Can we untangle a knot?

Math Circle Activities to Introduce Knot Equivalence

1/4/2026

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TEXAS A&M UNIVERSITY

TAMU Math Circle

Organized by Dr. Phil Yasskin, Dr. Kun Wang, Dr. Sinjini Sengupta, Dr. Xin Liu, Dr. Guoliang Yu and Dr. Sherry Gong

- Designed for three different groups based on students' ages/math background
- Discovery Sessions (in person)
- Problem solving Sessions (hybrid)

PReMa --- Program for Research in Mathematics

Organized by Dr. Kun Wang, Dr. Zhizhang Xie, Dr. Wencai Liu,
Dr. Sherry Gong and Dr. Michael Willis

- An online research program designed for high school students across the country
- Students meet regularly and do research in groups
- Research are led by faculty members at TAMU

pre

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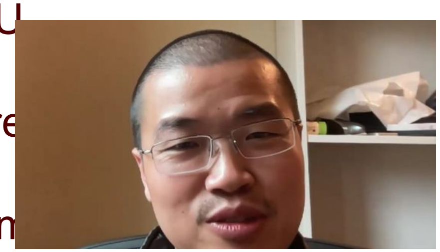
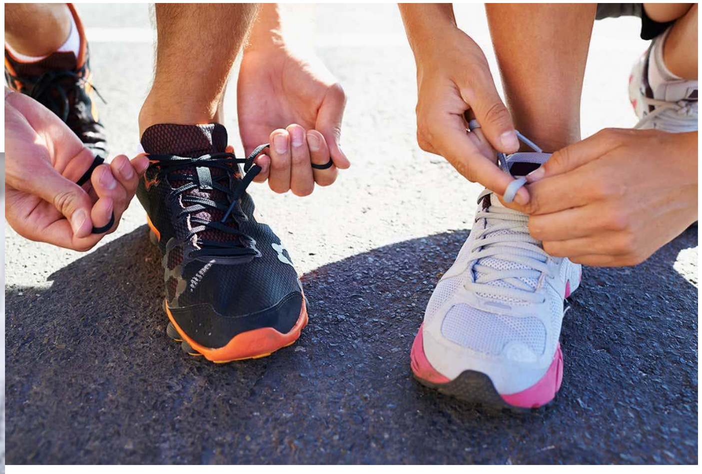
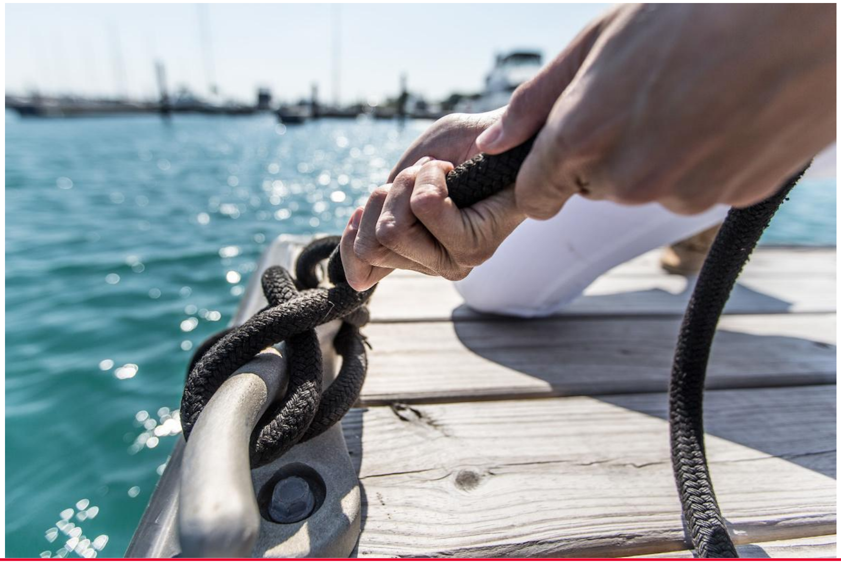


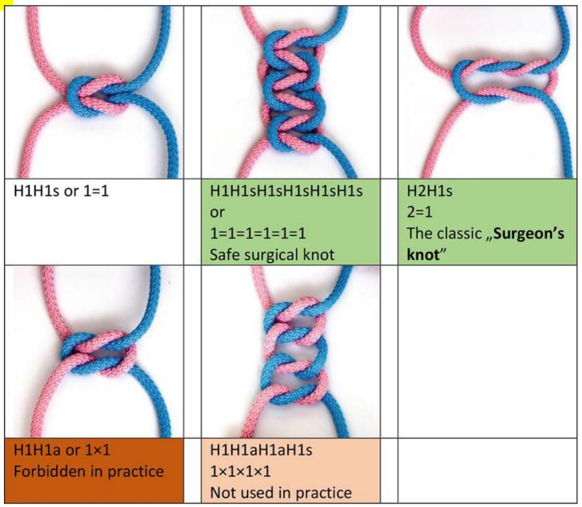

Emily Dad
Nik Rana

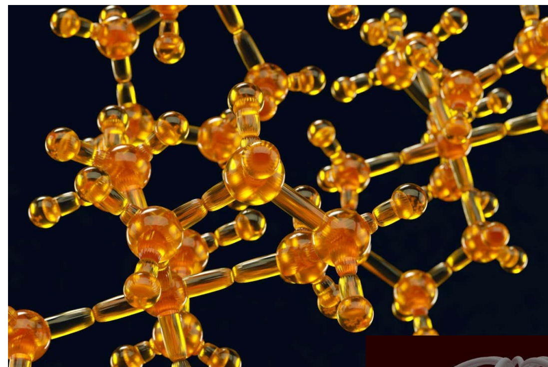
Knots in Daily Life

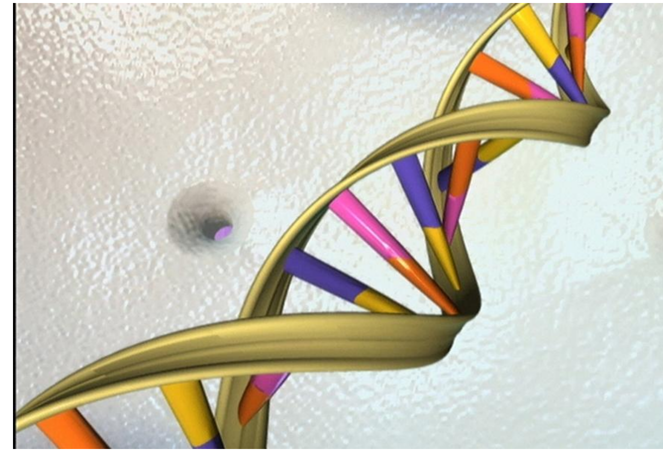


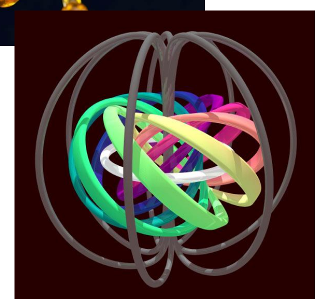


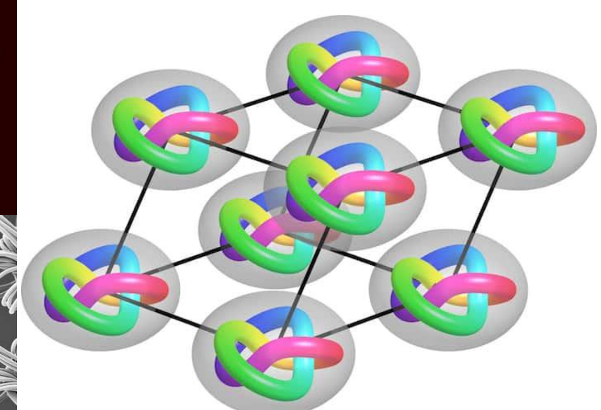
Knots in Chemistry, Biology, Physics, Computer Science, Medicine etc.

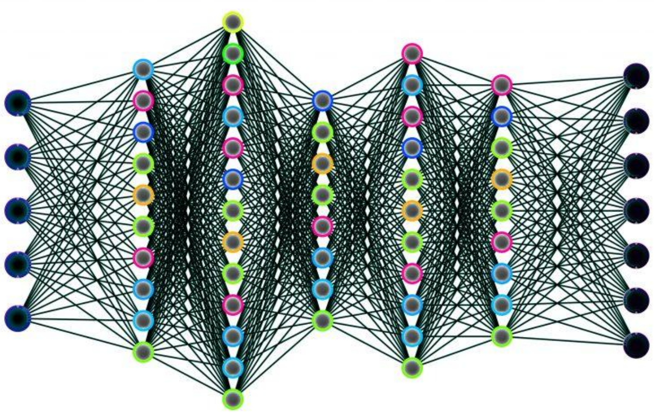


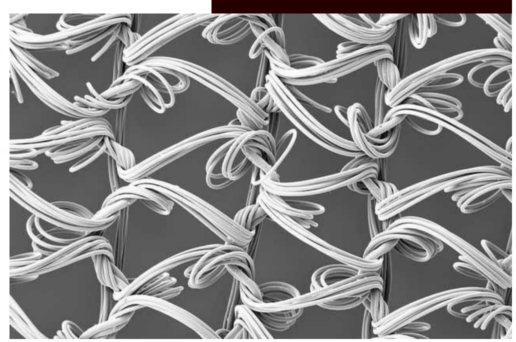












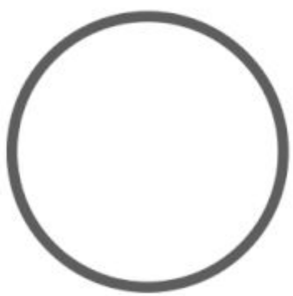
Definition of Knot in Topology

From a “tie”, we’d like to close the ends to make a mathematical knot.

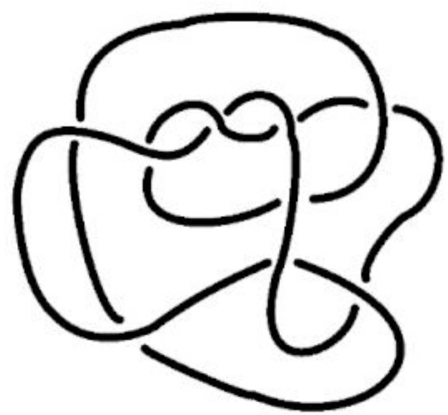
A knot is a closed loop in three-dimensional space that does not intersect itself. It can be represented mathematically as a simple closed curve.

Knot Equivalence

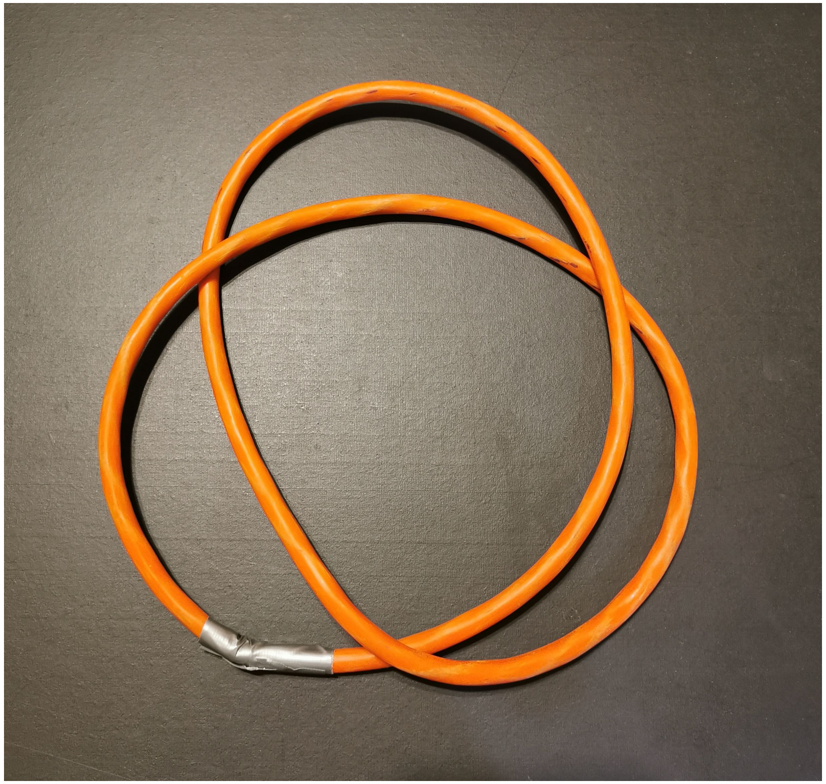
If two knots can be deformed into each other without cutting or passing through itself, they are equivalent.



Unknot



Activity #1: Judging whether two knots are equivalent or not



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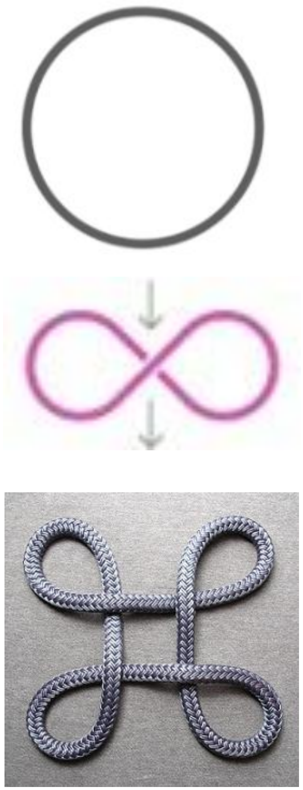
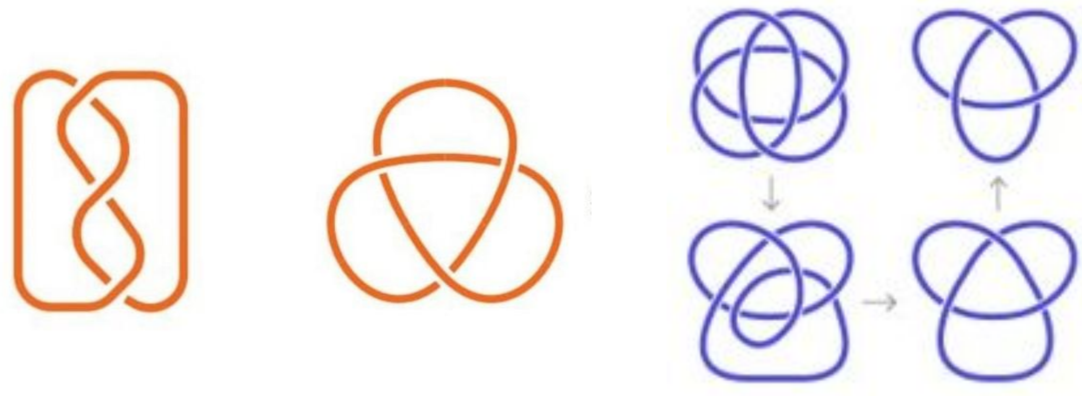
Knot Equivalence



Unknot



Trefoil

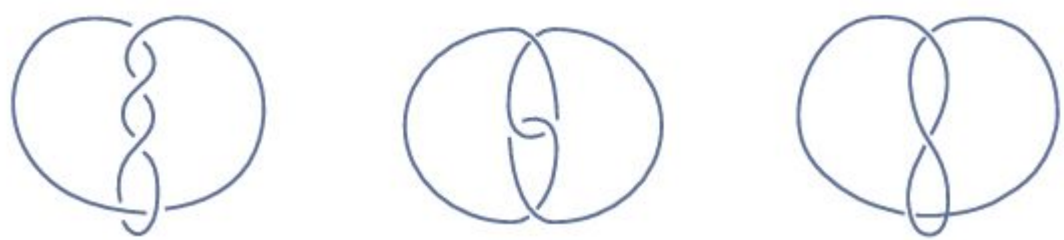
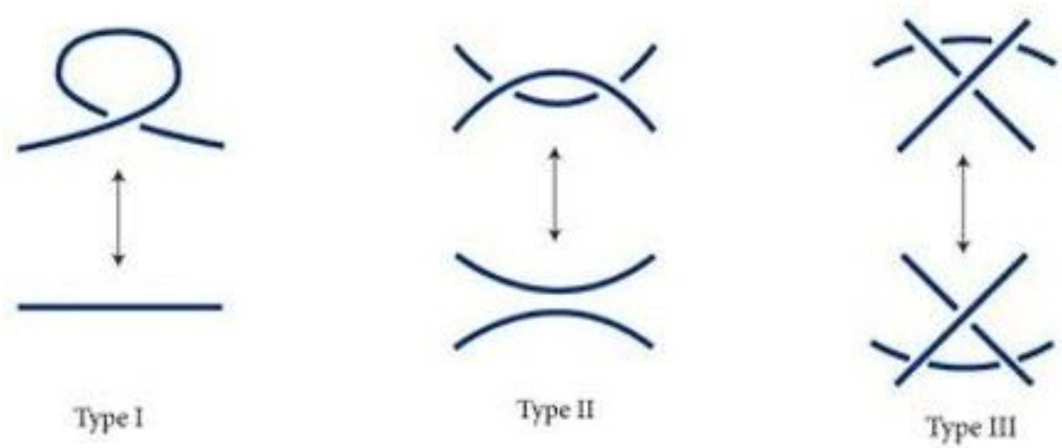


Figure 8 knot

Reidemeister Moves

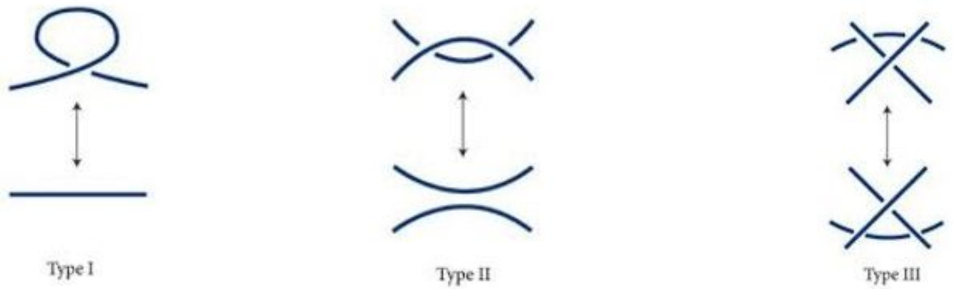
Reidemeister moves are moves that do not change the knot type, allowing for knot equivalence.

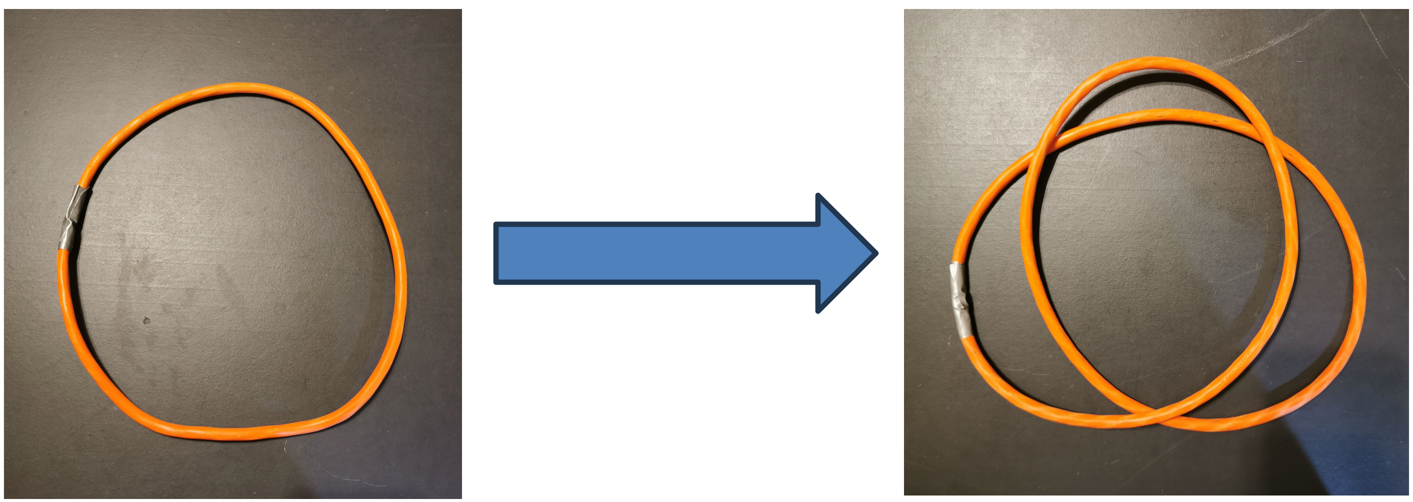


Kurt Werner Friedrich Reidemeister(1893-1971) was a German mathematician. Reidemeister's interests were mainly in combinatorial group theory, combinatorial topology, geometric group theory, and the foundations of geometry (Wikipedia)

- Reidemeister Move I (twist): adds or removes a crossing
- Reidemeister Move II (poke): adds or removes two crossings
- Reidemeister Move III (slide): slides a strand over or under a crossing

Activity #2: "Tangling" or "un-tangling" to change a knot to another form; identify the Reidemeister Moves





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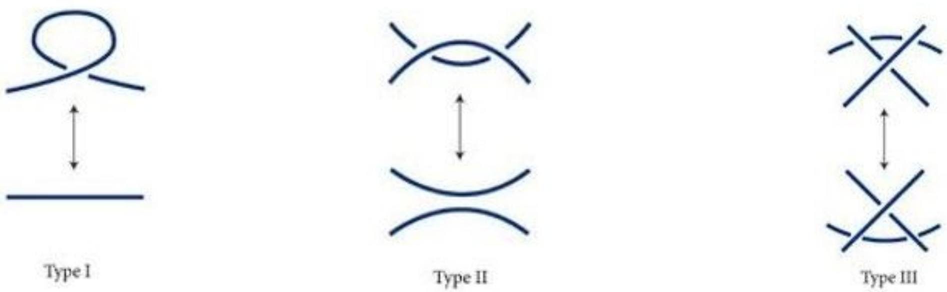

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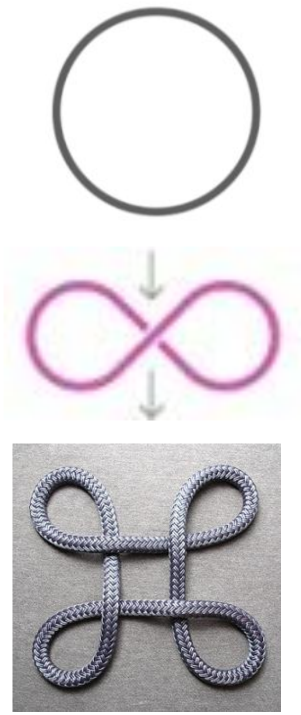
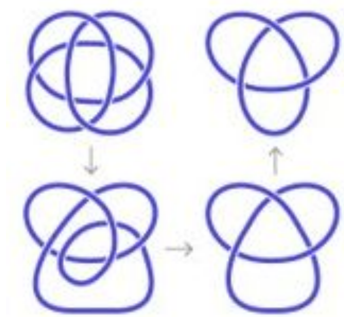
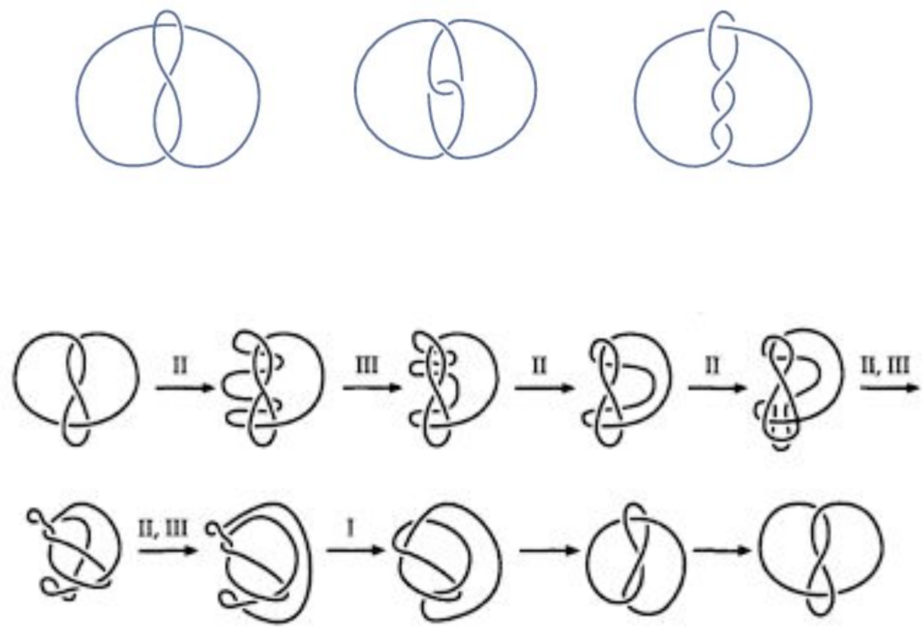




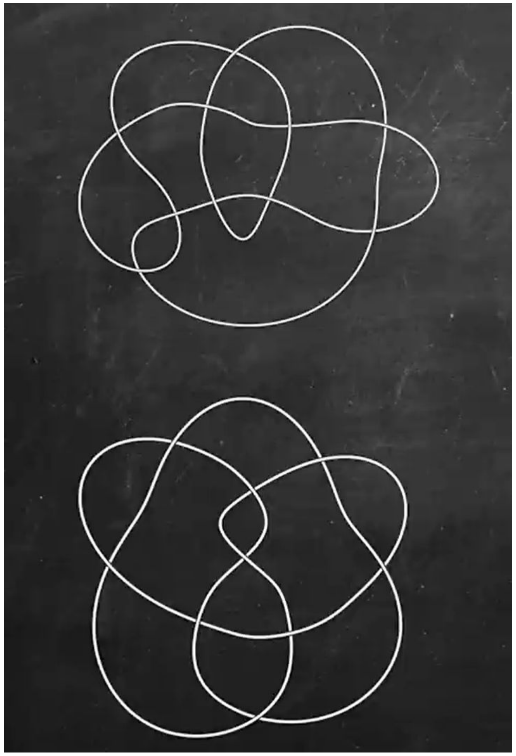
Trefoil

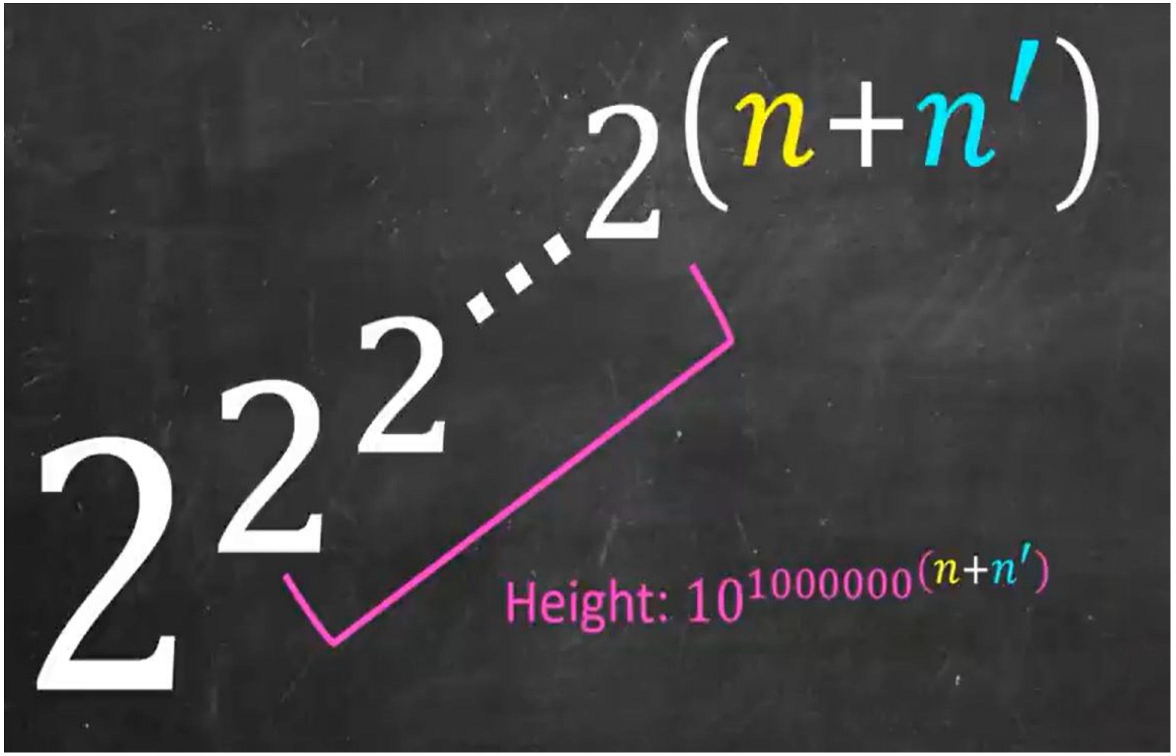


Unknot

Figure 8 Knot

How many Reidemeister Moves to identify the knot equivalence?





Knot Invariants

- Knot invariants are properties or values assigned to knots that remain unchanged under knot equivalence.
- Knot invariants help distinguish between different (non-equivalent) knots.
- All the discovered knot invariants are “incomplete”, that means they may assign the same value to different knots.

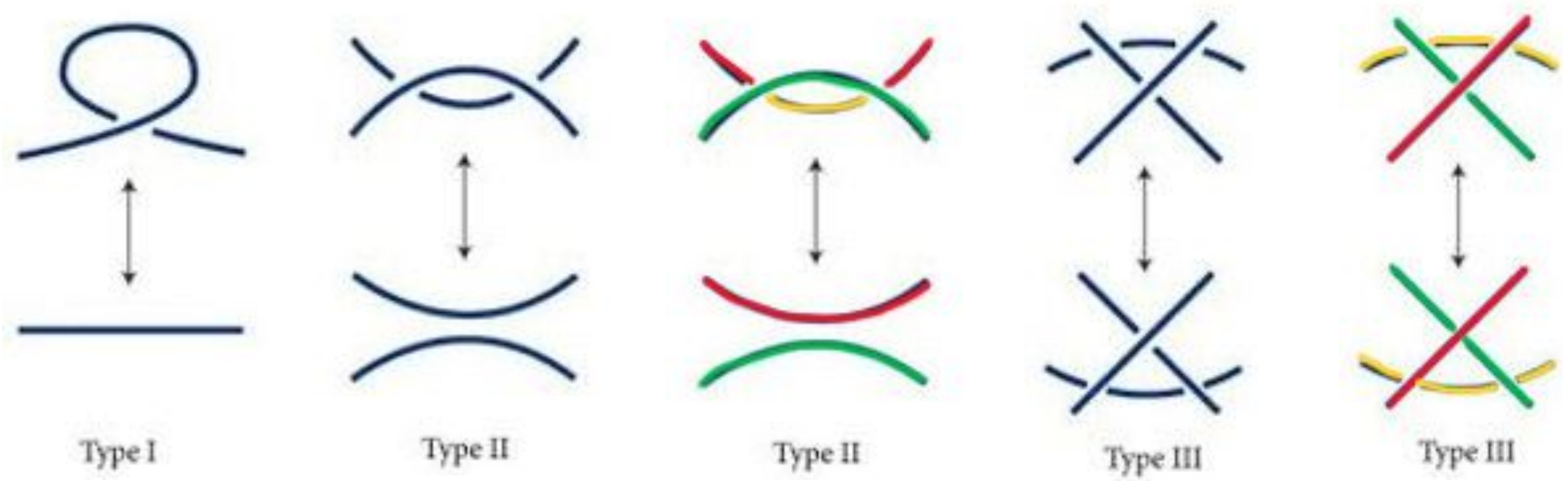
Tricolorability

A knot is **tricolorable** if each strand of the knot diagram can be colored one of three colors, subject to the following rules:

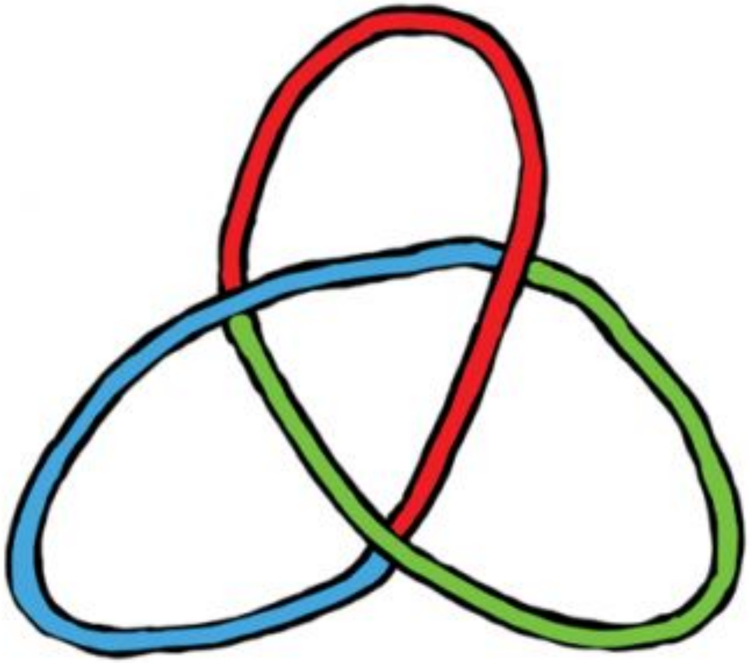
- At least two colors are used
- At each crossing, either
  (i) all three colors are used; or
  (ii) only one color is used.

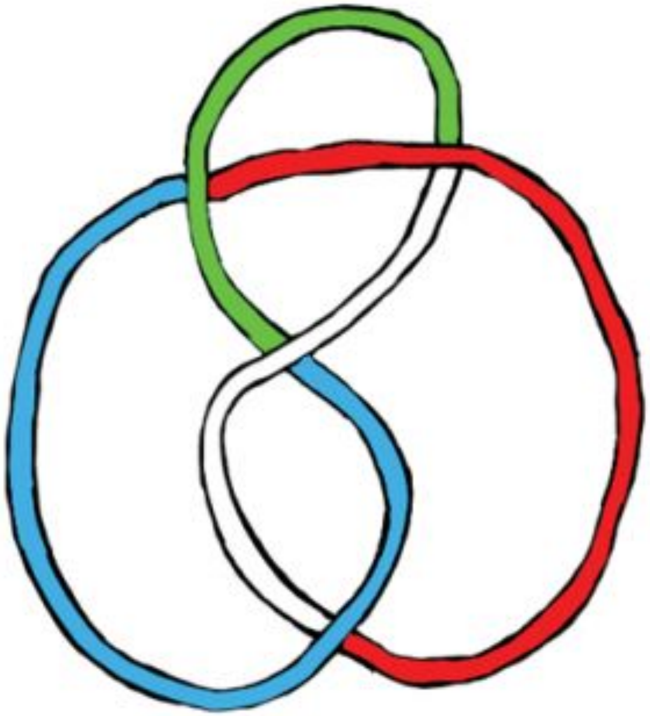
Theorem: Tricolorability is knot invariant.

Activity #3 (for the advanced group): Proving all three types of Reidemeister Moves reserves Tricolorability



Activity #4: Construct knots by wires; Judging whether two knots are equivalent by Tricolorability





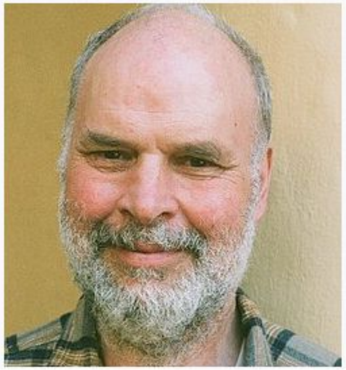
Activity #3: Judging whether two knots are equivalent by Tricolorability

Polynomial Invariants

- Alexander Polynomials,1923
- Jones Polynomials, 1984
- HOMFLY-PT Polynomials, 1984

James Waddell Alexander II

Sir Vaughan Jones
KNZM FRS FRSNZ FAA



Other activities

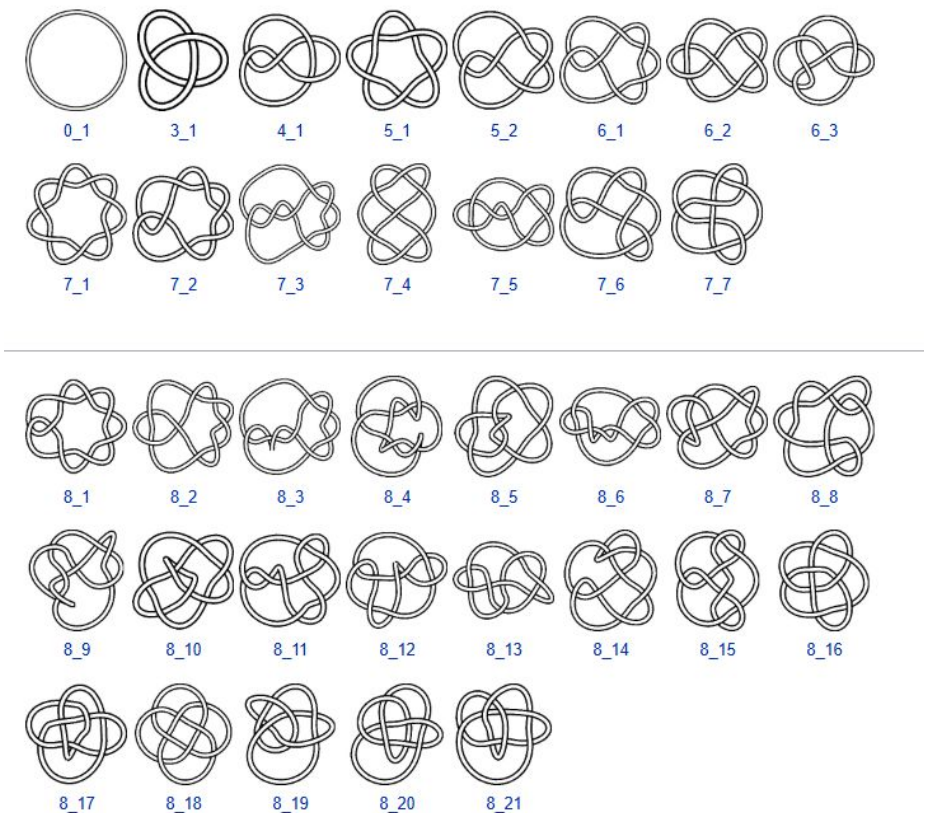
- Discuss why there isn’t any 1-crossing or 2-crossing knots, i.e. all 1-crossing and 2-crossing knots can be untangled to a circle(unknot)

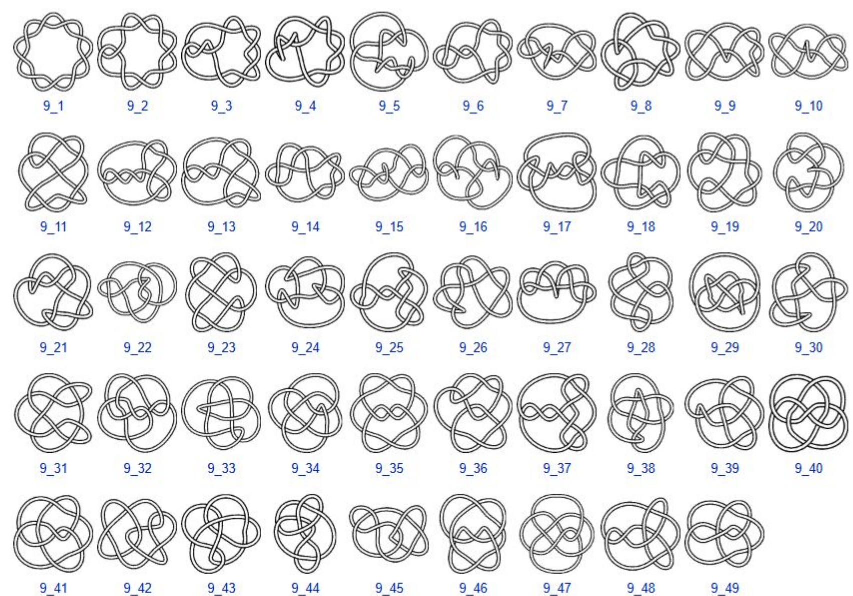
- Discover other knots with 3-crossing knots that is not Trefoil and 4-crossing knots that is not Figure-8 Knot (there will be no others)

- Discover all the different knots with 5 crossings, or 6 crossings, or 7 crossings etc.

The Rolfsen Knot Table, 1976

https://katlas.org/wiki/The_Rolfsen_Knot_Table





Thank you for listening!

Questions? Comments?

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