The BIG Notebook

A Newsletter of the MAA Special Interest Group for Mathematics in Business, Industry & Government

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I always felt that mathematics was applicable. I don't know where this feeling came from. I certainly didn't get it from growing up in the Mexican culture in Tucson, AZ. As an undergraduate I had almost no guidance from faculty. My first semester in college was all too typical of college students; I did horribly in mathematics and chemistry, earning Ds in both. My second semester was atypical. I looked at what I was doing and changed my behavior. By my third semester, I knew I was going to earn either a PhD in physics or mathematics. In 1968, I earned a BS degree with a major in mathematics and a minor in physics. I would have immediately begun graduate studies but I took a hiatus, it was called Vietnam. In 1970, I returned from active duty in the U.S Navy and began graduate school, earning a PhD in 1975 with a thesis in algebraic number theory.

Books on the history of mathematics and science took the place of faculty advice. The Rise of the New Physics by D'Abro introduced me to the applications of mathematics. I read biographies of mathematicians and scientists. This formed my motivation. These books transformed my view of the world. Graduate school was heaven. How fortunate I was to have the opportunity to think about mathematics non-stop. During the five years I was a graduate student, I taught two different courses each semester. Graduate students were given complete control of the class. I experimented with having students coming to of-

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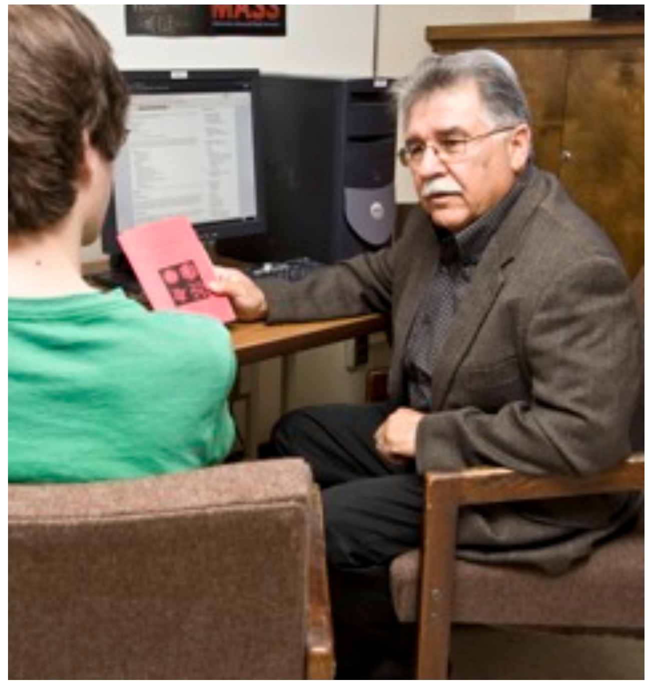


Professor Vélez advising a student.

Inside:

fice hours to give me a presentation on readings that I assigned. I insisted on "wasting" one class period each semester to talk about some mathematical idea that I found fascinating. Graduate school was real professional development, both as a research mathematician and as a teacher of mathematics.

In graduate school I found discrete mathematics more attractive and I decided to specialize in algebraic number theory. Number theory is often viewed as far from applied mathematics, but change was in the air. The world was being discretized. The ideas of algebra and number theory were now essential in modern communications systems. Look at a typical communication signal. It can be viewed as a finite sequence of 0's and 1's, but a better way of looking at it is to think of it as a finite dimensional vector over the field of two elements. Algebra!

As a graduate student I was busy with learning new mathematical ideas, but I was still intrigued with how mathematics mirrors the world. A course in algebraic coding theory showed me the applicability of linear algebra and number theory. In 1974 I obtained a summer internship at Bell Labs in Murray Hill, NJ, then a hotbed of activity in coding theory. It was an amazing place to visit. I wrote my first paper on a conjecture made by Ron Graham, then the director of the math group at Bell labs.

When I was completing my doctoral studies I decided that I did not want to be a university professor. Immediately after completing my degree, I accepted a position at Sandia National Laboratories in Albuquerque, NM. I worked on the Command and Control of Atomic Weapons Systems from 1975-77. This problem depended very much of the theory of finding solutions to systems of linear equations over finite fields. Algebra again!

After two years at Sandia I decided to return to academia and was fortunate to obtain a position

Editor's Note

In his profile, Professor Vélez describes a path familiar to many of us. An algebraic number theorist who has worked both inside and outside academia, he has blazed his own path guided by confidence in the usefulness of Mathematics. He finishes with a suggestion that mathematics professors often lack experience working outside the ivory tower that might help them advise their students on careers. This suggests another place where BIG mathematicians can find a role.

In his article, Barry Belkin, a long-time analyst at, and former president of, Daniel H. Wagner Associates, describes how he tells students what his career as a ``consulting mathematician'' involves. Daniel H. Wagner Associates is a firm that has provided analytical services to the U.S. Navy since the early 1960s. Wagner often hires graduating mathematicians and trains them in the applied discipline of Operations Research.

The artworks of Collin Carbno have helped enliven this newsletter for six years. For this issue, in addition, Collin provides guidance in how to determine how much money to save for retirement, and how to draw out payments, in his article entitled ``Magic Pension Formulas.''

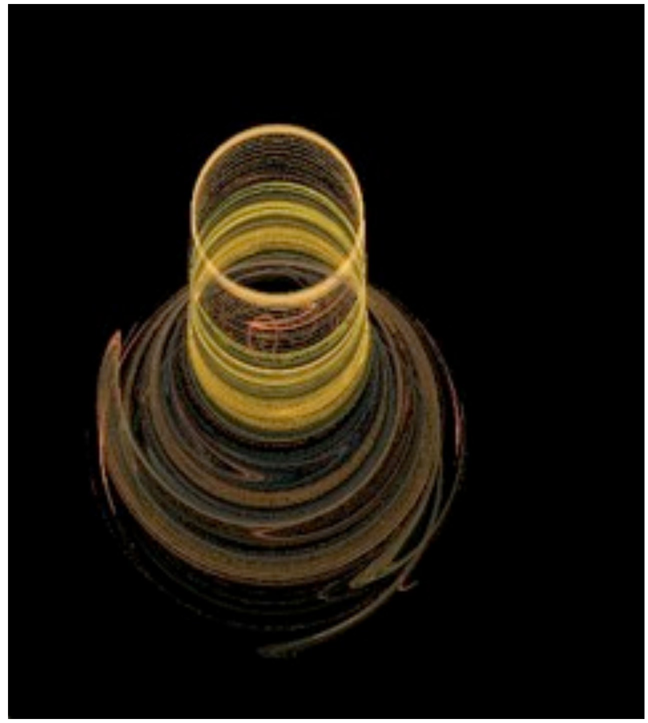
The poem ``Figures of Thought'' by Howard Nemerov (1920-1991) acquaints us with a mathematician in spirit if not in profession. Not only did Nemerov create a number of poems featuring mathematics, but in many of them the viewpoint is decidedly applied. The discovery of mathematics in the works of a skilled poet is surprising and can be partially explained by Nemerov's service as a U.S. Air Force pilot during World War II.

Check out the notices as well. There are exciting events this Fall and Winter.

back at the University of Arizona. I returned in 1977 and moved through the ranks. I am currently Professor of Mathematics and University Distinguished Professor. My research has been in algebra and number theory, but I have not forgotten the pleasure of applying mathematics. For four summers I worked at a naval lab on Communications Systems for submarines. In this work I earned four patents.

My teaching and advising have always been flavored with the applicability of mathematics and I have used my own experiences to motivate students to pursue further mathematical studies. In the late 1990's, I decided that I would devote myself to working to promote the study of mathematics among minority students. To this end I accepted an administrative position in 2004 to focus on increasing the number of mathematics majors. Since I accepted this administrative position I have doubled the number of mathematics majors (we ended the academic year, 2011, with 638 math majors, 21% of whom are minority students) and I give lectures across the country on how to increase the number of mathematics majors and on increasing diversity. The American Mathematical Society recognized this work with an Excellence award in May 2011. http://www.ams.org/news/ams-news-releases/ams-news-releases

A problem that plagues the professoriate is that most faculty have never had the opportunity to apply mathematics to problems that arise in industry and national labs. There are many opportunities for graduate students and faculty to apply for short-time positions in these sectors. If mathematics faculty had more industrial experience, we could better advise our students about opportunities for undergraduate mathematics majors. Increasing the mathematical literacy of our undergraduate students is our responsibility. If we don't take charge, who will?



Limit Cycle, by Collin Carbno

Quotation Corner

Only he who knows what mathematics is, and what its function is in our present civilization, can give sound advice for the improvement of our mathematical teaching.
-- Hermann Weyl

Every mathematician worthy of the name has experienced ... the state of lucid exaltation in which one thought succeeds another as if miraculously ... this feeling may last for hours at a time, even for days. Once you have experienced it, you are eager to repeat it but unable to do it at will, unless perhaps by dogged work ...
-- Andre Weil, in The Apprenticeship of a Mathematician.

Events at the Joint Mathematics Meetings in Boston, January 201

The deadline is coming up for the BIG SIGMAA paper session at the Joint Meetings in Boston January 4-7, 2012. In particular, we invite you to present papers or discuss projects involving the application of mathematics to BIG problems. The paper session is held Thursday, January 5.

To submit an abstract for this session, go to http://jointmathematicsmeetings.org/meetings/abstracts/abstract.pl?type=jmm and choose the MAA Session on Mathematics in Business, Industry, and Government. Abstracts are due by September 22, but due to the volume of abstracts, we encourage you to submit early!

Mathematicians, including those in academia, with BIG experience are invited to submit an abstract. The goal of this contributed paper session is to provide a venue for mathematicians with experiences in business, industry, and government to share projects and mathematical ideas in this regard. Anyone interested in learning more about BIG practitioners, projects, and issues, will find this session of interest.

We invite you to attend our guest lecture and reception the evening of January 5. Sommer Gentry, U.S. Naval Academy, will give the guest lecture. The title of her talk is "Rational Rationing in Healthcare: Observations from Organ Allocation" where she will share her experiences from the transplant community. She will explain how mathematics and operations research can increase the supply of live donor kidneys, maximize the number of life years gained from transplantation, or redistrict the geographic boundaries to make organ allocation more fair. Her work has attracted the attention of major media outlets including Time Magazine, Reader's Digest, Science, the Discovery Channel, and National Public Radio. It will be a talk not to miss.

Figures of Thought

by Howard Nemerov

from The Collected Poems of Howard Nemerov (University of Chicago Press, 1977). Permission requested.

To lay the logarithmic spiral on
Sea-shell and leaf alike, and see it fit,
To watch the same idea work itself out
In the fighter pilot's steepening, tightening turn
Onto his target, setting up the kill,
And in the flight of certain wall-eyed bugs
Who cannot see to fly straight into death
But have to cast their sidelong glance at it
And come but cranking to the candle's flame --

How secret that is, and how privileged
One feels to find the same necessity
Ciphered in forms diverse and otherwise
Without kinship -- that is the beautiful
In Nature as in art, not obvious,
Not inaccessible, but just between.

It may diminish some our dry delight
To wonder if everything we are and do
Lies subject to some little law like that;
Hidden in nature, but not deeply so.

Pinwheel Sudoku
by Laura Taalman, BrainFreeze Puzzles

A regular Sudoku puzzle requires finding a solution in which the integers 1 to 9 appear exactly once in each row, each column, and each of the nine 3x3 blocks indicated. The variant below is no different, except for the added requirement that each of the integers 1 to 9 appear exactly once in each of the colored 3x3 blocks. (warning: this one is not easy).

	7	3						
			1	8				3
					3			9
		5					4	
	9			1			3	
	3					2		
9			6					
4				7	2			
						9	2	

A Career as a Consulting Mathematician
by Barry Belkin,
Daniel H. Wagner Associates

I describe myself to mathematics students, particularly those who may be considering a non-academic career, as a consulting mathematician. Typically, this elicits a puzzled look on the part of the student, followed by a flurry of questions about exactly what it is that a consulting mathematician does and about what led me to choose the business world over academia. The substance of this article is my response to these inquiries.

In describing the types of mathematical problems that I work on, I begin with nonlinear state estimation. I first describe the application of nonlinear state estimation (at the microscopic scale) to the tracking of human sperm to study abnormal sperm motility as a source of male infertility. I then describe the application of nonlinear state estimation (at the cosmic scale) to the tracking of deep-space satellites as part of a project to maintain a satellite ephemeris catalogue. I then go on to identify how the mathematical techniques used to address these apparently very different problems are actually quite similar.

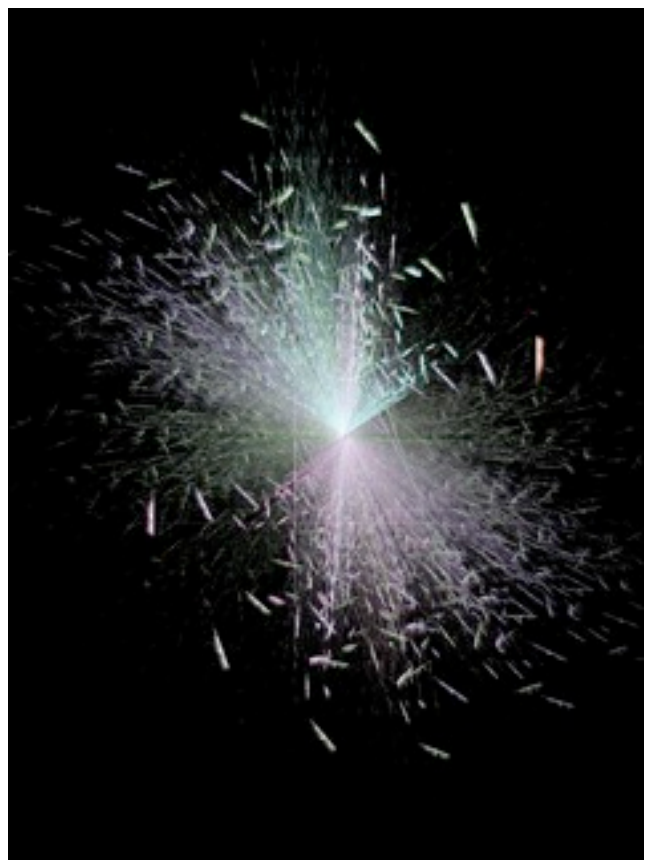
In the area of risk management, I describe a project to quantify the likelihood that passing icebergs might scour a power cable on the floor of the Strait of Belle Isle (between Newfoundland and Labrador). This required learning some physics related to the stability of floating bodies. I also give an overview of our firm's work on the application of modern financial theory to the setting of capital reserves to cover potential investment losses and to the pricing of derivative securities.

If the student I am speaking with has little interest in applications, the conversation generally ends at this point. Otherwise, I go on to note that I was fortunate enough to spend a summer working for Wagner Associates while I was in graduate school at Cornell. The project that I worked on had to do with anti-submarine warfare. The thought that I could pursue a career that would allow me to work on interesting and challenging mathematical problems with potentially important real-world application did it for me. After I completed my PhD thesis in 1967, I joined Wagner Associates and have been there ever since.

The next question that the student I am speaking with generally asks is what mathematics courses does one take to prepare for a career as a consulting mathematician. To answer, I recount my own experience when I was interviewed by Dan Wagner for a position on the Wagner Associates technical staff. After a few preliminaries, Dan asked me to outline the procedure for constructing the real and complex numbers starting from the Peano axioms. It took me a few seconds to recover from the initial shock of being asked such a question at a job interview for a non-academic position. Fortunately, in preparation for my PhD oral exam I had recently dusted off my real analysis text and reviewed the Dedekind cut scheme. When I was able to describe the required construction to Dan's satisfaction, he told me that I had a job.

Over the years I have learned that Dan had it right. He believed that the key to a successful career as a consulting mathematician is a strong foundation in the basics coupled with an equally strong interest in applications. Dan felt that completion of a PhD thesis in any area of mathematics demonstrated the requisite ability to work largely independently and successfully on a hard mathematical problem. The fact is that relatively few of the technical staff of Wagner Associates with advanced degrees in mathematics specialized in operations research or applied mathematics. Dan himself was an algebraist. My degree is in probability theory.

So, my final words of advice to the student are that if you are genuinely interested in applications and like to work on hard problems that lend themselves to mathematical formulation, then whatever your current area of specialization may be, you should consider a career as a consulting mathematician.



Kaleidoscope, by Collin Carbno

Conference Announcement and Call for Undergraduate Papers and Posters

SUMS Conference
Saturday October 22, 2011
James Madison University
Harrisonburg, Virginia (about two hours west of D.C.)

Advance registration deadline: October 7 Student talk/poster deadline: October 7 Travel funding deadline: September 22

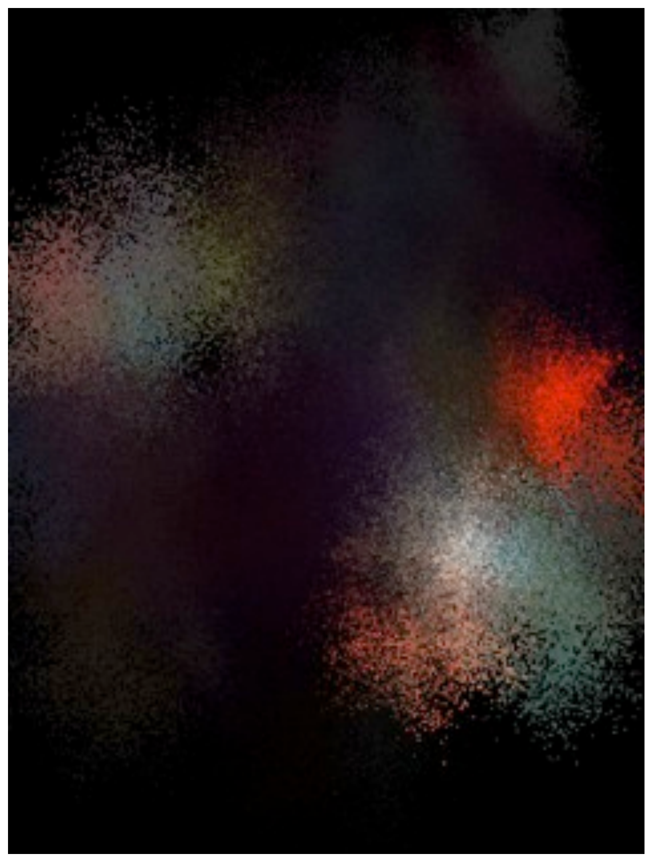
The seventh annual Shenandoah Undergraduate Mathematics and Statistics (SUMS) Conference at James Madison University is a
one-day undergraduate research conference that will feature:
* undergraduate contributed talks on mathematical research
* undergraduate and high school poster sessions on research and expository topics
* panel sessions on REU programs, graduate school, and industry
* a special AMC workshop for high school students and faculty
* invited opening and closing addresses from dynamic, engaging mathematicians

The SUMS conference has grown to include over 300 participants annually. Registration and lunch are free. Limited travel funds are available on a rolling application basis.

Opening Address: ``From Robotics to Geometry,'' Dr. Ruth Charney, Brandeis University
Abstract: Children build models with 3-dimensional cubes; mathematicians build them with higher dimensional cubes! Many physical systems can be represented by geometric models based on cubes of varying dimensions. Using an example from robotics, we will investigate how such models are constructed and what can we learn from their strange, but beautiful geometry.

Closing Address: ``Blown Away: What Knot to Do When Sailing'', Sir Randolph Bacon III, cousin-in-law to Dr. Colin Adams, Williams College
Abstract: Being a tale of adventure on the high seas involving great risk to the tale teller, and how an understanding of the mathematical theory of knots saved his bacon. No nautical or mathematical background assumed.

For more information and to register, submit a paper or poster, print a conference poster, or apply for travel funds, please visit
www.jmu.edu/mathstat/sums



Emergent, by Collin Carbno

Magic Pension Formulas

Collin Carbno, P. Phys.
Saskatchewan Telecommunications

Abstract

A simplified model of pension calculation is undertaken to provide a simple tool for retirement income planning based upon formulas for ``forever pensions'' (that is, capital-preserving) and capital-depleting pensions (over an expected remaining lifetime).

Introduction

With the rise of defined contribution plans, retirees essentially find that they have a fund of some value Q to manage and invest for their retirement. The question becomes how much money Q can generate in payments and how this should be taken out. The financial planners (the poor ones) often assume that the retiree wants to consume all their capital, and picks some expected age of life, say 80 years, and calculates the pension to include full consumption of the capital by age 80. This approach has the obvious difficulty of what the retiree will live on if he or she lives another 10 years beyond age 80.

Pensions also suffer from the "fish stock syndrome". The financial equivalent is when the retiree starts out only slightly overspending his or her resources, and maintains that rate, and the financial nest egg nonetheless collapses relatively quickly, leaving the retiree with serious pension shortfalls.

Real pension calculations of course are more complex in that one has to consider different kinds of earnings (interest, capital gains, dividend, tax free growth, tax free until withdrawal growth, wages, etc.) and their respective taxation rates. For this paper, however, we will ignore these considerations, and worry about only the pre-tax income.

Perpetual Pension with Capital Preservation

Given a retirement capital nest egg of size Q, the question becomes what yearly payment can be obtained while preserving the capital? In the absence of inflation, it should be clear that the yearly payment P will be simply $P = Q \cdot i$, where i is the return on investment. Of course this would be the before-tax payment. One also needs to be clear that at the start of the year before earning any interest one would probably be subtracting the payment in which case the formula is

$$P = \frac{Q \cdot i}{(1+i)}.$$

Unfortunately, in real life there is also the complicating factor of inflation. In this case one cannot spend all the interest and hence must slowly grow the capital nest egg so that the increase in capital is enough to cover the increasing pension payments required to conserve buying power. Let us guess that the correct form would be given by $P = Q \cdot \alpha$ for some constant α. The new capital Q_1 after the first period is given by subtracting the first payment P from Q, and then applying interest growth. The new capital Q_1 multiplied by constant α then should equal the inflated Payment P needed for the second period. The constant α can be determined from this condition by solving

$$(Q - Q \cdot \alpha) \cdot (1+i) \cdot \alpha = (1+j) \cdot Q \cdot \alpha$$

for constant alpha α yielding a payment

$$P = \frac{Q \cdot (i-j)}{(1+i)}. \tag{1}$$

Equation (1) gives the amount P that can taken with a Quantity Q so that in next payment period an amount P · (1+j) can be taken, and so on forever. Since this reduction works between any two periods, it must work over all periods.

From formula (1) it should be clear that in order to have a forever pension, the investment rate of return must be greater than rate of inflation. Historically, financial planners like to assume that rate of return i will be 2% greater than j. While this might be average situation over the long term, it often not true over periods as long as decades.

Capital-Depleting Pension

The following gives the amount of capital after 1,2 and so on periods looking at the pattern. For a depleting pension after n periods, the capital must be zero.

$$(Q-P)\cdot(1+i)-P\cdot(1+j)$$
$$((Q-P)\cdot(1+i)-P\cdot(1+j))(1+i)-P(1+j)^2$$
$$\ldots$$
$$(Q-P)(1+i)^n-P\cdot(1+j)\cdot(1+i)^{n-1}-$$
$$P\cdot(1+j)^2(1+i)^{n-2}\ldots-P\cdot(1+j)^n \quad . \tag{2}$$

Solving for P with k = (1+j)/(1+i) yields

$$P=\frac{Q\cdot(1-k)}{1-k^n} \quad . \tag{3}$$

Letting 1+λ = k, one obtains, for small λ,

$$P\approx\frac{Q}{n\cdot(1-\dfrac{(n-1)\cdot(i-j)}{2\cdot(1+i)})} \quad .$$

One can see when i=j that initial payment becomes just Q/n, exactly as one would intuitively expect. Similarly if i>j, the initial payment would be larger and if i<j, that payment would be smaller.

Capital-Maximizing Pension

A pension based on the depleted formula raises the question of what value to use for number of years n over which capital is expected to be depleted. Guess too short and the retiree risks running out of money, guess too long and the retiree risks ``leaving a lot of money on the table''. The perpetual formula has the advantage that the pension never runs out and is guaranteed to leave an inheritance for the estate but also has the disadvantage that the pension amounts are naturally much smaller. The advantages of both pension types can be partially realized by splitting the capital between the two strategies.

Letting $Y = (1-k)/(1-k^n)$ and $M = (i-j)/(1+j)$, the achievable annual Salary S, over n years, with a combination pension, allocating x percent to capital preservation and (1-x) percent to depletion, is given by

$$S=x\cdot Q\cdot M+(1-x)\cdot Q\cdot Y \quad . \tag{6}$$

Alternatively, solving for x, the fraction of capital preserved for given desired S, one obtains

$$x=\frac{S-QY}{Q\cdot(M-Y)} .$$

The factor x is a measure of the safety of the pension. The factor x should also account for the expected life remaining given what it might be possible to achieve. For example, using mortality tables (1941 Actuarial Society of America), although at age 50 expected remaining life is 21, it may be possible that life continues to age 100 years, meaning in reality 50 years to live. Respective numbers at age 70 are (9,30), at age 80 (5,20), at age 90 (3, 10) (the first number is expected number of years to live, the second is maximum number). Ideally, one would have x large enough that with a capital-depleting pension one would have a reasonable pension to age 100. What period of time should one use? One might think that the best time period would be the expected remaining life, but I have found through simulation that if one is in good health that it is better to pick a period equal to half the maximum possible age. Doing so means that choosing x = 0.50 ensures that there will be enough capital to survive to the maximum human life span. If one's health is not good, then using the expected remaining life is probably sufficient.

A new simulation at start of each year of the retirement, adjusting at the start for actual rate of return achieved, and amount of capital consumed, aids staying on track. If investments have done well, extra capital can be divided between perpetual and depleted portions. However, if earnings have been lower, or expenses higher, the retiree will need to lower expenses.

Summary

A completely capital-preserving pension is an estimate of the minimum achievable pension while a completely capital depleting pension is an estimate of the maximum achievable pension. A compromise pension based on preserving half of the initial capital over a period of approximately one half of one's possible remaining lifetime results in a fairly stable compromise between being excessively conservative and overly optimistic. The capital-depleting portion of the pension is somewhat less subject to effects of interest rates and lets one maintain some pension even in the face of very low rates of return.

About the author: Collin Carbno is a specialist in process improvement and methodology. He completed his bachelor degree for Mathematics and Physics in 1974, Master's in theoretical physics in 1977, and completed course work for Ph.D. in theoretical physics in 1979. He has been employed for 30 years in various IT and process work at Saskatchewan Telecommunications and currently holds a Professional Physics Designation from the Canadian Association of Physicists, and the Information System Professional designation from Canadian Information Processing Society.

Mixed Manifolds, by Collin Carbno

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