

# Math Circle on the Lakeshore

Istvan Lauko and  
Gabriella Pinter  
University of Wisconsin- Milwaukee

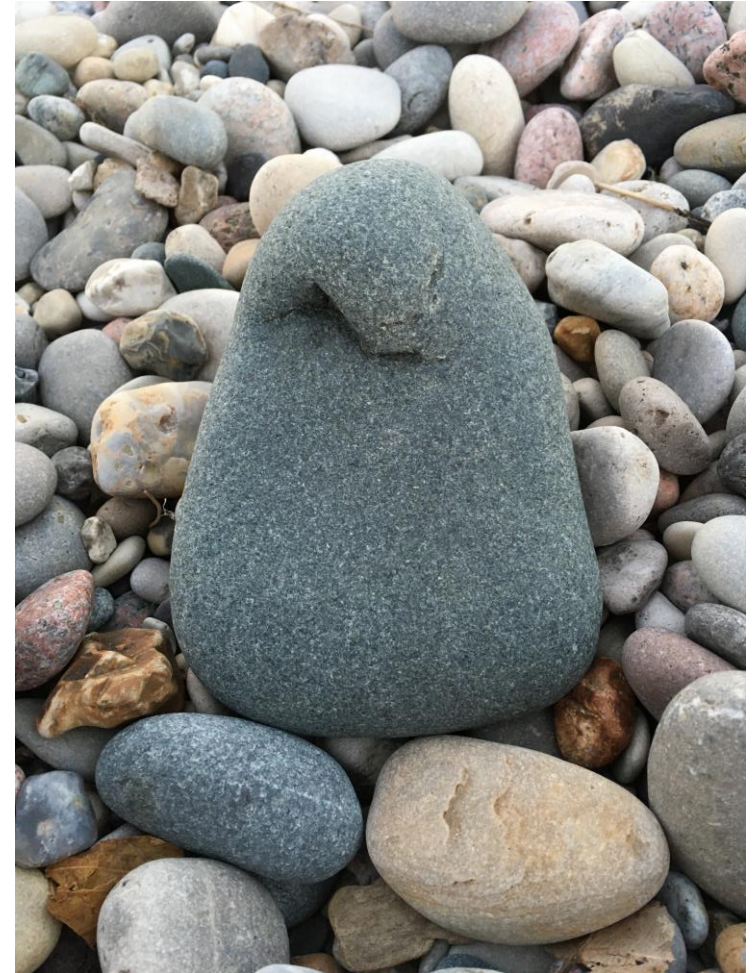
# Lake Michigan



# Lake Michigan now



# Lots of rocks



# Rocks and games



- Countdown
- NIM games
- Cookie Jar

# Countdown

A wonderful low-floor high ceiling JRMF activity

One pile of rocks – two people taking turns removing rocks from the pile according to some rule. The person making the last legal move wins.

- Take 1,2,3,4,5 or 6 rocks at a time
- Take 1,2,...,n rocks at a time
- Take 1 or n rocks at a time
- Take 1,3 or 6 rocks at a time

# Cookie Jar

There are jars with cookies in them, say 15 jars with 1,2,3,...,15 cookies in them.

Each day you can choose some jars and then take the same number of cookies from each jar.

Example: Day 1: from jars with 10,11,12,13,14 and 15 cookies, take 10 cookies each

Questions to ask?



# How can we empty 15 jars?

- 15 jars, 1,2,3,...,15 cookies in them
- E.g. take 1 from 1,3,5,7,9,11,13,15  
take 4 from 4,5,6,7,12,13,14,15  
take 8 from 8,9,10,11,12,13,14,15  
take 2 from 2,3,6,7,10,11,14,15

4 moves – is this the least?

- What is the minimum number of moves needed to empty the jars?
- What if the jars contain an arbitrary set of numbers?
- How could you place cookies in the jars so that it takes the maximum number of moves to empty them?
  
- Lower the floor: two jars, three jars, four jars
- Make it a game: two jars, two players taking turns; take some cookies from just one jar or the same number of cookies from both jars; player taking the last cookie wins!

# The Greedy Pickle Monster

Variation by Gord Hamilton (MathPickle)

- Pickle Monster chooses a number, then eats that many pickles from each jar that has at least that many in it.

- Example:

no. of pickles in jars: 2,3,4,5,7 – number chosen: 3

no. of pickles left: 2,0,1,2,4

How does the problem change?

# Pouring sand



# Pouring sugar

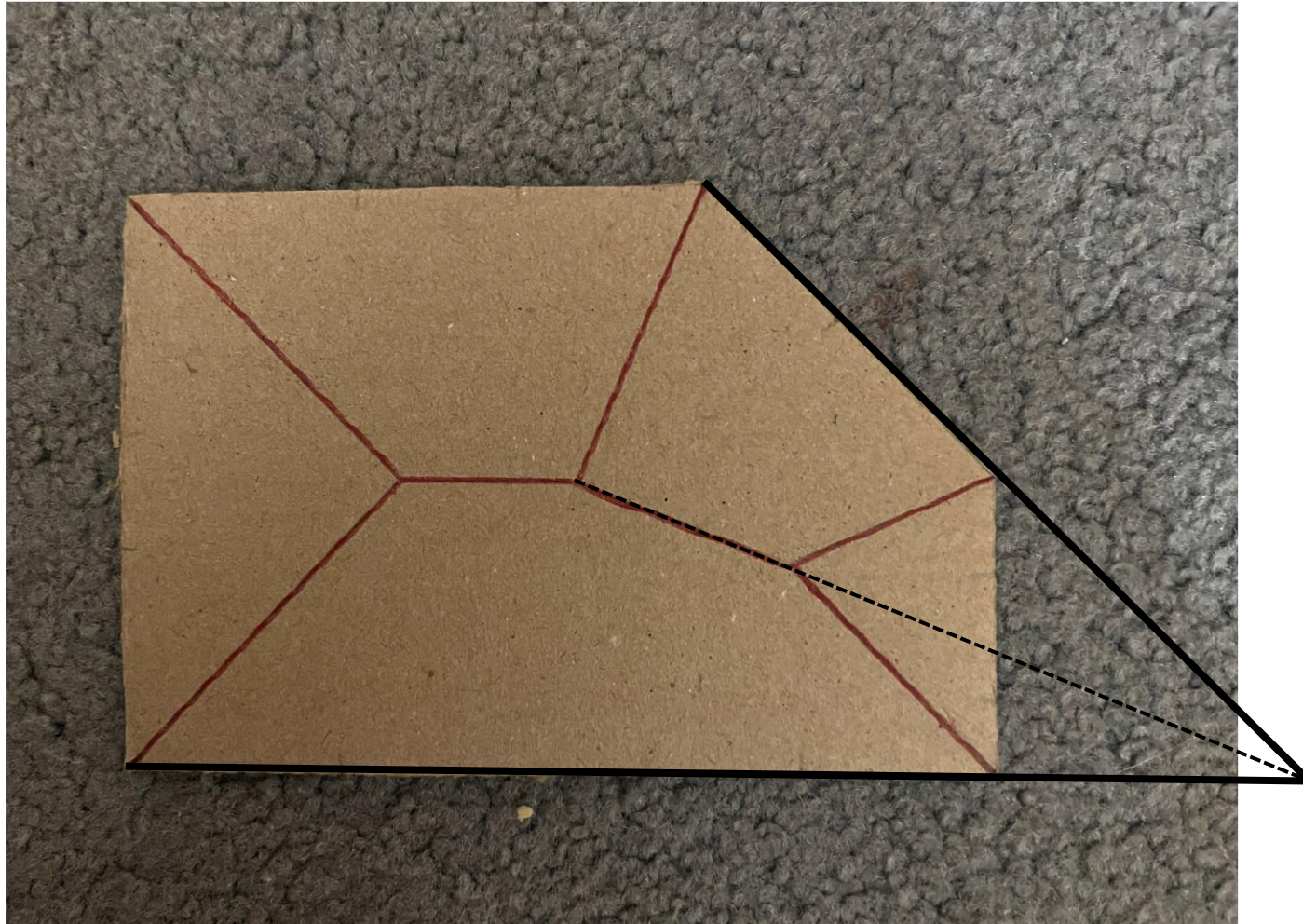




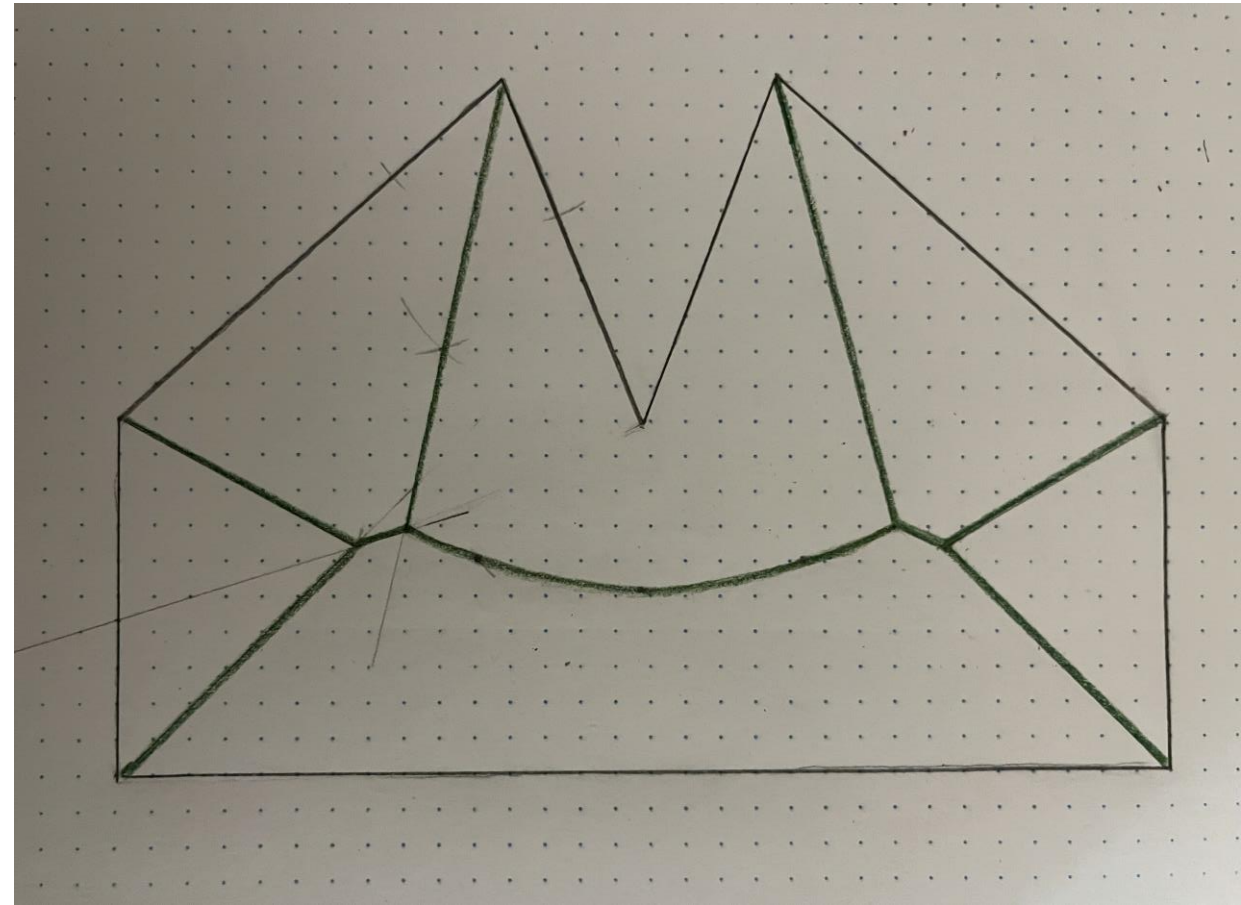
## Sand/sugar mountains

- Ridges – closure of points that have two or more closest points on the boundary
- Facets slope at the same angle with respect to the polygon
- Medial axis

# Convex polygons



# Concave polygons



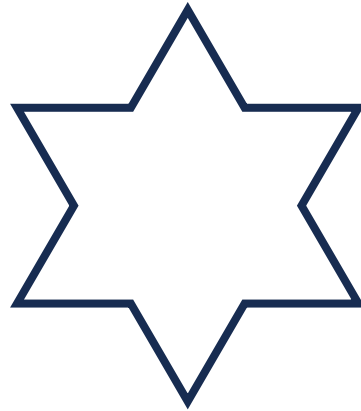
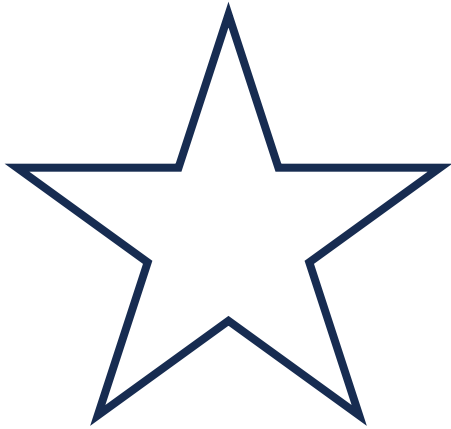
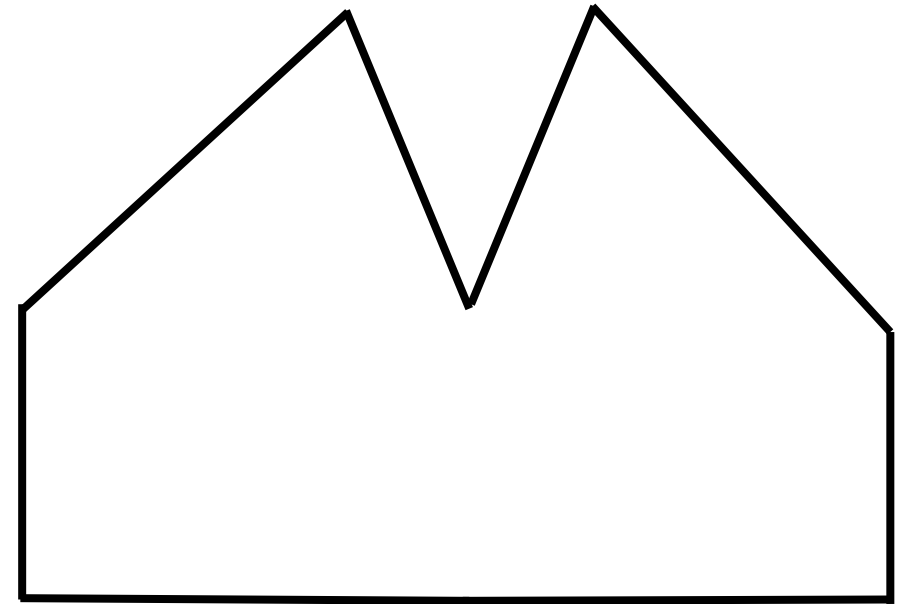
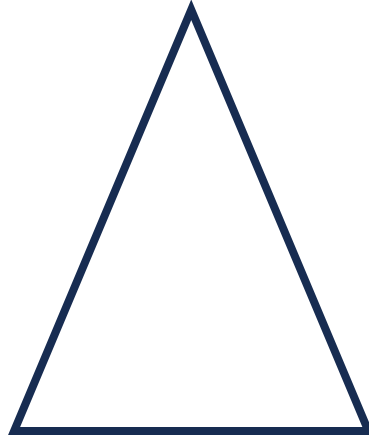
# Folding and cutting

Given one of these polygons on a piece of paper, you want to cut the polygon out (and only the polygon)

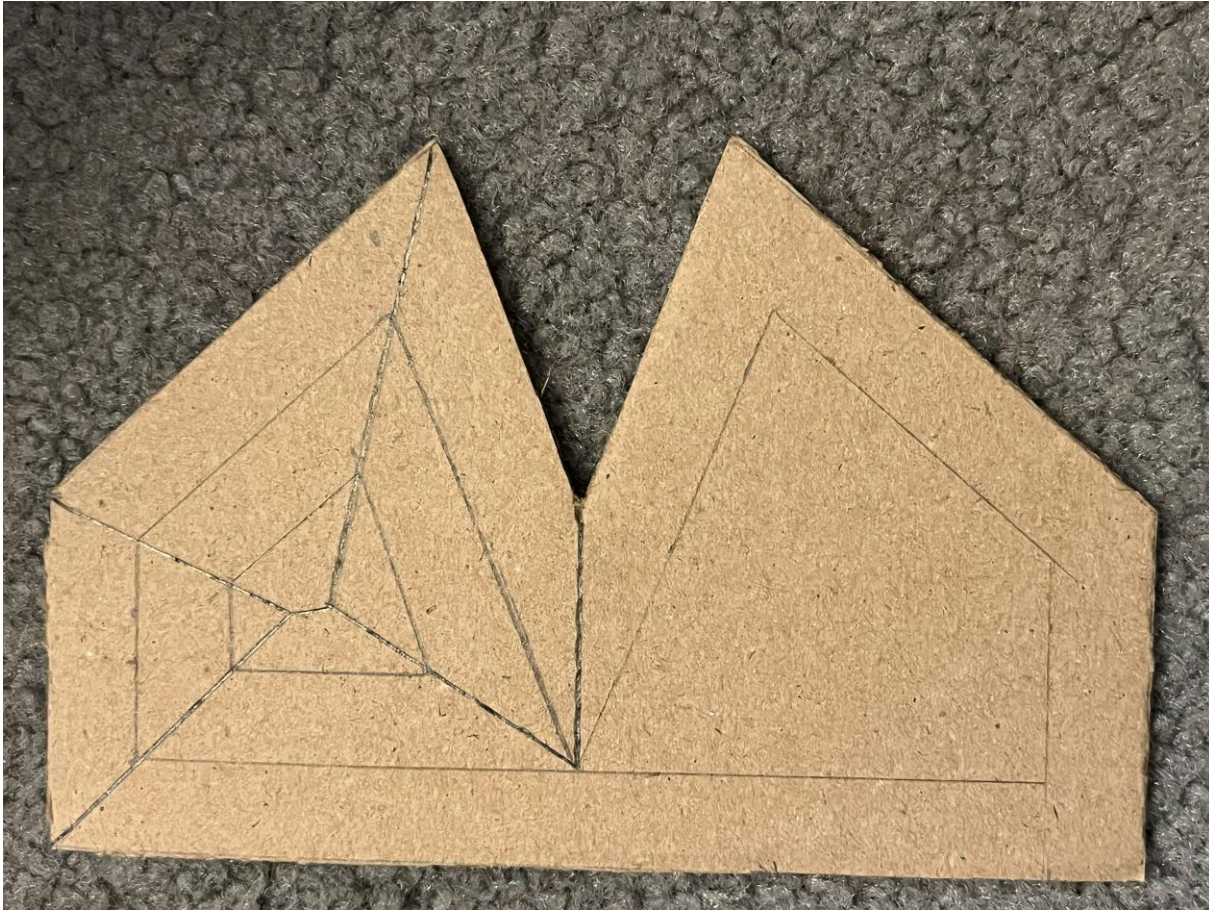
The catch: you can only use one straight cut

Try it!

# Can you cut out these shapes with one straight cut?

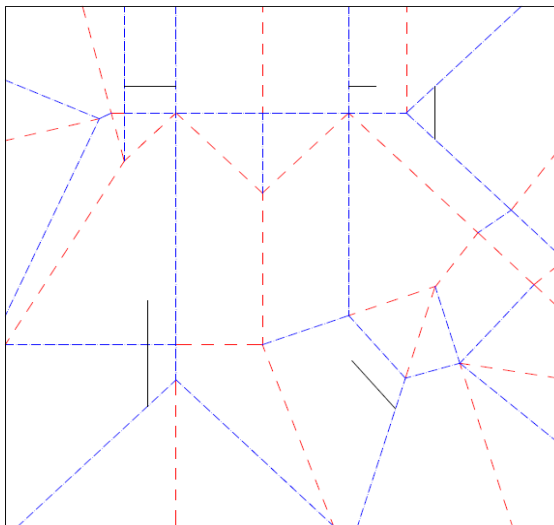


# Straight skeleton

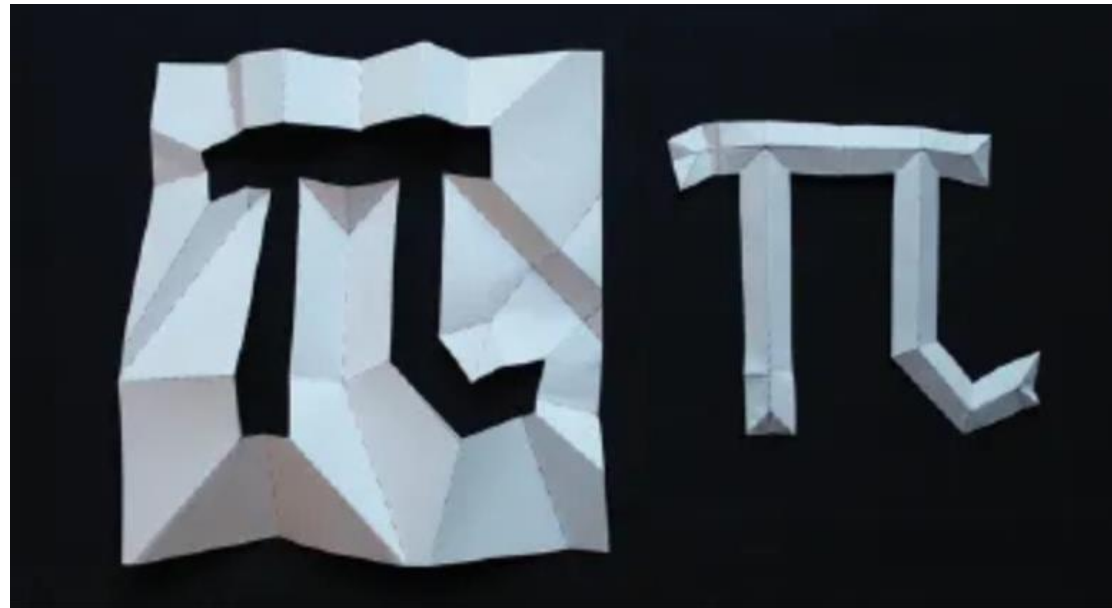


# Fold and One-Cut Theorem

Any straight-line drawing on a sheet of paper may be folded flat so that one straight scissors cut completely through the folding cuts all the segments of the drawing and nothing else. (E. Demaine et.al.)



The dashed red lines are valley folds and the blue dot-dashed lines are mountain folds. Once all creases are folded, the black line segments will line up, and a single cut can be made along this line. Unfold and see what you've made!  
Created by: Todd Nelson (2021)



# Thank you!

## References:

<https://jrmf.org/puzzle/countdown/>

Paul Vanderlind, Loren Larson, Richard Guy: The Inquisitive Problem Solver, MAA Problem Book Series, 2002;

MathPickle – Gord Hamilton <https://mathpickle.com/project/cookie-monster-problem/>;

Troy Jones, Darryl Yong: Salt Mountain Geometry, Math Horizons, November 2018;

Joseph O'Rourke: How to Fold It, Cambridge University Press, 2011;