A BRIEF INTRODUCTION TO THE WORLD OF INQUIRY-BASED LEARNING

SIGMAA IBL Leadership Team

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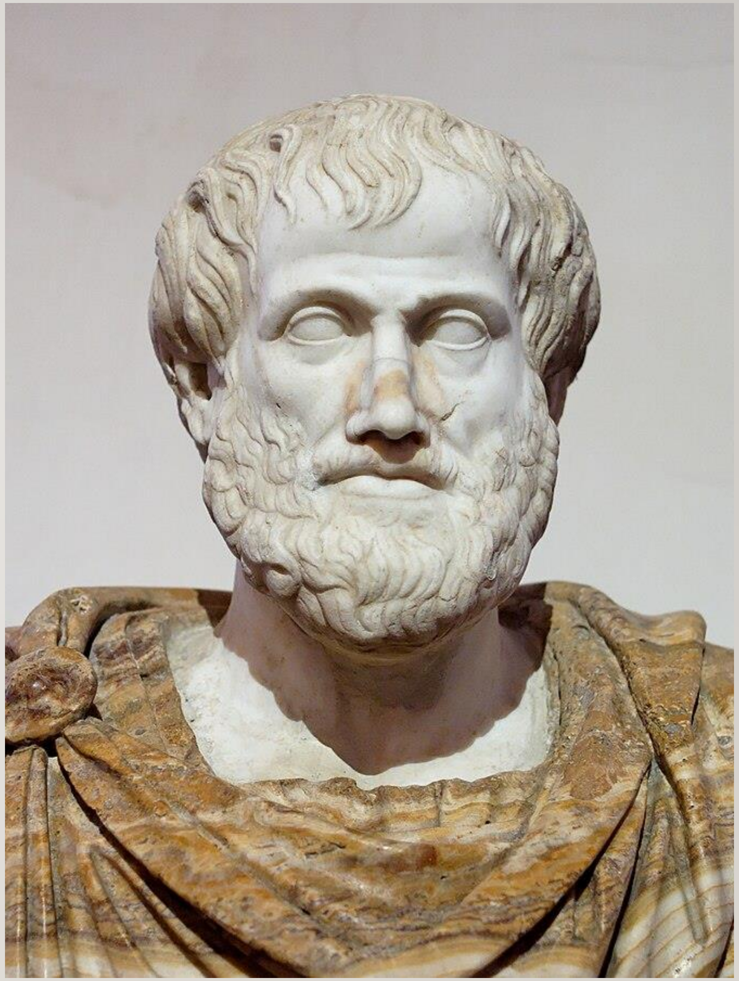
“For the things we have to learn before we can do them, we learn by doing them.”

— **Aristotle, The Nicomachean Ethics**



https://commons.wikimedia.org/wiki/File:Aristotle_Altemps_Inv8575
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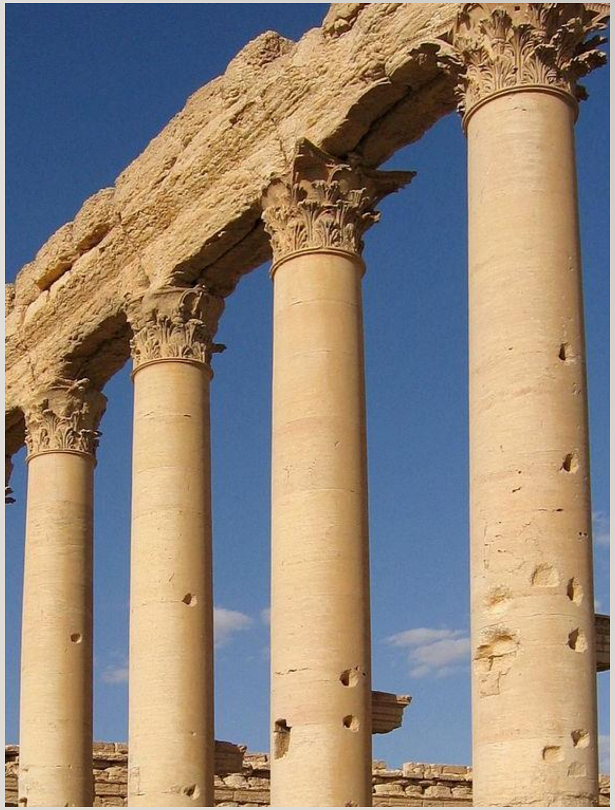
WHAT IS INQUIRY-BASED LEARNING (IBL) IN MATHEMATICS?

Four Pillars:

1. Student engagement in meaningful mathematics,
2. Opportunities for student collaboration,
3. Instructor inquiry into student thinking,
4. Equitable instructional practice

Laursen, S. L., & Rasmussen, C. (2019)



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STUDENT ENGAGEMENT

Students are actively engaging in mathematics:

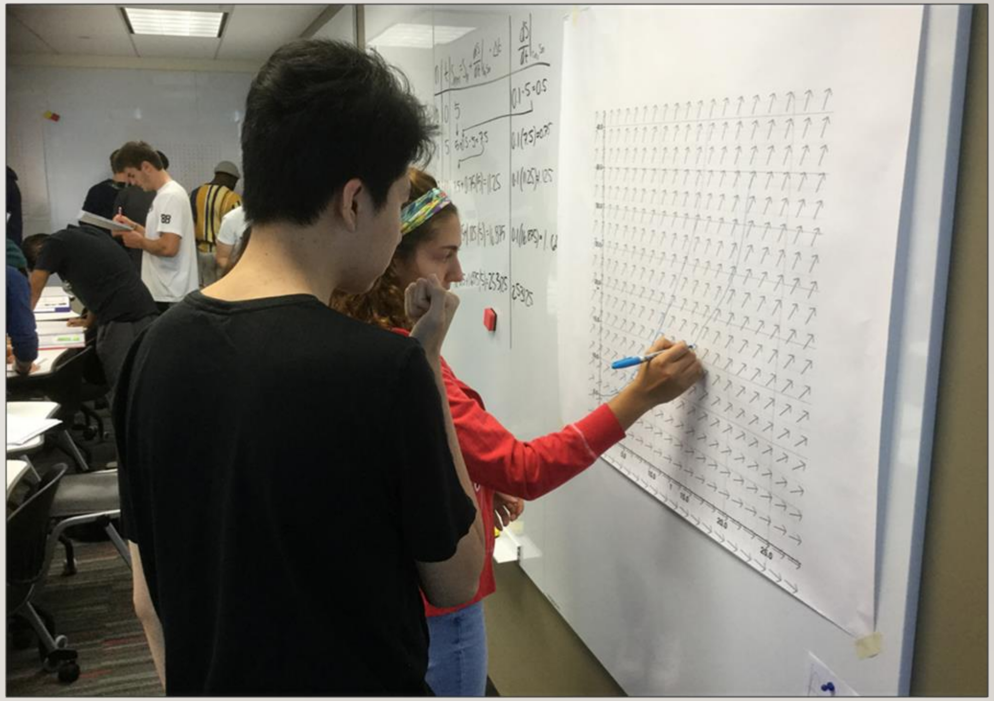
- Working on challenging mathematics problems
- Presenting solutions/proofs
- Reflecting on the problem and their work
- Relating the current problem to their previous knowledge

Ernst DC, et al, 2017

STUDENT COLLABORATION

Students work together in the classroom:

- In small groups or whole class discussions
- At the whiteboard, windows
  Jamboard, MS Whiteboards, Limnu (online)
- Class presentations



- Sharing thoughts and ideas with each other

INSTRUCTOR INQUIRY INTO STUDENT THINKING

- Eliciting, listening to, and interpreting students’ thinking
- Building classroom community
- Elaborating student ideas
- Moving the learning agenda forward

Laursen, S. L., & Rasmussen, C. (2019)

EQUITABLE INSTRUCTIONAL PRACTICE

- Grouping students of varying abilities:
  - Collaboration, cooperation, growth mindset
- Students take ownership of their learning
- Considering not just what students think, but what they feel and experience

Laursen, S. L., & Rasmussen, C. (2019)
Boaler (2006)

GET STARTED USING IBL IN YOUR CLASSROOM

You can begin small:

1. One or two IBL-oriented (or Active Learning) lessons
2. Have students work together
3. Have students engage in mathematics in the classroom

ACTIVE LEARNING STRATEGIES

1. Work on a vertical erasable surface
    a) Each student in the group can see their work
    b) Groups can see other groups' work
    c) The instructor (and LAs) can see each group's work.

Online:

- Use Google Jamboard, MS Whiteboard, or Limnu with Zoom breakouts rooms

ACTIVE LEARNING STRATEGIES

1. Work on a vertical erasable surface

Liljedahl, 2014

ACTIVE LEARNING STRATEGIES

2. Select groups using strategic randomness

a) Pay attention to group dynamics with regards to gender, language, strengths, etc.

b) Cards, counting off, group mixing

Liljedahl, 2014

ACTIVE LEARNING STRATEGIES

3. Begin with your current material

a) Focus on questioning techniques
   - Elicit thinking, generate ideas, clarify/revise explanation, justify claims

b) Give time to think and respond

c) Value student responses and questions

Hallman-Thrasher. & Spangler (2020)

ACTIVE LEARNING STRATEGIES

4. Move to more Inquiry-Oriented Material

a) Use established materials (JIBL, TBIL, IODE, Active Learning Materials for Calculus, etc)

b) Develop your own
   - Turning Routine Exercises into Activities that Teach Inquiry: A Practical Guide, Suzanne Dorée

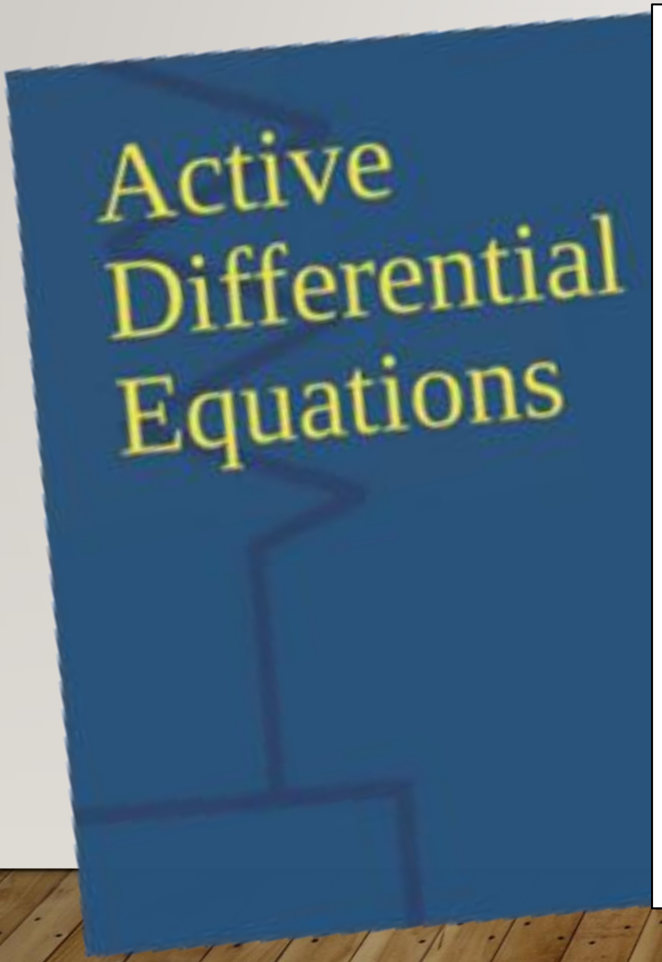
Dorée (2017)

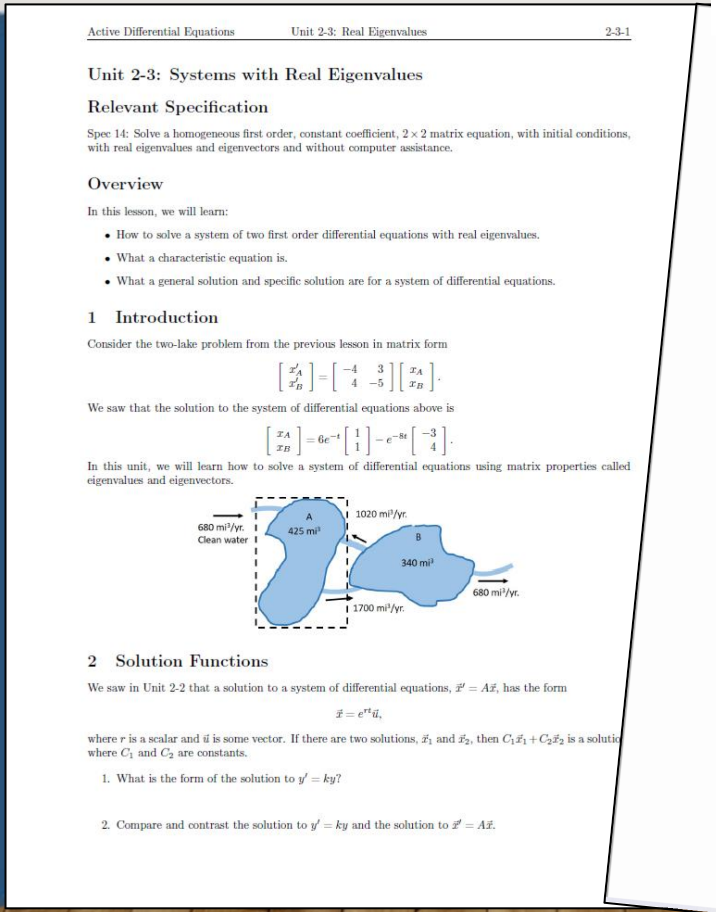
ACTIVE LEARNING – TWO STUDENTS' PERSPECTIVES

IBL COURSE MATERIAL EXAMPLES

- IODE – Inquiry Oriented Differential Equations - SDSU
- IOLA – Inquiry Oriented Linear Algebra – Virginia Tech
- JIBLM - Journal of Inquiry-Based Learning in Mathematics

IBL COURSE MATERIAL EXAMPLES





Unit 2-3: Systems with Real Eigenvalues

Relevant Specification

Spec 14: Solve a homogeneous first order, constant coefficient, 2×2 matrix equation, with initial conditions, with real eigenvalues and eigenvectors and without computer assistance.

Overview

In this lesson, we will learn:

- How to solve a system of two first order differential equations with real eigenvalues.
- What a characteristic equation is.
- What a general solution and specific solution are for a system of differential equations.

1 Introduction

Consider the two-lake problem from the previous lesson in matrix form

$$\begin{bmatrix} x_A' \\ x_B' \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x_A \\ x_B \end{bmatrix}.$$

We saw that the solution to the system of differential equations above is

$$\begin{bmatrix} x_A \\ x_B \end{bmatrix} = 6e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - e^{-8t} \begin{bmatrix} -3 \\ 4 \end{bmatrix}.$$

In this unit, we will learn how to solve a system of differential equations using matrix properties called eigenvalues and eigenvectors.

2 Solution Functions

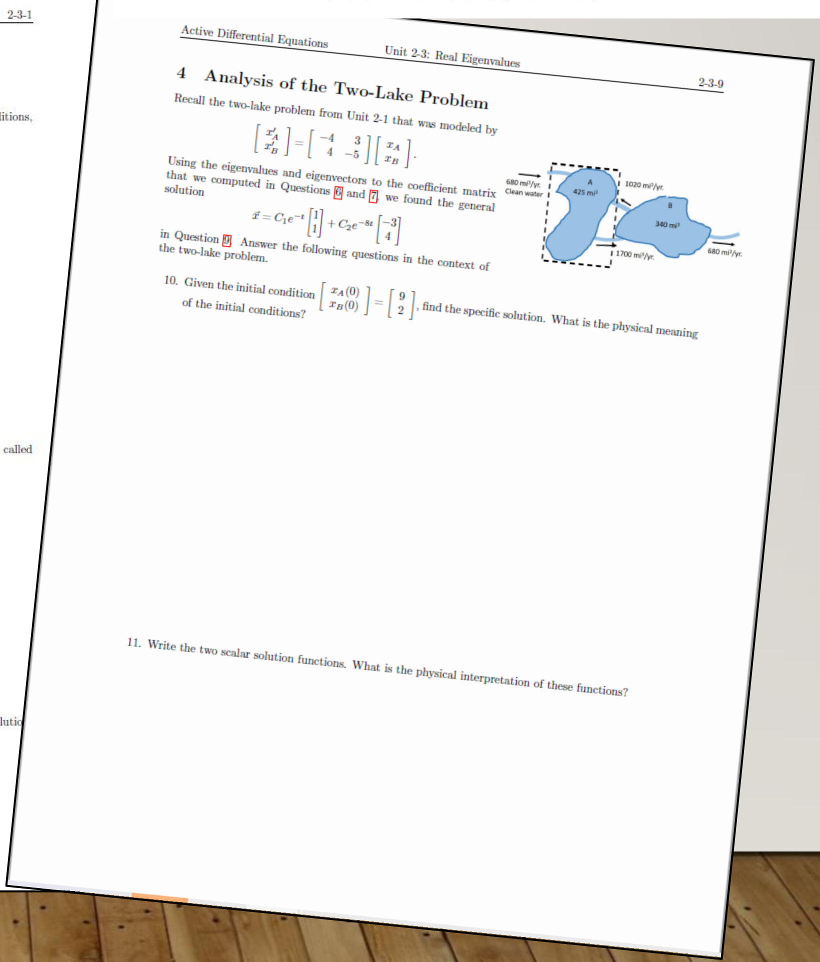
We saw in Unit 2-2 that a solution to a system of differential equations, $\vec{x}' = A\vec{x}$, has the form

$$\vec{x} = e^{rt}\vec{u},$$

where r is a scalar and $\vec{u}$ is some vector. If there are two solutions, $\vec{x}_1$ and $\vec{x}_2$, then $C_1\vec{x}_1 + C_2\vec{x}_2$ is a solutio where C_1 and C_2 are constants.

1. What is the form of the solution to $y' = ky$?

2. Compare and contrast the solution to $y' = ky$ and the solution to $\vec{x}' = A\vec{x}$.



4 Analysis of the Two-Lake Problem

Recall the two-lake problem from Unit 2-1 that was modeled by

$$\begin{bmatrix} x_A' \\ x_B' \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x_A \\ x_B \end{bmatrix}.$$

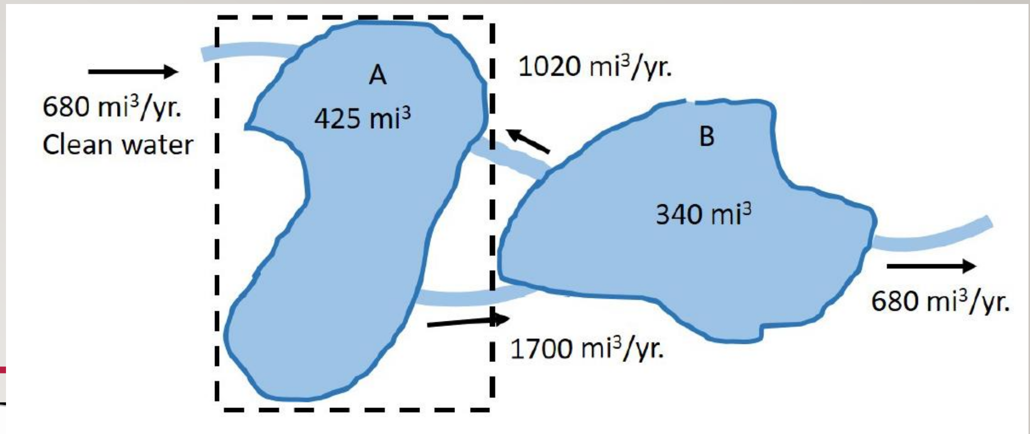
Using the eigenvalues and eigenvectors to the coefficient matrix that we computed in Questions 6 and 7, we found the general solution

$$\vec{x} = C_1 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{-8t} \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

in Question 9. Answer the following questions in the context of the two-lake problem.

10. Given the initial condition $\begin{bmatrix} x_A(0) \\ x_B(0) \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \end{bmatrix}$, find the specific solution. What is the physical meaning of the initial conditions?

11. Write the two scalar solution functions. What is the physical interpretation of these functions?



IBL COURSE MATERIAL EXAMPLES

Math 1776 FTC-1 In-class Assignment
Adapted from *Active Learning in the Calculus Classroom* by Mil

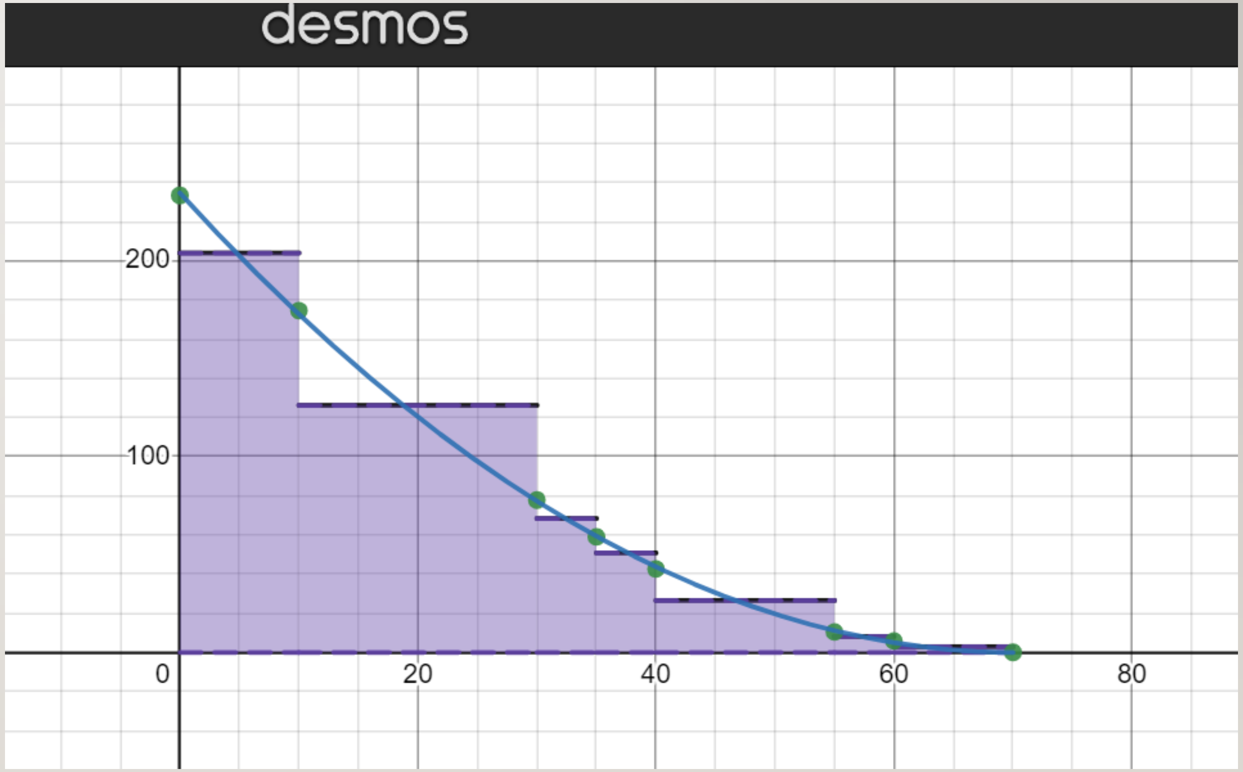
Japan Airlines flight 008 is arriving from Tokyo, and its leaking fuel! The pilots are
landing on the first available runway at Logan. That runway is runway 9, but run
only 7,000 feet. Data under similar stressful conditions was collected on the landin
of aircraft and is tabulated below.

Time (sec)	Speed (ft/sec)
0	233.4
10	174.6
30	77.8
35	59.0
40	42.6
55	10.4
60	5.8
70	0

1. Will the pilot be able to stop the plane before it plunges into Boston Harbor?
   (a) Develop with your group a strategy for answering the question, then im
   (b) Explain in a few sentences how you arrived at your conclusion.
   (c) Include one or more visual representation to illustrate your method.
   (d) You may find that using Excel and/or Desmos will hep in developing and visualizing your solution. (As necessary, refer to the Excel and Desmos tutorials posted on Brightspace.)

1

Time (sec)	Speed (ft/sec)
0	233.4
10	174.6
30	77.8
35	59.0
40	42.6
55	10.4
60	5.8
70	0



Adapted from Active Learning in the Calculus Classroom by Miller, E.

RESEARCH FINDINGS

Studies demonstrate improved outcomes on:

- Problem-solving
- Conceptual Understanding
- Modeling
- Retention
- Beliefs and attitudes about mathematics

Yoshinobu & Jones (2011)

RESEARCH FINDINGS

All while maintaining similar achievements in procedural knowledge

Compared to traditional lecture-based courses, for example:

- Instructor demonstrates solutions
- Assigns similar problems for practice

Yoshinobu & Jones (2011)

SELECTED RESOURCES

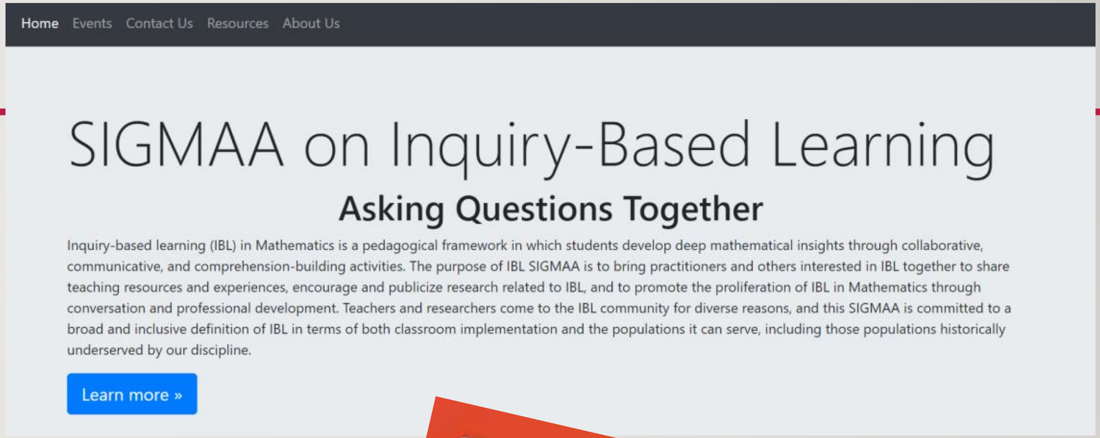
14 Regions

4 Corners (Utah, New Mexico, Colorado, Arizona)	NE COMMIT (New England)
Alliance for Michigan IBL (AMiIBL)	Pacific NorthWest (PNW) COMMIT
COMMIT-CaN (California & Nevada)	San Antonio COMMIT
ChiCOMMIT (Chicago)	SINE (South dakota, Iowa, NEbraska) COMMIT
Hawai'i COMMIT	Southern Arizona COMMIT
MD-DC-VA COMMIT	Southeast COMMIT
MITN COMMIT (Minnesota)	Greater Upstate NY IBL (UNY IBL) Consortium

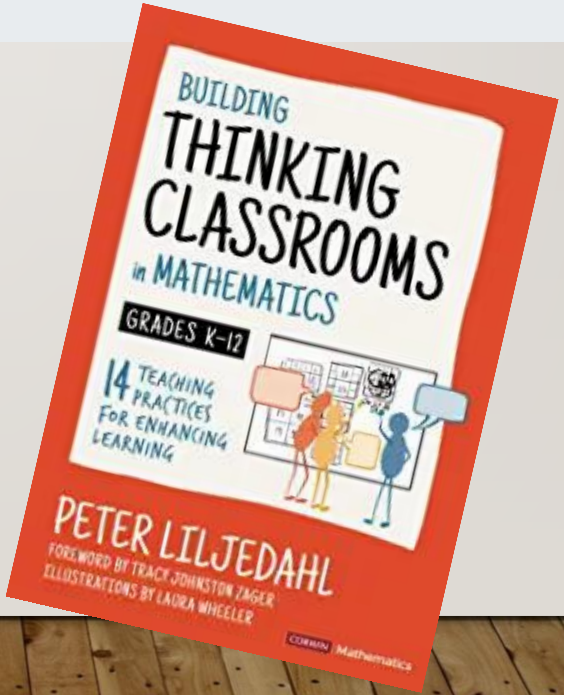
- Regional COMMIT Groups (Community for Mathematics Inquiry in Teaching)
- The Academy of Inquiry Based Learning

SELECTED RESOURCES

- SIGMAA IBL Website / Workshops

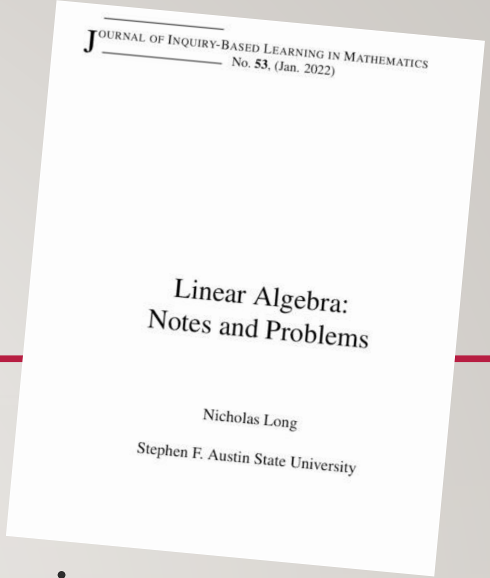


- *Building Thinking Classrooms in Mathematics* - Peter Liljedhal



SELECTED RESOURCES

- Blog: Math Ed Matters – Dana Ernst
- Journal of Inquiry-Based Learning in Mathematics
- *Mathematical Mindsets* - Jo Boaler
- Significant Body of Research…

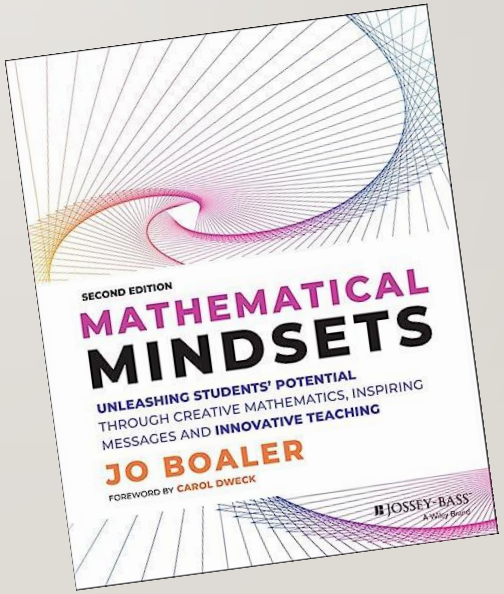


JOURNAL OF INQUIRY-BASED LEARNING IN MATHEMATICS
No. **53**, (Jan. 2022)

Linear Algebra:
Notes and Problems

Nicholas Long

Stephen F. Austin State University



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THANK YOU!

SIGMAA IBL WEBSITE

FALL SPEAKER SERIES

SIGMAA IBL

Fall Speaker Series 2023

WORKSHOP SERIES

SIGMAA IBL hosts workshop series each fall and spring. Each interactive workshop is 50 minutes long. All levels of IBL practitioners (including beginners) are welcome! Please visit our website for more details including abstracts and speaker bios.

AUGUST 29 @ 3 PM EDT

Stan Yoshinobu

"IBL and Large Classes"

SEPTEMBER 26 @ 1 PM EDT

Valerie Peterson

"Inquiry-Oriented Instruction: What It Is (and Isn't) and Where to Start"

OCTOBER 27 @ 1 PM EDT

David Clark and Robert Talbert

"Supporting IBL with Alternative Grading"

NOVEMBER 29 @ 1 PM EST

Christine Andrews-Larson

"Aligning *how* we teach with IBL with *why* we teach with IBL"

http://sigmaa.maa.org/ibl/   Lee.Roberson@colorado.edu