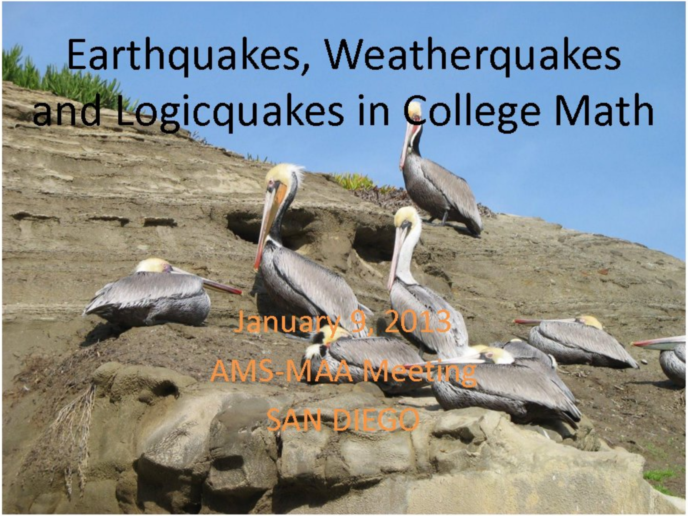
Earthquakes, Weatherquakes
and Logicquakes in College Math
January 9, 2013
AMS-MAA Meeting
SAN DIEGO

Earthquakes, Weatherquakes, and Logicquakes

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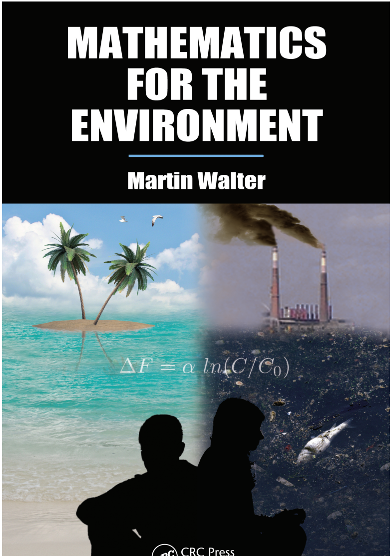
American Mathematical Society, San Diego Meeting, January 9, 2013

The Book/Class: 19 Years in the Making

Teaching Mathematics as if Our Survival Matters



Earthquakes and Weatherquakes: Mathematics and Climate Change

Martin E. Walter

In memory of Roger Gallet

In 1824 the French mathematician Jean Baptiste Joseph Fourier (1768–1830) in order to describe certain observations, created the term "greenhouse effect", [7, 8]. In modern language this effect occurs when visible spectrum sunlight passes through an enclosure-creating barrier, such as glass or an atmosphere, and the enclosure heats up because the barrier absorbs/emits infrared spectrum radiation or otherwise traps heat. This paper is one (small) mathematical step on the journey that Fourier began. Our main goal is to describe a plausible model wherein the proportion of extreme weather events, such as tornados, among all weather events, can be expected to increase as the concentrations of greenhouse gases, such as carbon dioxide, increase in the atmosphere.

In 1896 Swedish scientist Svante August Arrhenius (1859–1927), 1903 Nobel Prize winner in chemistry, was aware that atmospheric concentrations of CO_2 (and other gases) had an effect

where C is CO_2 concentration measured in parts per million by volume (ppmv); C_0 denotes a baseline or unperturbed concentration of CO_2, and ΔF is the radiative forcing, measured in Watts per square meter, $\frac{W}{m^2}$. The Intergovernmental Panel on Climate Change (IPCC) assigns to the constant α the value 6.3; [14] assigns the value 5.35. Radiative forcing is directly related to a corresponding (global average) temperature, by definition radiative forcing is the change in the balance between radiation coming into the atmosphere and radiation going out. A positive radiative forcing tends on average to warm the surface of the Earth, and negative forcing tends on average to cool the surface. (We will not go into the details of the quantitative relationship between radiative forcing and global average temperature.)

Qualitatively his CO_2 thesis, which Arrhenius was the first to articulate, says: *increasing emissions of* CO_2 *leads to global warming.* Arrhenius predicted that doubling CO_2 concentrations would

Gutenberg-Richeter Law for Earthquakes

Assuming that the Californian shocks are representative of general conditions, and attaching the results from California to those found directly for the whole world in the higher magnitude levels, the conclusions follow:

	Magnitude	Annual number
Great earthquakes	8 or more	1
Major earthquakes	7–7.9	10
Destructive shocks	6–6.9	100
Damaging shocks	5–5.9	1000
Minor strong shocks	4–4.9	10000
Generally felt	3–3.9	100000

The total number of shocks potentially strong enough to be perceptible to persons in a settled area (magnitudes 2 and over) must be of the order of several hundred thousand per year. Including aftershocks and swarms of small shocks, the total may be well over a million.

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$$f[x] = \beta\, x^{\alpha} \qquad \text{(Power Law in General Form)}$$

Thus our axiom, that nature has no preferred size (or scale) for earthquakes, implies that earthquakes are geometrically distributed if the number of earthquakes is plotted versus magnitude, i.e., the logarithm of intensity.

I close this section by mentioning that there is a wide variety of natural phenomena described by power laws [3, 15].

Weatherquakes and Global Warming

Climate is, by definition, weather statistics. Thus one might suppose that any number of mathematical tools might be applicable to the study of climate and weather. Dr. Roger Gallet, our friend and a scientist with the National Oceanic and Atmospheric Administration (NOAA) many years ago initiated a sophisticated statistical analysis of weather events using every tool of which we were aware and more, including, for example, Gumbel's work, [11]. He was trying to demonstrate what we refer to as *Gallet's Conjecture*, viz., that the proportion of extreme weather events among all weather events increases as the atmosphere (troposphere) becomes warmer. Our colleague, Dr. Holley briefly joined the effort to statistically verify Gallet's Conjecture. About the time it was becoming evident to us that the results would likely not be definitive, sadly, Dr. Gallet became ill and passed away. Add to this the fact that we were familiar with the now famous work of Jerzy Neyman on the statistics of smoking and health from the last century, and how in the early years it was ignored/attacked by some—with some success since exact mechanisms by which cigarettes impacted health were not then well understood. Any analogous statistical analysis of weather events by this author would likely be greeted with even less enthusiasm. Finally we were (are) of the opinion that should Gallet's Conjecture become statistically, obviously, unassailable, it might be too late to do anything about it. Thus we were motivated to bypass statistics and look for a fundamental mechanism and/or principle (or principles) that would imply the truth of Gallet's Conjecture.

have made the same hand-waving argument about earthquakes, which we have seen are distributed geometrically when plotted as a function of the logarithm of event intensity. Ultimately this question is to be answered by empirical observation of weatherquakes, some of which has been done; cf. [4].

The alert reader will have noticed that we have not given a precise definition of weatherquake other than a tautological one. Neither have we given a definition of intensity of a weatherquake or how to go about measuring same. These considerations are actually part of our weatherquake hypothesis.

The Weatherquake Hypothesis

There exists a definition of *weatherquake* and there exists a definition of *intensity of weatherquake* such that nature has no preferred size (or scale) of intensity of weatherquake. Significant nonempty classes of such weatherquakes exist.

Although some statistical measures of hurricanes, for example, are not analogous to that of earthquakes, [6], we have found no arguments supporting the negation of the weatherquake hypothesis, i.e., that there is a reasonable definition of weatherquakes or their intensity such that nature prefers some intensities more than others. Furthermore, one could disprove the weatherquake hypothesis by showing that our conclusion about extreme weatherquakes (in the next section) implied by the weatherquake hypothesis is false. However, empirical evidence thus far is tending to confirm, not contradict, this conclusion [9, 10, 4].

From the point of view of pure mathematics we could remain silent on any proposed definitions of weatherquakes and their intensities, but we owe the reader some discussion of these topics. Thus virtually no one who has been in the presence of a tornado would deny that such is a weatherquake—same for a hurricane. These and some other classes of weatherquake follow power laws [4]. The data tell us that two different classes of weatherquake can (and often do) follow

Quakes Follow Power Laws

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Different classes of weatherquakes, e.g., hurricanes, tornados, follow different power laws (but of same general form).

Theorem on Extreme Weatherquakes

Given $0 < p < 1$ *and the normalized, geometric probability distribution* $F_p[x] = (-\ln p)\ p^x$, *the mean,* M_p, *of* F_p *satisfies*

$$M_p = \frac{-1}{\ln p}$$

. *The "tail past a" of* F_p, $T_p[a] = -\ln p \int_a^\infty p^x dx$, *satisfies*

$$T_p[a] = p^a.$$

If $0 < p < q < 1$, *then we have the following formula for the fractional increase in the mean of* F_q *relative to the mean of* F_p*:*

$$\frac{M_q - M_p}{M_p} = \frac{\ln p}{\ln q} - 1.$$

We have the following formula for the fractional increase in $T_q[a]$ *over* $T_p[a]$*:*

$$\frac{T_q[a] - T_p[a]}{T_p[a]} = \left(\frac{q}{p}\right)^a - 1.$$

Qualitative Statement of the Weatherquake Theorem

A MODEST increase in global temperatures,

Qualitative Statement of the Weatherquake Theorem

A MODEST increase in global temperatures, likely yields an IMMODEST increase in **EXTREME WEATHER**.

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Humans are but one part of the BIOSHPERE,
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Logicquakes=Changes in Patterns of Thought

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Human history becomes more and more a race between education and catastrophe.

H.G. Wells

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