching 10 classes a week.

The trajectory continued as a graduate student at UC Berkeley. My thesis work was on spaces of holomorphic functions on the unit disk of the complex plane. This combined methods from functional analysis (the spaces), complex analysis (the functions), and real analysis (functions on the unit circle, induced by limits from inside the disk). It was fun having so many angles from which to attack a problem. Teaching continued to be enjoyable, as leading discussion sections gave way to being in charge of whole courses.

The trajectory began slowing in speed after graduate school, if not changing direction. I had first a one-year appointment at Middlebury College, with a large enough teaching load that it was difficult to make research progress. Then I got a three-year postdoc at the University of Illinois at Chicago, but an untimely faculty

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Dr. Rick Chartrand

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Editor's Note

Christopher Tong has contributed another terrific book review to the Notebook, this time on M. Grae Worster's book *Understanding Fluid Flow*. Unfortunately, this issue appears as Chris is mourning the loss of his father, Ts'ing-Hi Tong. His father was an emeritus professor of Mathematics at Illinois College and a member of MAA for 40 years.

We are thankful to Rick Chartrand of Los Alamos National Labs for relating his story in this issue. He came to our attention when he served on a five-member panel discussing "Non-Academic Hiring" at the Joint Mathematics Meetings in San Francisco. His unique background provided an interesting extra dimension to the discussion. When a question about job market challenges for foreign workers was posed by an Australian student in the audience, Rich took the question and related his experience as a Canadian citizen pursuing a position with Los Alamos.

Also in this issue is a description of the billiards problem by David Mazel. Having known Dave a while, as a colleague at my firm, I have learned over time that Dave is able to view a wide range of problems in terms of various forms of the billiards problem. For example, he can use billiards to illustrate quantization effects, or to explain chaos, or to understand properties of spectra or the behavior of light. Furthermore, his interest in billiards leads him in myriad directions of mathematical interest. Look for a follow-up article in future issues.

Thanks also to Collin Carbno for new artworks he provided for this issue, and to Bill Haloupek for searching in his archives of mathematical quotation for a quotation for this issue.

departure left me rather isolated. The reduced teaching load (1 and 1) was itself enough to provide some research impetus, enough for a quick new result. But from there things seemed increasingly difficult. That the field of holomorphic spaces provided so many weapons for attacking problems seemed to make the research frontier less accessible, with many of the open problems seeming to be those that are very difficult, or of such narrow scope that they hadn't been contemplated. Teaching was also becoming less enjoyable for a variety of reasons. Now married and with the job market still tight, I was furthermore not happy about not being in control over where we might end up living. So after a year, we decided that I would get off the academic roller coaster, and that we would choose where we wanted to live, and seek work there.

Earlier hiking trips left us wanting to live in the Southwest, particularly the Four Corners area. The natural target became Los Alamos National Laboratory. I began looking for other areas of research that might make it easier to find a job at LANL. I considered quantum computation, then a new field with recent exciting results, most notably Peter Shor's demonstration of a quantum computer algorithm that can factor integers in polynomial time. However, I couldn't make the connections between that field and my expertise, and my inquiries to people in the field were not proving fruitful.

I had better luck getting some interest from some people at LANL working in something they called "sequential dynamical systems," which attempted to provide an axiomatic theory of computer simulation. The axiomatic aspect was appealing, and I managed to come up with a modest "new" result (which turned out to have been first noticed decades earlier). But this path didn't end up producing a job.

By this time my wife, with no Ph.D. but with marketable computer skills, had managed to find a job as a staff member at LANL (their equivalent of a tenure-track position) doing 3-D visualization. So we moved to Los Alamos at the end of my second year at UIC. My job search now focused on in-person networking, mostly in the form of showing up at LANL-hosted conferences and striking up conversations (not something I find easy). But in the end this paid off, with one person I spoke to introducing me to Kevin Vixie who, the following year, invited me to attend some discussions on image processing he was having with some summer students. My analysis background was at least somewhat relevant to discussions of the space of functions of bounded variation and its dual. Shortly thereafter, Kevin offered me a postdoc position, after almost 18 months of unemployment.

One of my first projects (necessarily unclassified, as I was still a Canadian citizen) used cosmic-ray muons to form images of the contents of cargo containers. These charged particles stream down from the sky, passing through most matter but being deflected by atomic nuclei. With detectors above and below a container, one can get a track in and a track out for each particle, and use the scattering angle to make inferences about the atomic number of what they pass through. This is a challenging data analysis problem, to which I attempted to apply machine-learning methods. The highlight for me was organizing a session on muon tomography at the annual AAAS meeting, which received substantial media coverage. I was quoted (often misquoted) in newspapers and websites all over the world. But some of these reports got me and the project in hot water, primarily due to political competition from X-ray--based technologies.

Subsequently I worked on several projects involving total-variation (TV) regularization, mostly applied to radiographic imaging. Penalizing TV does very well at suppressing noise, while allowing discontinuous solutions (such as edges in images). The non-smoothness of TV makes for a nontrivial optimization problem, with new research ideas still emerging after almost 20 years since its invention. I was by now unquestionably an applied mathematician, a complete changeover from my upbringing.

Based on my tomography experience, I was flummoxed to see Emmanuel Candes's presentation (in a talk he gave at LANL) of perfect reconstruction of a test image using radiographs along 22 angles, one of the early examples from the then-brand-new field of compressive sensing. A few months later I was experimenting with non-convex objectives, and managed to reproduce this example, using only 10 radiographs! I knew I was onto something big, and I spent the next few years pushing non-convex methods as hard as I could. I still have the best results in this direction, but in this popular field nothing stays novel for long.

The repercussions of the end of the Cold War are still being felt at LANL, with one of the indirect consequences being a marked decline in basic research funds. I find myself moving more into applications than fundamental science. While Los Alamos remains a very nice place to live, the other advantages of a position here are eroding, and I find myself contemplating other options. From beginning to end, careers seldom transpire exactly as expected.

Book Review of *Understanding Fluid Flow*, by M. Grae Worster (Cambridge University Press, AIMS Library Series, 2009).

Christopher Tong, PhD, Center for Veterinary Biologics, U.S. Department of Agriculture.

Wouldn't it be nice if, before a student embarks on a course on fluid dynamics, he or she has the opportunity to tour the subject and get a flavor of what it will be like to study it? This brief, informal book provides just that, and there is nothing else like it in the fluids literature.

At about 100 pages in length, the book provides neither depth nor breadth in the field. Rather, it offers a whirlwind tour of many of the major concepts and flows in fluid mechanics. Examples include parallel shear flows, boundary layers, vorticity dynamics, potential flow, separation and D'Alembert's paradox, aerodynamic lift, surface waves, ship wakes, and the Kelvin-Helmholtz instability. There is a strong emphasis on intuition, dimensional analysis, and scaling arguments, and an avoidance of excessive mathematical formalism. The Navier-Stokes equations are presented and motivated, but not given a thorough derivation.

There are 25 exercises included; several are solved within the text itself. The book focuses on theory, but there are also two rather involved "Assignments" included, one experimental and the other computational. Both deal with a viscous gravity current, generated by a spreading pool of syrup, poured onto a horizontal disk.

The author is an applied mathematician and Professor of Fluid Dynamics at Cambridge University. He is also the co-chief editor of the most important journal in the field, the Journal of Fluid Mechanics. This book is one of the first two published in the AIMS Library Series, affiliated with the African Institute of Mathematics (Muizenberg, South Africa), and is based on lectures given by the author there. Several movies associated with the book are available on the corresponding website from the publisher.

Some basic facility with vector calculus and partial differential equations is required by the reader. The writing style is very conversational, and readability is high (although the material becomes more difficult towards the end). There is no shortage of diagrams to illustrate the text; only a handful of photographs are included.

Unfortunately, the book does not attempt to bring the reader to the forefront of research. Turbulence earns a single paragraph. Given that the transition to turbulence is considered the most important unsolved problem in fluid dynamics, the intended reader might have benefited from more discussion. More mathematically inclined students might also appreciate a mention of the Clay Mathematics Prize offered for an existence and smoothness proof for solutions of the 3D Navier-Stokes equations. Finally, the book does very little to develop the thermodynamics of flow, an arguably fundamental aspect. Of course, one could go on and on about other missing topics and concepts, which underlines the richness and diversity of fluid dynamics. This book pushes the door open on only a small sliver of the field. However, the brevity of the book is a major factor in its friendliness and accessibility.

The book is ideal for an upper level undergraduate or beginning graduate student in physics, applied mathematics, the geosciences, or engineering, for whom fluid dynamics is a potential future course of study or research. It will also benefit more mature physicists and applied mathematicians who are new to fluid dynamics, and would like a taste of the field and a start on achieving some literacy in it. I would recommend this book, along with the two chapters on fluids in The Feynman Lectures on Physics, to any beginner.

Dedicated to the memory of Ts'ing-Hi Tong (1923-2010)

Flower of Symmetry, by Collin Carbno

Editor: Greg Coxson
Artwork: Collin Carbno
Copy Editing: Allen Butler
Quotations: Bill Haloupek

A Look at Some Trajectories of Billiards

David Mazel,
Technology Service Corporation

Several years ago, I was reading a book on dynamical systems when I came across the concept of billiards. The idea is fairly straightforward.

Imagine that there is a point, like a ball, that moves within a bounded region of the plane, say a square. You give the point a starting location and a velocity. Then, let the point move such that when it encounters a wall, the point is reflected with the angle of reflection equal to the angle of incidence; pictorially:

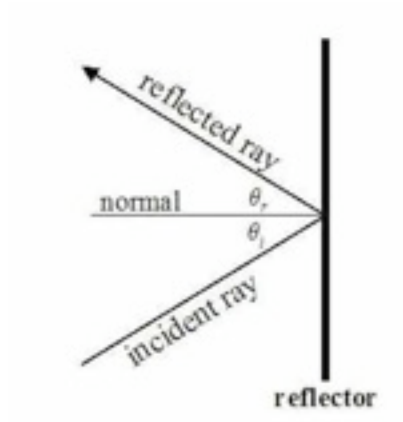


Figure 1: The reflection (or "bounce") rule for billiards.

As the point moves, we trace its path to see where it goes. That path is the trajectory of the billiard. some examples of trajectories inside different regions are shown below:

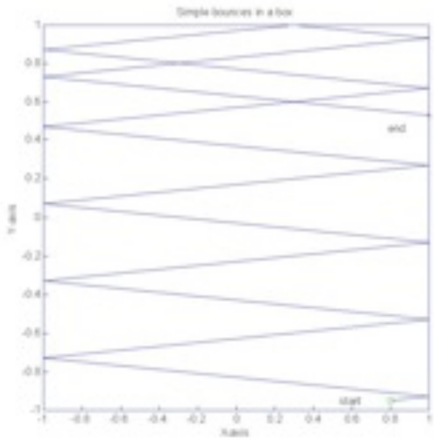


Figure 2a: A boundary of a box. The dynamics are simple here.

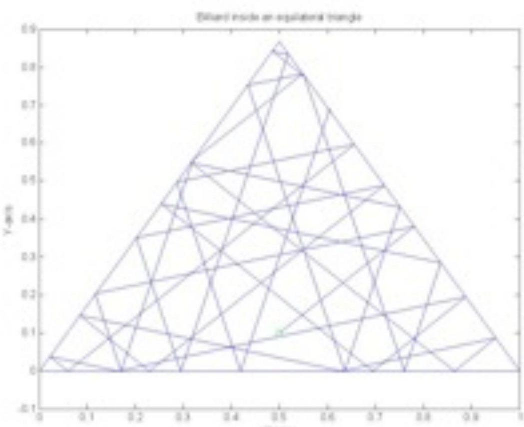


Figure 2b: A boundary is an equilateral triangle.

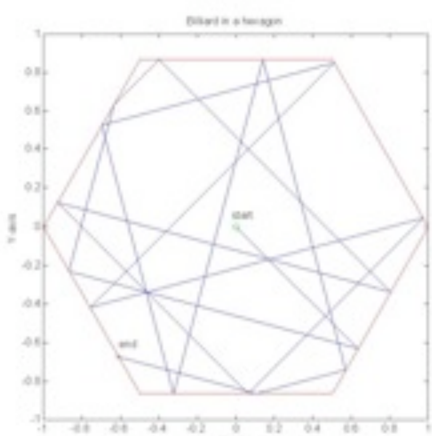


Figure 2c: A hexagon as a boundary.

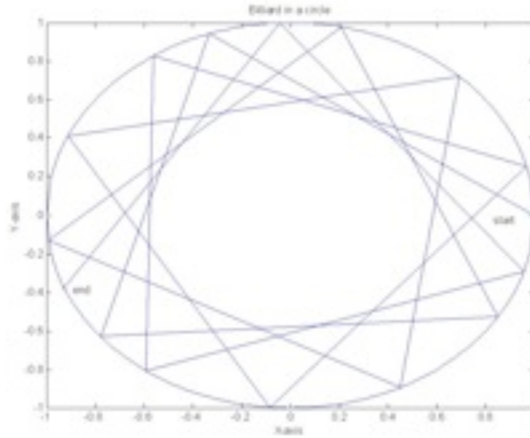


Figure 2d: Dynamics inside a circle.

The trajectories can be quite lovely, a sort of knitting pattern with lines criss-crossing each other for a square boundary. If we let the point bounce around in the box we will see the path repeat itself. If we select an initial velocity at a slight angle relative to the horizontal, the trajectory will crawl up the box. The trajectory inside the circle, for instance, produces an envelope of another circle. I have spent many hours playing with this sort of thing and examining the resulting patterns.

There is a wonderful book entitled *Geometry and Billiards* by Serge Tabachnikov in which he explores the dynamics of billiards inside polygons of different types with various internal angles.

Now let's make things a little more complicated and add a bumper inside the box. When we do this, we find that the orderly behavior we observed earlier disappears and the behavior is now chaotic. This set-up is the Sinai billiard, after Yakov Sinai who was the first to study it. In fact, it is this chaotic behavior in billiards that makes them such an interesting object of study. Here is one possible trajectory:

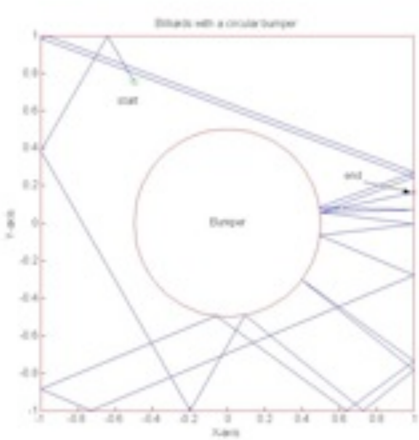
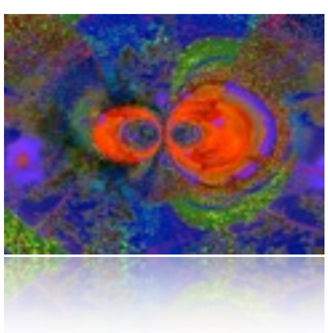


Figure 3: The Sinai billiard.

Current research into billiards looks at the dynamics of the trajectories with bumpers or with some complicated boundaries such as the stadium billiard.

It is worth mentioning that the coding is straightforward for all these cases. One could, for example, easily include multiple circular bumpers, or perhaps a few straight lines as bumpers. The trajectories are quite beautiful with visually interesting patterns.

A future article will explore the chaotic properties and look at more examples.



Mathematics and MAA Section Borders

Greg Coxson

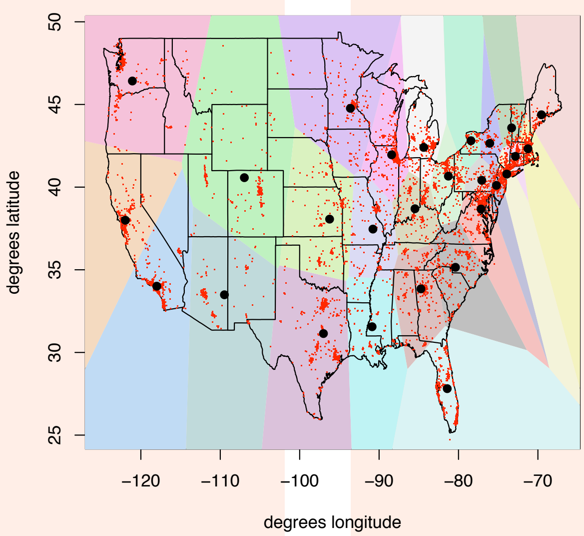
The membership of the Mathematical Association of America is divided into 29 sections of varying sizes. Some conform to state boundaries, others spread across state lines, or across large bodies of water. Some even include parts of neighboring countries. One might ask (as I did, to myself, during an MAA board meeting at MathFest last year), whether participation by the average MAA member in section meetings could be facilitated by a redesign of section borders.

Take for example the Northwest section of MAA. It includes the states of Washington, Oregon, Idaho, Montana and Alaska. As a consequence of bordering several provinces of Canada, it also includes British Columbia, Alberta, Manitoba and the Yukon Territories. This creates a challenge for section officers desiring to schedule meeting locations so that as many MAA members as possible can attend sections meetings.

It is a hypothetical question, of course, since it is unlikely that the section boundaries will be redrawn any time soon. However, if one deigns to try and formulate it as an optimization problem, complexities soon present themselves. For example, how should distance be defined -- as the crow flies, or using shortest paths along available roads? Should the centroid of a given section stand for the geographical centroid, or should the locations of typical MAA section meetings (often the colleges and universities within the various sections) be considered? What should be done about serious transportation obstacles such as mountains or bodies of water?

Last fall, I decided to use the BIG SIGMAA listserv to solicit help in formulating and solving this problem. Two volunteers stepped forward and began to attack it separately: BIG SIGMAA officer Kurt Tekolste (retired from Lockheed Martin) and Grant Izmirlian, a mathematical statistician with the National Cancer Institute. Interestingly, they each chose to apply variations of cluster analysis to the MAA database. Kurt chose a clustering method running in Mathematica and Grant chose K-means cluster analysis running in R, the open-source statistical package.

Ultimately Grant and Kurt learned of each others' efforts and began collaborating. Comparison of results led to agreement that the K-Means approach was returning more reasonable results (Kurt's approach tended to form many tiny sections in the Midwest, while Grant's results resembled more closely the current sections). They began working together to refine the approach and work out remaining details.



Despite its technical-sounding title, K-Means Cluster Analysis (or more precisely, K-Means Cluster Lloyd) is easy to describe. The algorithm begins with an arbitrary selection of K points termed centroids. Once this is done, a process is initiated involving two steps repeated iteratively. The first step is to take each point (or member) and determine the closest centroid among the current set of centroids, with respect to Euclidean distance (using local flattening of the Earth surface). The point is then assigned to the grouping (or section) associated with that closest centroid. Once all the points have been processed, we have a partition of the points into K groups. The second step is to recompute the centroids for the K groupings. The process now repeats with the new centroids, and continues iteratively until there is no change in partitions. The result is a Voronoi Tessellation in which groupings are delimited by straight line borders.

The chair of the MAA Sections Committee, Rick Gillman, generously provided a list of MAA members, represented by zip codes, including multiplicities, with the stipulation that any reporting of results make clear that this is a theoretical exercise. The zip codes were converted to latitude and longitude coordinates. To deal with the use of the Euclidean metric for an oblate Earth, a scaling was applied preliminary to running the algorithm. To deal with the geographical isolation of Alaska and Hawaii, separate sections were reserved for each, leaving 27 sections to be determined.

Given a set of K=27 initial centroids, K-Means Cluster Lloyd returns a locally optimal tessellation. Grant and Kurt ran the method for 450 Monte Carlo trials. Two options they tried for ranking results are Variance and Mean-Square Error (or bias plus variance). The MSE approach seemed to give slightly better rankings. The highest-ranked solution with MSE is shown in the above illustration.

One of the limitations of K-Means Cluster Analysis is that there is no easy way to control the number of points per cluster. The MAA has traditionally aimed for roughly uniform membership numbers per section. It is also difficult to align resulting clusters with geopolitical boundaries, or to account for obstacles to travel by automobile such as large bodies of water.

Grant and Kurt's results were presented to the MAA Sections Committee during the Joint Math Meetings in San Francisco. They are considering developing their results further, possibly with an MAA publication in mind. Meanwhile, a good reference for K-Means Cluster Lloyd is "Least Squares Quantization in PCM", in *IEEE Transactions on Information Theory*, volume 28, pages 128 to 137.

Ripples in Time and Space, by Collin Carbno

Contributed Papers Presented at the BIG SIGMAA Session at the JMM in San Francisco, January 2010

"A Robust, Multi-Criteria Modeling Approach for Optimizing Aeromedical Evacuation Asset Emplacement", Nicholas Bastian, U.S. Army.

"Probability in Solutions for Assembly in Earth Orbit of a NASA Spacecraft for Travel to Mars", Richard Jarvinen, Winona State University

"Using Radar to Identify Persons Carrying Wires", William Fox, John Visecky and Kip Laws, Naval Postgraduate School.

"Scoring Line of Best Fit Test Questions", James Fife, Educational Testing Service.

"A Green's Function Technique for Radiation Transport in Three Dimensions", Candice Rockwll and Dr. John Tweed, Old Dominion University.

"Searching for the Home Base of a Serial Criminal", Mike O'Leary, Towson University.

"An Alternative Distributional Model for Control of Process Particle Counts in the Semiconductor Industry", Robert Henderson.

"The US Blood Supply, Bioterrorism and Mathematics", Sonja Sandberg.

"Movie Recommendation Systems", Erich Kreutzer, Davidson College.

"Calculating the Greeks via Malliavin Calculus for Variance Gamma Process in Poisson-Wiener Space", Dervis Bayazit and Craig Nolder, Florida State University.

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(Contributed papers, continued)

Statistical Inconsistency in the Audit Risk Model", Rick Cleary, Bentley College.

"Why Companies Need Mathematicians Even During Tough Times", Carla D. Martin, James Madison University.

Pi Day Sudoku

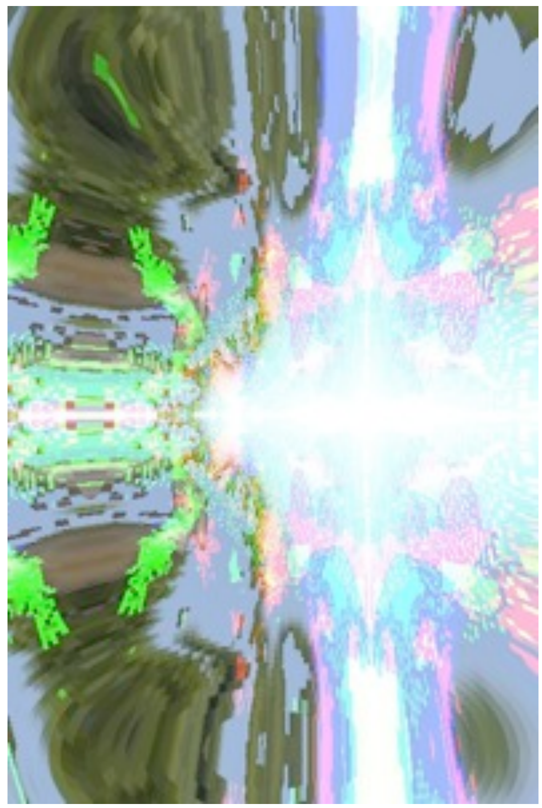
Dr. Laura Taalman of James Madison University and BrainFreeze Puzzles

It is believed that 18 clues are the minimum for a Sudoku puzzle to yield a unique solution. For Pi Day 2010, Laura Taalman crafted a Sudoku puzzle seeded with the first eighteen digits of pi. The puzzle is provided below.

7	2							
	5				9			
				3	8			
			4			5		
		3				9		
		1			3			
			2	5				
			6				3	
							1	9

Sudoku puzzles are typically designed with "rotational symmetry" as this is thought to be eye-pleasing. This means that if the puzzle is rotated 180 degrees the pattern of clue locations remains the same. It is an open question whether 18 clues is the minimum for such a puzzle to yield a unique solution. Nobody has found a 17-clue example with unique solution. For general puzzles (i.e., regardless of symmetry) the conjectured minimum is 17; nobody has yet found a 16-clue unique-solution puzzle, according to Laura.

The rules for solving the puzzle are the usual ones. That is, each row, column and bordered 3x3 block must have one and only one copy of each of the integers from 1 to 9. Enjoy!



Moment of Insight, by Collin Carbno

After you know enough facts about groups, you can easily do Exercise 35.

-- Serge Lang, *Algebra* (Third Edition), p. 9.