Environmental Mathematics: The Unifying Theme in an Introduction to Scientific Computing Course

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Talk Outline

Computing Requirement for Math Majors . . .

- Prior to 2000: **CS 150 - Problem Solving and Programming I**
  Problem solving and algorithm development using Java.

- Early 2000's: **CS 340 - Introduction to Scientific Computing**
  Condensed introduction to numerical analysis with programming as needed, very similar to another existing course.

- Post 2006: **Revised CS 340/MATH 340 - Introduction to Scientific Computing**
  Survey mathematical software and programming languages; applications in modeling and simulation.

What is the course about?

Course Learning Objectives:

By the end of the course students should be able to

- model problems mathematically and use mathematical software (numerical, symbolic, graphical, statistical, and system dynamics) to solve or simulate these problems;
- develop algorithms and implement them in the appropriate software or programming language;
- draw pertinent examples from a variety of mathematical models;
- present professional documents, algorithms and solutions to problems in a mathematically sophisticated manner; and
- know the benefits and drawbacks of each of the computational tools used during the semester.

Who takes the course?

Required for

- Mathematics – Applied, Traditional, and Education concentrations (B.S.);
- Mathematics Education (B.S.Ed)

Additional students from

- Chemistry
- Computer Science
- Electrical Engineering
- Engineering Technology
- Mathematics minor

Spring 2013 Students/Majors:

9 Mathematics Education, 7 Mathematics, 4 Computer Science, 4 Chemistry, 1 Electrical Engineering, and 1 Anthropology

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2. Using Excel in Modeling
   - Some Excel Basics and Working with Data
   - Modeling with Difference Equations
   - Incorporating Stochasticity
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   - Number Theory in Mathematica
   - Differential Equations and Modeling

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   - Introduction to MATLAB
   - Programming and Algorithm Development

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6. Individual (Agent) - Based Modeling with NetLogo

Spring 2013: An Environmental Mathematics Theme

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        - Predator-Prey Data
        - Predator-Prey Difference Equations

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   - Some Excel Basics and Working with Data
     - Predator-Prey Data
     - Predator-Prey Difference Equations
   - Modeling with Difference Equations
     - Spread of a Disease Simulation
     - Susceptible-Infected-Recovered Difference Equations
     - Single Species Population Growth (with and without competition, harvesting, etc.)
     - Interacting Population Models

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     - Interacting Population Models
   - Incorporating Stochasticity
     - Demographic Stochasticity (natural variability)
     - Environmental Stochasticity (catastrophes)

Spring 2013: An Environmental Mathematics Theme (continued)

3. Dynamical Systems Modeling with Vensim

Spring 2013: An Environmental Mathematics Theme (continued)

4. Dynamical Systems Modeling with Vensim
    - Population Modeling with Dynamic Systems (revisiting the following:)
        - Predator-Prey Model
        - SIR Model
        - Single Population Model

Spring 2013: An Environmental Mathematics Theme (continued)

5. Dynamical Systems Modeling with Vensim
    - Population Modeling with Dynamic Systems (revisiting the following:)
        - Predator-Prey Model
        - SIR Model
        - Single Population Model
    - Additional Dynamical Systems Models
        - Carbon Cycle
        - Water Cycle
        - Pesticide Accumulation

Spring 2013: An Environmental Mathematics Theme (continued)

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   - Population Modeling with Dynamic Systems (revisiting the following:)
     - Predator-Prey Model
     - SIR Model
     - Single Population Model
   - Additional Dynamical Systems Models
     - Carbon Cycle
     - Water Cycle
     - Pesticide Accumulation
   - Incorporating Stochasticity (the mechanics)

Spring 2013: An Environmental Mathematics Theme (continued)

7. Programming, Simulations, and Modeling with MATLAB/Octave
    - Introduction to MATLAB

Spring 2013: An Environmental Mathematics Theme (continued)

8. Programming, Simulations, and Modeling with MATLAB/Octave
    - Introduction to MATLAB
    - Stage-Based Modeling
        - Age- and Stage-Based Modeling with Leslie and Lefkovitch Matrices
        - Markov Chains and Ecological Succession

Spring 2013: An Environmental Mathematics Theme (continued)

9. Programming, Simulations, and Modeling with MATLAB/Octave
   - Introduction to MATLAB
   - Stage-Based Modeling
     - Age- and Stage-Based Modeling with Leslie and Lefkovitch Matrices
     - Markov Chains and Ecological Succession
   - Programming and Algorithm Development
     - Implementing Difference Equations of Population and Epidemiology Models
     - Simulations

Spring 2013: An Environmental Mathematics Theme (continued)

10. Computer Algebra Systems with Mathematica

Spring 2013: An Environmental Mathematics Theme (continued)

10. Computer Algebra Systems with Mathematica
    - Mathematica Basics and Calculus
    - Differential Equations and Modeling
    - Probabilistic Simulations and Markov Chains

Spring 2013: An Environmental Mathematics Theme (continued)

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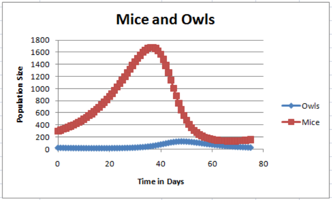
    - Differential Equations and Modeling
    - Probabilistic Simulations and Markov Chains
11. Individual (Agent) - Based Modeling with NetLogo
    - Population Models (predator-prey, single and interacting)
    - Epidemiology Models
    - Environmental Succession
    - Invasive Species
    - Patterned Behavior

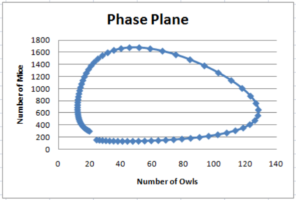
Owls and Mice

Owls are natural predators and mice are one of their main prey. Suppose mice have a linear growth rate (given two values for growth rate, students calculate the equation), and typical interactions between predator and prey.

B3 =B2+OwlsPerDeadMouse*RatioInt*Lethal*B2*C2-OwlDR*B2

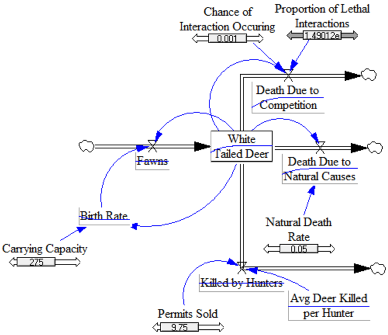
Time	Owls	Mice		
0	20	300	Mice GR	0.08
1	19.00769	313.5	Owl DR	0.09
2	18.09916	328.1519	Ratio Lethal Interactions	0.7
3	17.26975	344.0103	Chance Interaction Occurs	0.0025
4	16.51522	361.1344	Baby Owls Per Dead Mouse	0.0769231
5	15.83173	379.5878		
6	15.21585	399.4382		
7	14.66459	420.7571		
8	14.17538	443.6198		
9	13.74612	468.1045		
10	13.37517	494.2923		
11	13.06138	522.266		
12	12.80414	552.1096		
13	12.6034	583.9071		
14	12.45976	617.7411		
15	12.3745	653.6908		
16	12.34971	691.8301		
17	12.38838	732.2247		
18	12.49453	774.9283		
19	12.67342	819.9784		
20	12.93172	867.3908		

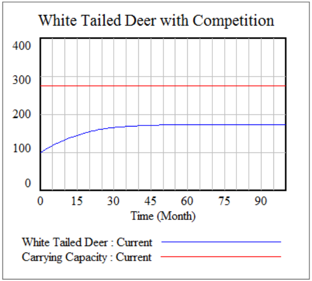




White Tailed Deer

Model a population of white-tailed deer in a confined habitat where food is limited and fighting for food and mates takes place. Suppose hunting is allowed with a maximum number of deer killed per hunter. How many permits should be issued to maintain a population of a certain size?

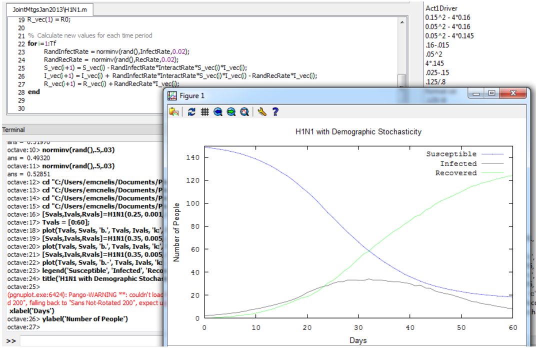




Sample Assignment Topics

Spread of H1N1

Adjust the SIR model to model the spread of H1N1 in the freshman class of the US Air Force Academy at training camp. Assume that the infection and recovery rates are normally distributed with means 45% and 11% respectively, and standard deviations of 2%.



Possum Populations

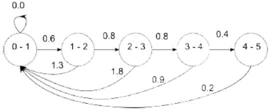
Given the birth and survival rates of female possums, divided into five age-groupings, draw the state diagram that represents this situation and give the difference equations and matrix equation that models the population growth over time.

Birth and Survival Rates for Female Possums

Age (years)	Birth Rate	Survival Rate
0 - 1	0.0	0.6
1 - 2	1.3	0.8
2 - 3	1.8	0.8
3 - 4	0.9	0.4
4 - 5	0.2	0.0

Let x_1 represent the number of possums in the 0-1 year age group; x_2 the number of possums in the 1-2 year age group; x_3 the number of possums in the 2 - 3 year age group; x_4 the number of possums in the 3 - 4 year age group; and x_5 be the number of possums in the 4 - 5 year age group.

1. [10 points] On a clean sheet of typing paper, draw the state diagram that models this situation. Make sure to label your components *neatly* and put your name on the paper.

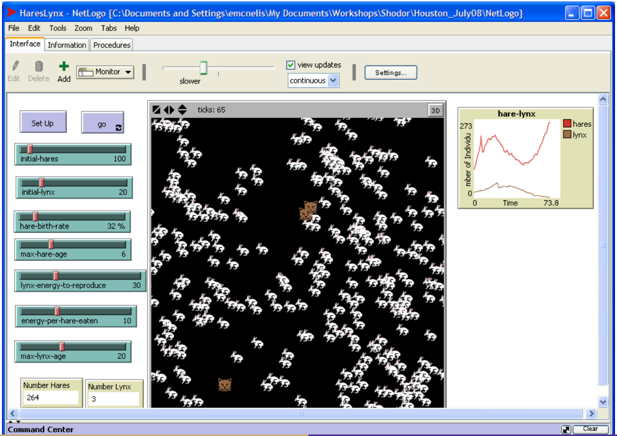


2. [10 points] In a Word document, give the system of equations for $x_1(n+1)$, $x_2(n+1)$, $x_3(n+1)$, $x_4(n+1)$, $x_5(n+1)$ in terms of $x_1(n)$, $x_2(n)$, $x_3(n)$, $x_4(n)$, $x_5(n)$.

$$\begin{aligned}
x_1(n+1) &= 0.0x_1(n) + 1.3x_2(n) + 1.8x_3(n) + 0.9x_4(n) + 0.2x_5(n)\\
x_2(n+1) &= 0.6x_1(n)\\
x_3(n+1) &= 0.8x_2(n)\\
x_4(n+1) &= 0.8x_3(n)\\
x_5(n+1) &= 0.4x_4(n)
\end{aligned}$$

Individual Based Model of Hare and Lynx Interactions

Given information on the dynamics between an isolated hare and lynx population, use NetLogo to create an individual based model for these populations and compare these results to your aggregate population model results.



Long-Term Project Ideas

- A Predator-Mesopredator-Prey System with Three Species
- Global Warming and the Carbon Cycle [14]
- Mathematics and Gardening with Limited Water Supply [3]
- Experimenting with and Modeling a Pan Water Cycle [14]
- Glacial Retreat and Rising Sea Level
- Modeling Oil Leakage in the Gulf Oil Spill
- Optimization and Land Use Models
- Pollution in a Series of Lakes
- Estimating Parameters in a Spread of Disease Model Using Real Data
- Deforestation on Easter Island and a Predator-Prey Model [10]
- The Great Waste Debate: What Contributes More Nitrogen and Phosphates to Local Streams [14]

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Thank you for your attention.

Questions?