Fibonacci Patterns and Parastichy Numbers: Playing with Clay and Pinecones in a Math Modeling Course

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Mathematical Models: Course Overview

- Applied math course focusing on the construction, analysis, and interpretation of mathematical models of real world phenomena.
- Interdisciplinary course with applications to biology, climate science, ecology, finance, medicine, physics, astronomy, and sociology. Many applications with an environmental theme (course counts for credit toward ENVS major/minor).
- Project Course: in lieu of a final exam, students do a group final project and presentation modeling a topic of their choice.
- Textbook: Topics in Mathematical Modeling, K. K. Tung, Princeton University Press (2007).
- Mathematics: Wide range — counting, induction, geometric series, elementary ODEs, bifurcations, PDE climate models

The Fibonacci Numbers

Definition

The Fibonacci numbers are the numbers in the sequence

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots .$$

This is a recursive sequence defined by the equations

$$F_0 = 1, \; F_1 = 1, \quad \text{and} \quad F_{n+2} = F_{n+1} + F_n \quad \text{for all } n \geq 0.$$

Also known as the Hemachandra numbers. Indian scholars such as Gopala (c. 1135) and Hemachandra (1089–1173) discovered the sequence in their studies of cadences (long and short syllables) in Sanskrit poetry.

Fibonacci Numbers in Art

Figure: The chimney of Turku Energia in Turku, Finland, featuring the Fibonacci sequence in 2m high neon lights (Mario Merz, 1994).

Figure: Fibonacci Cubes (Petra Paffenholz, 2014), a sculpture situated on two meadows near Lake Dümmer (Germany), consisting of nine iron cubes with dimensions based on the Fibonacci numbers.

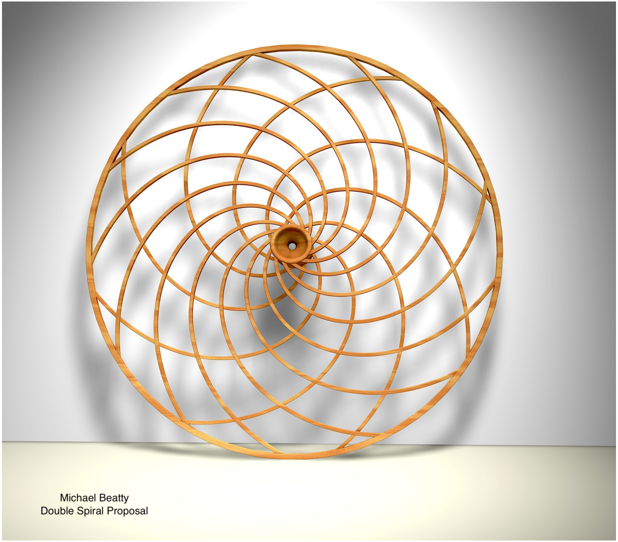


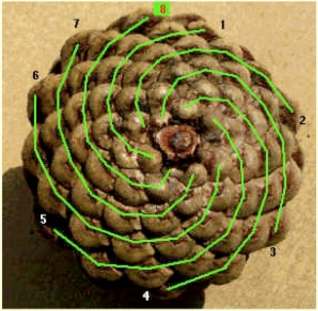
Figure: Double Spiral (proposal) (Michael Beatty). What do you notice about the number of spirals in each direction?

Fibonacci Numbers in Nature

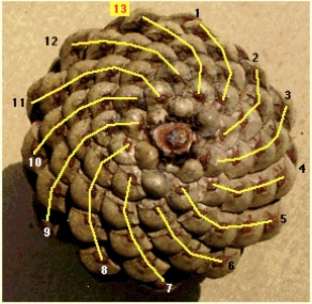
- Number of petals in many, many flowers: e.g., three-leaf clover, buttercups (5), black-eyed susan (13), daisies (21 or 34).
- Number of spirals, called parastichies, of a pinecone or pineapple in opposite directions are typically consecutive Fibonacci numbers.
- In many plants, the number of leaves and the number of turns between two leaves growing almost directly over each other are Fibonacci numbers (e.g., 13 leaves in 5 turns).
- Number of spirals in the seed heads on daisy and sunflower plants.
- This is not a coincidence! Some of these facts can be modeled mathematically using continued fractions and the golden ratio.


Figure: Red Columbine (left, 5 petals, source: ljhimages/iStock/Thinkstock); Black-eyed Susan (right, 13 petals, source: herreid/iStock/Thinkstock)

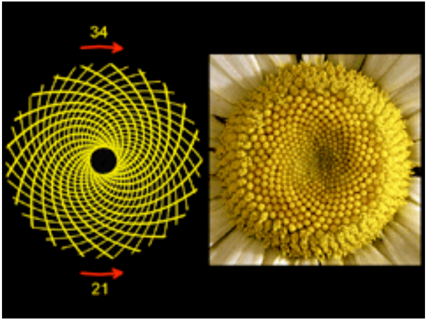

Figure: Chicory (left, 21 petals, source: ArminStautBerlin/iStock/Thinkstock); Sunflower (right, 34 petals, source: Racide/iStock/Thinkstock)



Adjacent Fibonacci numbers, 8, 13



The parastichy numbers for this pinecone are (8, 13).



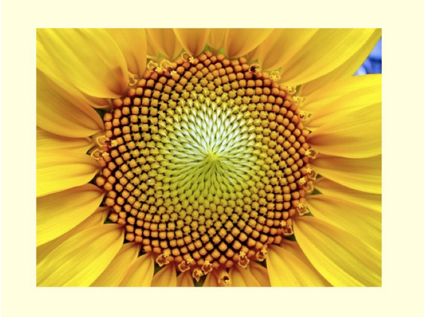
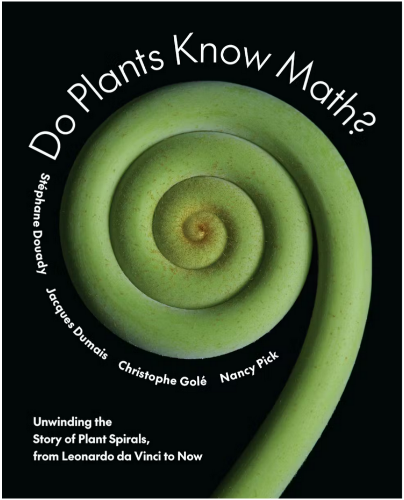


Figure: In the seed heads of most daisy or sunflower blossoms, the number of spirals in opposite directions are consecutive Fibonacci numbers.

The study of these mathematical patterns in plants and flowers is called Fibonacci phyllotaxis. It has a long and fascinating history (e.g., Leonardo da Vinci, Alan Turing).



Princeton University Press (2024)

How can mathematics help explain the prevalence of Fibonacci numbers in nature?

Example: The Golden Angle

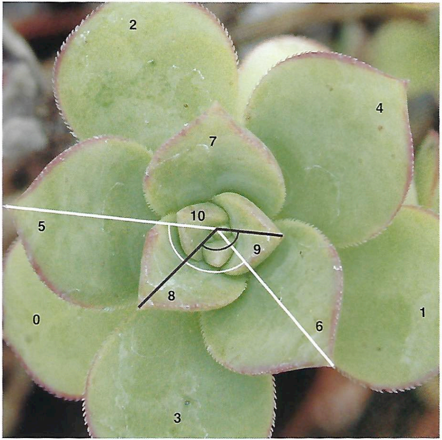


Figure: The divergence angle between leaves 5 and 6 (white) and between leaves 8 and 9 (black) in this Aeonium plant are each very close to 137.5°, the golden angle. Source: *Do Plants Know Math?* Douady, Dumais, Golé, and Pick

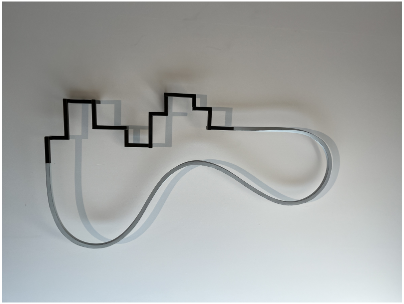
Book features several hands-on activities for readers to explore and gain a deeper understanding of Fibonacci phyllotaxis.

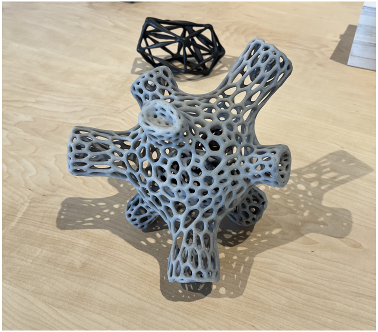
Above: Fill in the missing numbers as you count the seeds of this dahlia. Within red spirals, add/subtract 21; within green spirals, add/subtract 13. Amazingly, you get the same lattice either way.

Special Math Models Class

- Coordinated with Michael Beatty (HC art professor) and Christina An (Assistant Director for Education and Engagement in the HC Cantor Art Gallery).
- Beatty incorporates geometry, mathematical shapes (e.g., Möbius strip), polyhedra, and the golden ratio in his work. Uses computer software and 3D printing to produce intricate sculptures.
- At the time of the class, Beatty had an exhibit in the gallery entitled Fabrications: Selections from 1992 to present.
- Note: I had used Beatty's art in a previous offering of the course, asking students to find evidence of Fibonacci patterns and the golden ratio in the gallery for a different exhibit (HW problem).

Samples of work by Michael Beatty






L BEATTY
oricactions
Selections from 1982 to present
HOLY CROSS

Activity: Unrolling pinecones to find parastichy numbers

1. Find some pinecones that are fairly symmetric and closed up.
2. Roll out dough or modeling clay using a rolling pin, slightly wider than the height of your cone.
3. Mark a reference point on your cone with a sharpie or nail polish.
4. Roll the cone over the dough for at least one revolution making sure the marked points are visible.
5. On the clay, trace the parastichies in opposite directions by drawing parallel lines between reference points.
6. Count the number of lines in opposite directions to determine the parastichy numbers of the pinecone.

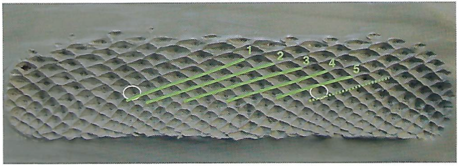


FIG. 0.12 Tracing the first set of parastichies.

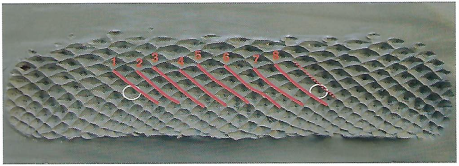


FIG. 0.13 Tracing the second set of parastichies.

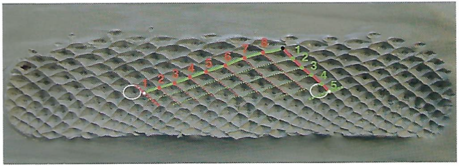


FIG. 0.14 Counting the steps reveals the parastichy numbers.

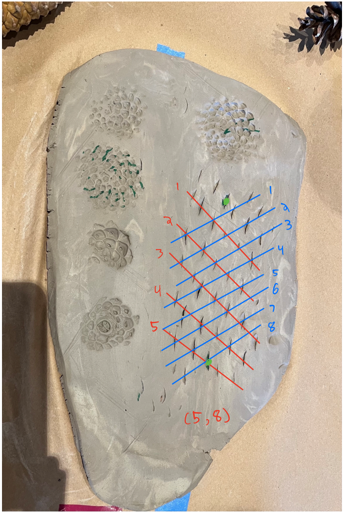
Students grouped into 3 tables, each covered with parchment paper.

Ingredients: Sample pinecones, modeling clay, rolling pin, sharpie, straight edge.


HOLY CROSS


HOLY CROSS

Results from two of the groups. Notice the multiple orientations and approaches students used to study different cones.



A cone with parasticky numbers (5, 8).

Surprise: At the end of class students started constructing 3D model clay pinecones — true mathematical modelers!

Some Reflections and Suggestions

1. Conducting creative activities outside the traditional classroom is fun and memorable. Can be a lot of work to organize, but definitely worth it!

2. Keep it fun. Doesn't have to be high level of difficulty. Can always follow up later with more in-depth material.

3. Reach out to your gallery and/or art department. Make connections — ask for help.

4. Surprises will occur. Things will definitely NOT go as planned. Practice the activity beforehand.

5. Find good pinecones! "You can make your cone close up by leaving it in a humid environment." Really?

Never leave a burning stove unattended!

Thank you for your attention!