

Lights Out! - Dimensions

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Texas A&M Math Circle

Founded in 2011 by PY, Frank Sottile, and Alex Sprinston

55 students signed up with 30 showing up each Saturday (numbers down significantly from pre-COVID years)

1-3pm Discovery Learning activities, 3-4pm Problem Solving (AMC/MATHCOUNTS/etc competition prep)



Rules of Lights Out

Start with some number of nodes called “lights”

They begin either turned on or off, and this data is called a “starting configuration”

The goal is to get all the lights turned off by “toggling” a switch at each light

Here’s the twist: toggling a switch doesn’t change the light where it’s located; it changes the lights adjacent to it!

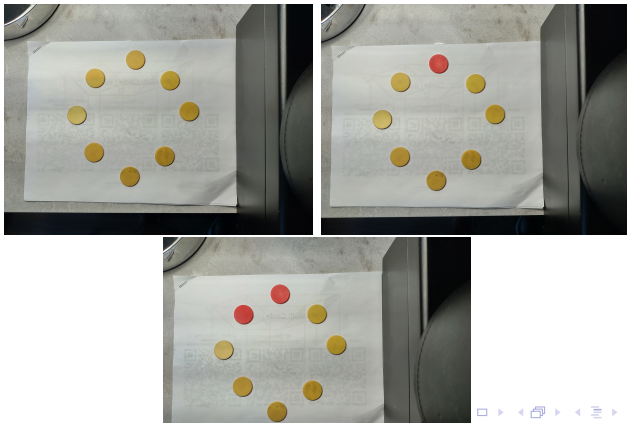
Example

This GIF only plays in Adobe Reader; [click here for link to GIF](#)

Play Around With It

Try it yourself! Take *eight* tokens and make a random starting configuration (turn some number of lights on and leave others off). Can you find a way to turn *all* your lights off?

Here are some starting configurations to try in case you run out of ideas yourself. *About what percentage* of starting configurations can be solved?

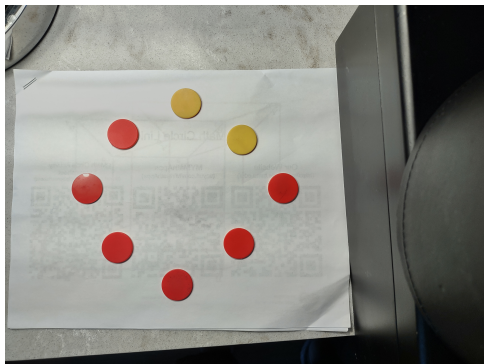


What Did We Learn?

Starting configuration with odd number of lights on does not work

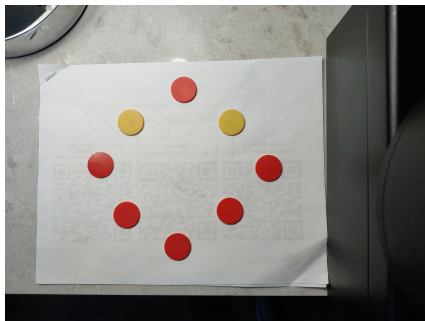
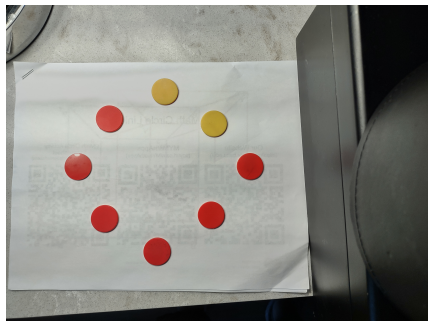
What Did We Learn?

Starting configuration with odd number of lights on does not work
Some starting configurations can be turned all the way to the configuration below, but no sequence of moves can turn these off



If Only...

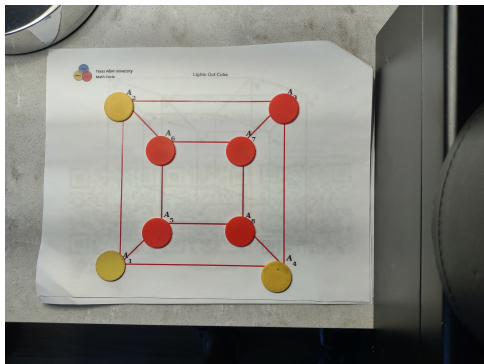
If only there had been a way to turn these two adjacent lights into lights with one off between them :'(



On To the Cube

Place one of each light on each of the vertices of your cube

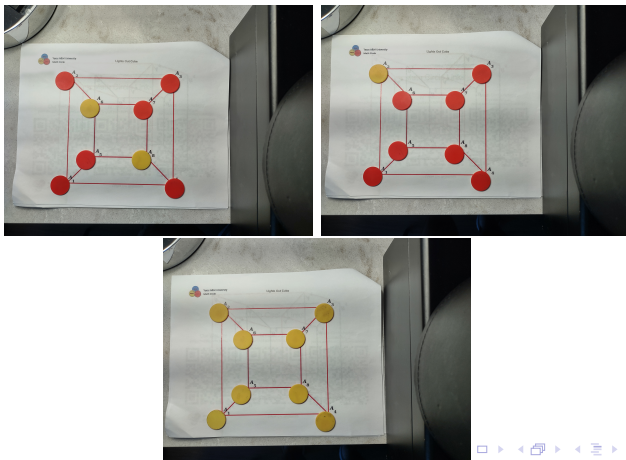
The rules of the game are now the same, except you notice that there are *three* adjacent lights to each light (instead of *two* in the previous game)



Play It Again

Remember: *toggling a light doesn't change the light itself; it changes the lights adjacent to it!* The goal is to get all of the lights off.

About what percentage of starting configurations can be solved on the cube? Previous answer for octagon was $\sim 30\%$



What Do You Think the Answer Is?

About what percentage of starting configurations can be solved on the cube?

What Do You Think the Answer Is?

About what percentage of starting configurations can be solved on the cube? It's 100%.

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What Can We Discover?

The octagon and cube are only some of the many Lights Out! puzzles we can discover. The hypercube (shown in your handout) is an example of another puzzle on which Lights Out! can be played.

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The following are characterizations of a Lights Out! puzzle where every starting configuration is solvable:

All starting configurations with exactly one light on are solvable (if the arrangement of lights is symmetric, like with the octagon or cube, this is simplified greatly)

Schwinghamer, Peters, and Cathain (Math Monthly No. 7, 2022) - the adjacency matrix of the puzzle is invertible as an element of matrices with elements in \mathbb{Z}_2

Other Ideas for Lights Out

What about the hypercube?

What about other shapes? Grids? *Graphs*?

What if lights didn't go on then off? What if they went *red, green, then off* (three toggled states)?

Math Circle Links

Our Website

(<http://mathcircle.tamu.edu/>)



MYMathApps

(tinyurl.com/MYMathApps)



Math Circle Activity
Spreadsheet

(tinyurl.com/MathCircleSpreadsheet)

