IBL and Large Classes

TL;DR Mostly Similar*

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Briefly about me as a teacher

1. Started teaching via IBL in 2003.
2. Class size under 35 in the Cal State University system.
3. Real analysis, Calculus, Math for Prospective Teachers, etc.
4. Ranging from students presenting at the board to students working in groups and making public ideas that worked and ideas that didn’t work and why.

Today

1. University of Toronto
2. Large lectures 150-200 or more
3. Coordinated courses of 1000+ students across multiple sections

First-year Linear Algebra at UofT (Context)

1. 1500 students
2. 8 lecture sections of 190 students
3. 35 tutorials/recitations
4. 25 TAs (undergraduate and graduate students)
5. 8 instructors (grad students and postdocs)
6. Common assessments

IBL Pillars

1. Students engaged in rich mathematics
2. Frequent opportunities for collaboration
3. Instructor inquiry into student thinking
4. Instructor focus on equity

Sharing via Padlet

Name

What are you teaching?

How many students?

What are your challenges?

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How I setup my IBL large classes

Perspectives and context

1. My context includes new graduate and postdoc instructors, who are new(ish) to IBL.
2. **Starting simple**, expanding as people go.
3. There are more sophisticated approaches.
4. Course coordinator as facilitator, supporter.


10:30


What is one of your hobbies?
How did you get better?

Guiding philosophy/mantra

1. Keep it simple!
2. Implement simple things well.
3. Quality over quantity or fancy.
4. Something **new instructors** can do with “light” training.

Typical class

- Instructor and a lecture TA are in the room.
- Divide the room in half and visit groups on the same side during the class. Switch sides each class.
- Half the time or more is student activities.

IBL Handouts used in lectures

- Create a pdf with tasks written on them and send them to students.
- We work through these in class.
- Use a mix of small group work (pairs) and riffs on think-pair-share.

MAT 223 Module 3 and Appendix 2 Lecture Handout

1. The first concept we learn about in Module 3 is the idea of span. Read the definition.

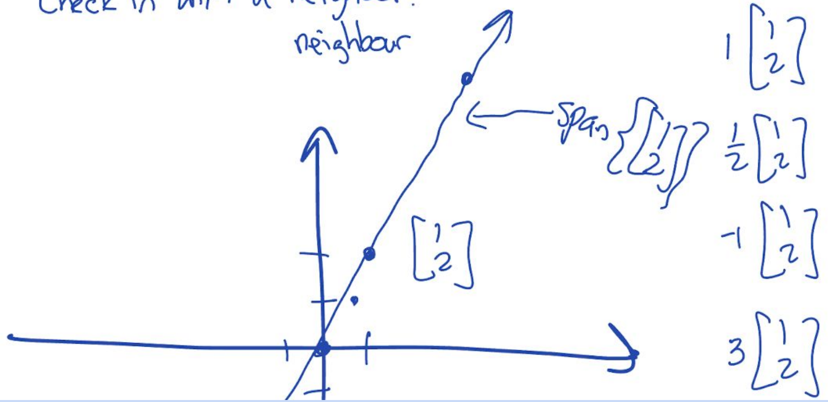
> **Span.** The *span* of a set of vectors V is the set of all linear combinations of vectors in V. That is,
>
> $$\operatorname{span} V = \{\vec{v} : \vec{v} = \alpha_1\vec{v}_1 + \alpha_2\vec{v}_2 + \cdots + \alpha_n\vec{v}_n \text{ for some } \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n \in V \text{ and scalars } \alpha_1, \alpha_2, \ldots, \alpha_n\}.$$
>
> Additionally, we define $\operatorname{span}\{\} = \{\vec{0}\}$. $\quad \operatorname{span}\phi = \{\vec{0}\}$

Read.

2. The span is a set of vectors, so we can represent them with graphs. Graph $\operatorname{span}\left\{\begin{bmatrix}1\\2\end{bmatrix}\right\}$. $\quad \mathbb{R}^2$

Think

Check in with a neighbor.
neighbour



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2. The span is a set of vectors, so we can represent them with graphs. Graph $\operatorname{span}\left\{\begin{bmatrix}1\\2\end{bmatrix}\right\}$.

3. The context for this problem is $\mathbb{R}^2$. In general, if two vectors are not multiples of each other, then the span of those two vectors is all of $\mathbb{R}^2$. In this task we will study a specific case of the above idea. Justify why $\operatorname{span}\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\right\}$ is equal to $\mathbb{R}^2$.

4. The next group of ideas is about connecting lines, planes, volumes, etc. to spans.

Takeaway. Lines and planes through the origin, and only lines and planes through the origin, can be expressed as spans.

What about lines and planes that do not go through the origin? Please read the definition of set addition.

Set Addition. If A and B are sets of vectors, then the ***set sum*** of A and B, denoted $A+B$, is

$$A+B=\{\vec{x}\ :\ \vec{x}=\vec{a}+\vec{b}\text{ for some }\vec{a}\in A\text{ and }\vec{b}\in B\}.$$

Lines and planes that do not go through the origin are translated spans, so they are shifted by a point (vector), $\vec{p}$.

Takeaway. All lines and planes, whether through the origin or not, can be expressed as translated spans.

Task: The context is $\mathbb{R}^2$. Let ℓ be the line with equation $y=x-4$. Express ℓ as a translated span. (Hint: consider finding the vector form of the line.)

5. We now turn our attention to a fundamentally important topic, linear independence and dependence. Carefully read the geometric version of the definition.

Linearly Dependent & Independent (Geometric).
We say the vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are ***linearly dependent*** if for at least one i,

$$\vec{v}_i \in \operatorname{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{i-1}, \vec{v}_{i+1}, \ldots, \vec{v}_n\}.$$

Otherwise, they are called ***linearly independent***.

Task: Let $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix} \right\}$. Show that S is linearly dependent.

6. Let $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$. Explain why S is linearly independent.

7. There is another equivalent definition of linear independence. Please read the following two definitions.

> **Trivial Linear Combination.**
> The linear combination $\alpha_1\vec{v}_1 + \cdots + \alpha_n\vec{v}_n$ is called ***trivial*** if $\alpha_1 = \cdots = \alpha_n = 0$. If at least one $\alpha_i \neq 0$, the linear combination is called ***non-trivial***.

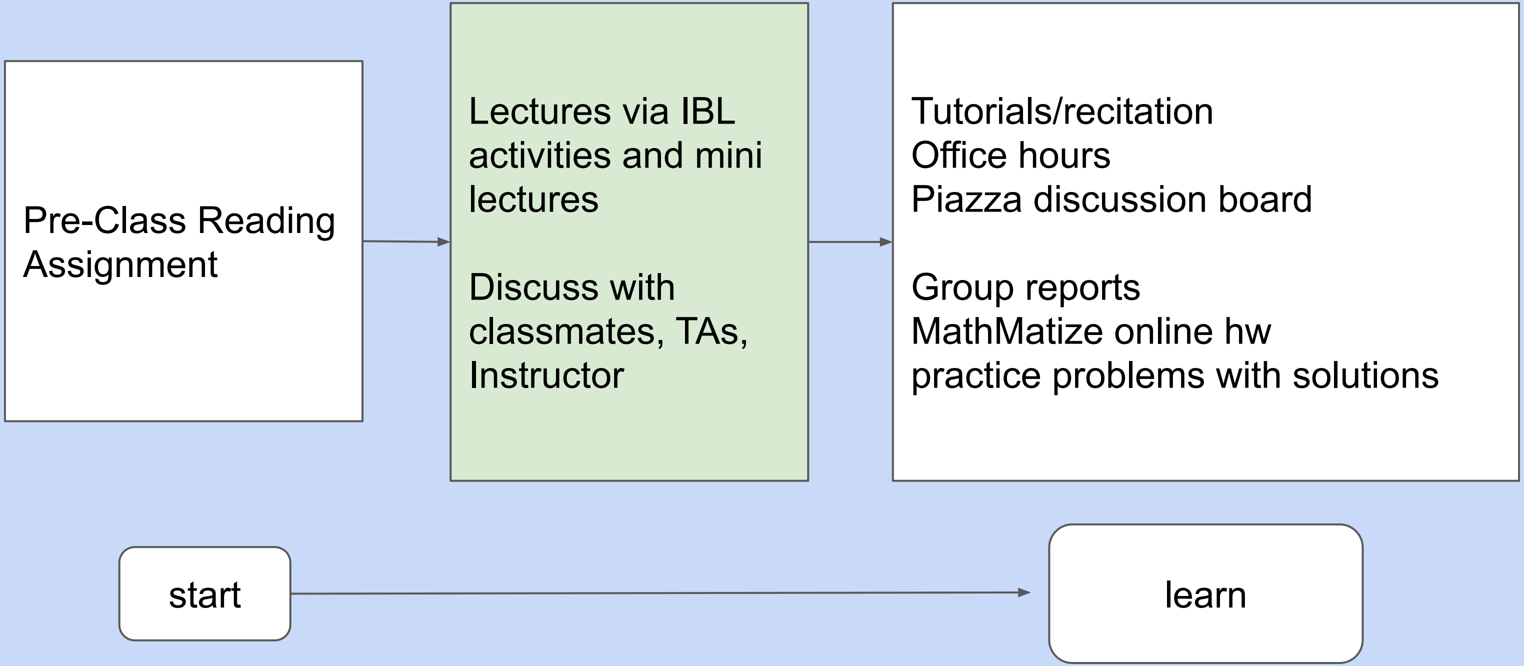
> **Linearly Dependent & Independent (Algebraic).**
> The vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are ***linearly dependent*** if there is a non-trivial linear combination of $\vec{v}_1, \ldots, \vec{v}_n$ that equals the zero vector. Otherwise they are linearly independent.

Task: Let $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix} \right\}$ Find a nontrivial linear combination of the vectors in S (which shows S is linearly dependent).

Implementation details

1. Tasks designed for thinking and helping students understand the key ideas, methods.
2. Instructor roams during longer activities, stays at front during shorter activities.
3. Students encouraged to share OR instructor forages for student ideas.
   a. “That’s a great idea [name]! Would you like to share it or could I share it with the rest of class?”
   b. Raising up softer spoken students, and managing those who want to share all the time.
4. Think-Pair-Share and riffs* (1-2-all, 1-2-4-all, 1-all)

Zoom out to get a wider view



What is different? Same?

- Students are not presenting at the board or leading discussions with whole class.
- Harder to learn names.
- Class can feel less personal.
- Hard to get to some students in the middle of room.
- Less opportunity for “deep dive” conversations.
- More high stakes for students to talk in front of the whole class.

- Students have time to think, discuss and share.
- Group work runs similarly.
- Instructors can still inquire about student thinking.
- Good tasks design is similar.
- Waiting time, visiting groups feels similar.
- Pre-class reading -> check what students need help with.

Tutorials/Recitations

1. 45 students
2. IBL Handouts with planned activities
3. Answering questions
4. Working on group reports

Assessment

1. Final exam 35% (required by the university, in-person)
2. Three group reports 30% with one resubmission
3. Reading assignments 14% (completeness)
4. Standards-based homework 15% (infinite attempts)
5. Reflections 6% (completeness)

Student comments

“The structure of this course was very nice. Some readings were a little hard to get through but they were all manageable. The mathematize homework also improved my understanding of the material. I also liked how content in lectures were created based off of what students needed help with and how we could resubmit group reports.”

Student comments

“The MAT223 [teaching team] is the only math [instructors] that does not make students cry every day. This course gives you room to fail and gives you a chance to learn and grow.”

Student comments

“I'm enjoyed in talking to other students and feel super pleasant after solving a question by teamwork.”

Student comments

“I feel like the instructor designed the course to promote our understanding.”

Overall

- Mostly similar*
- Students can still grapple with questions and collaborate. (IBL Pillars)
- Instructors can still inquire about student thinking, focus on equity in the classroom. (IBL Pillars)
- Logistics of large classes means there are more things to manage, less personalized, fewer tools or more challenging to implement tools.
- Large classes are not a substitute for smaller classes. Harder to implement mastery-grading/standards-based grading.
- Don’t have those deep dive discussions in a small IBL class generated by students.

Zoom out to get a wider view

Pre-Class Reading Assignment

Lectures via IBL activities and mini lectures

Discuss with classmates, TAs, Instructor

Tutorials/recitation
Office hours
Piazza discussion board

Group reports
MathMatize online hw
practice problems with solutions

start

learn

Questions, Comments, Discussion

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