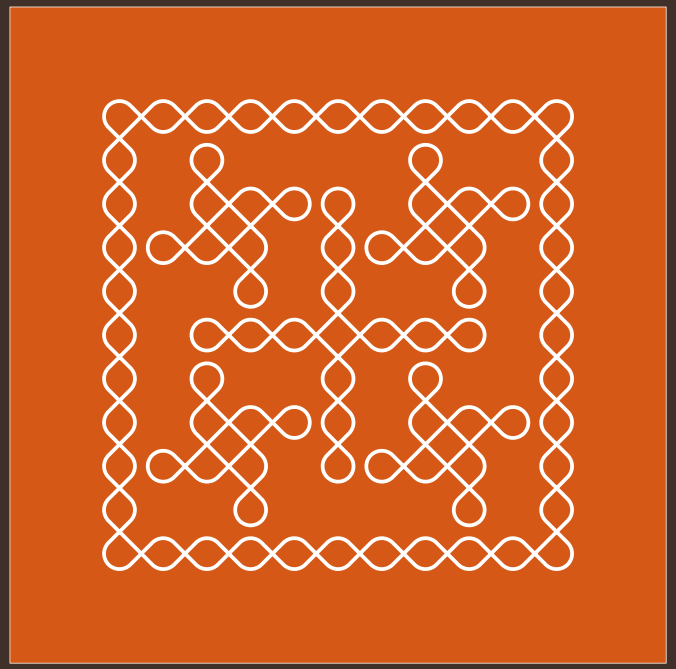
Deconstructing Rangoli



Harish Kulkarni & Tanmay Kulkarni

AIM Special Session on Circles in Motion

JMM 2026

Lines of Color

- "***Rangavali***" in Sanskrit – *Lines of Color*
- **2000-year-old folk art**
- Ephemeral floor art

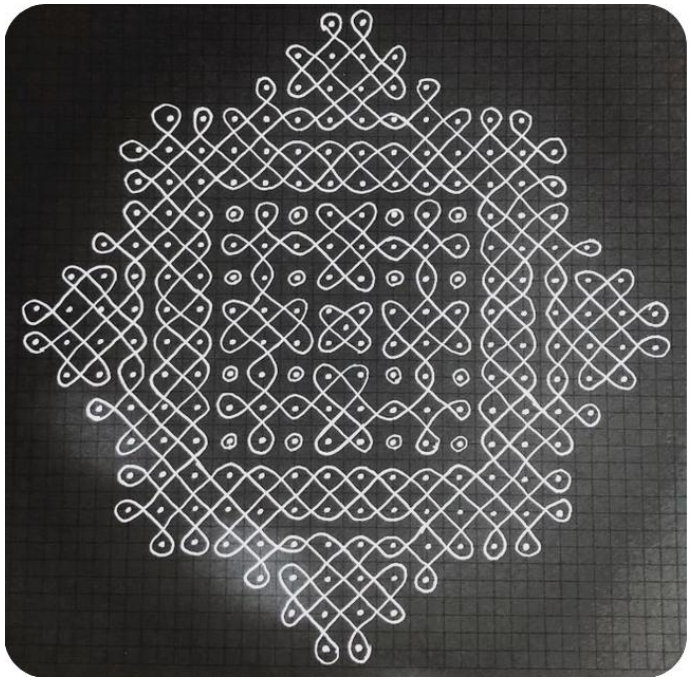
Lines of Color

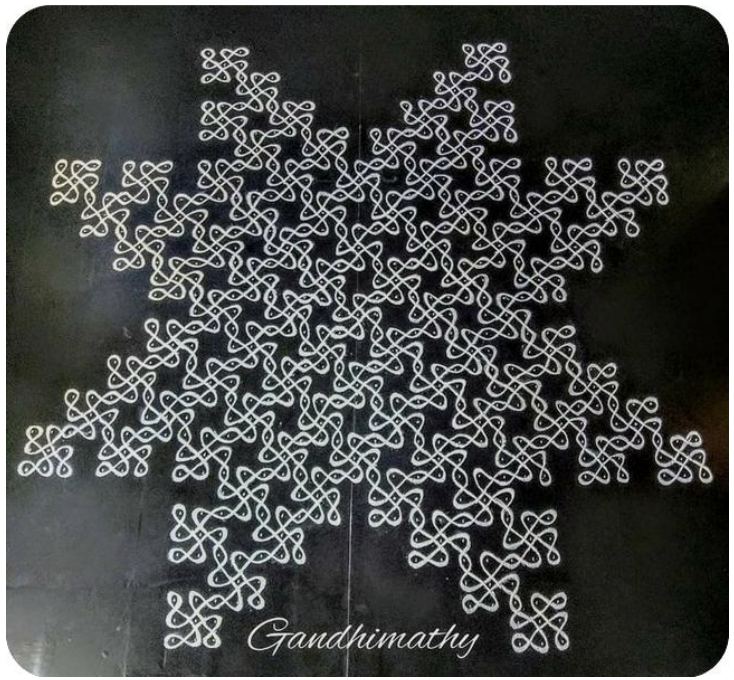
- **2000-year-old folk art**
- Colored rice powder or colored sand
- Flowers, lamps

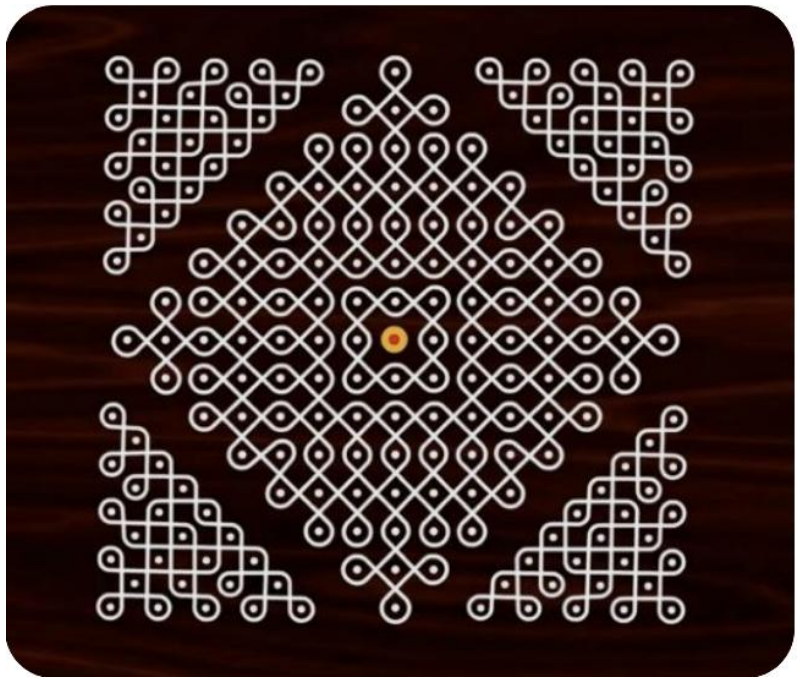
- **2000-year-old folk art**
- Still ubiquitous
- Thresholds

Lines of Color

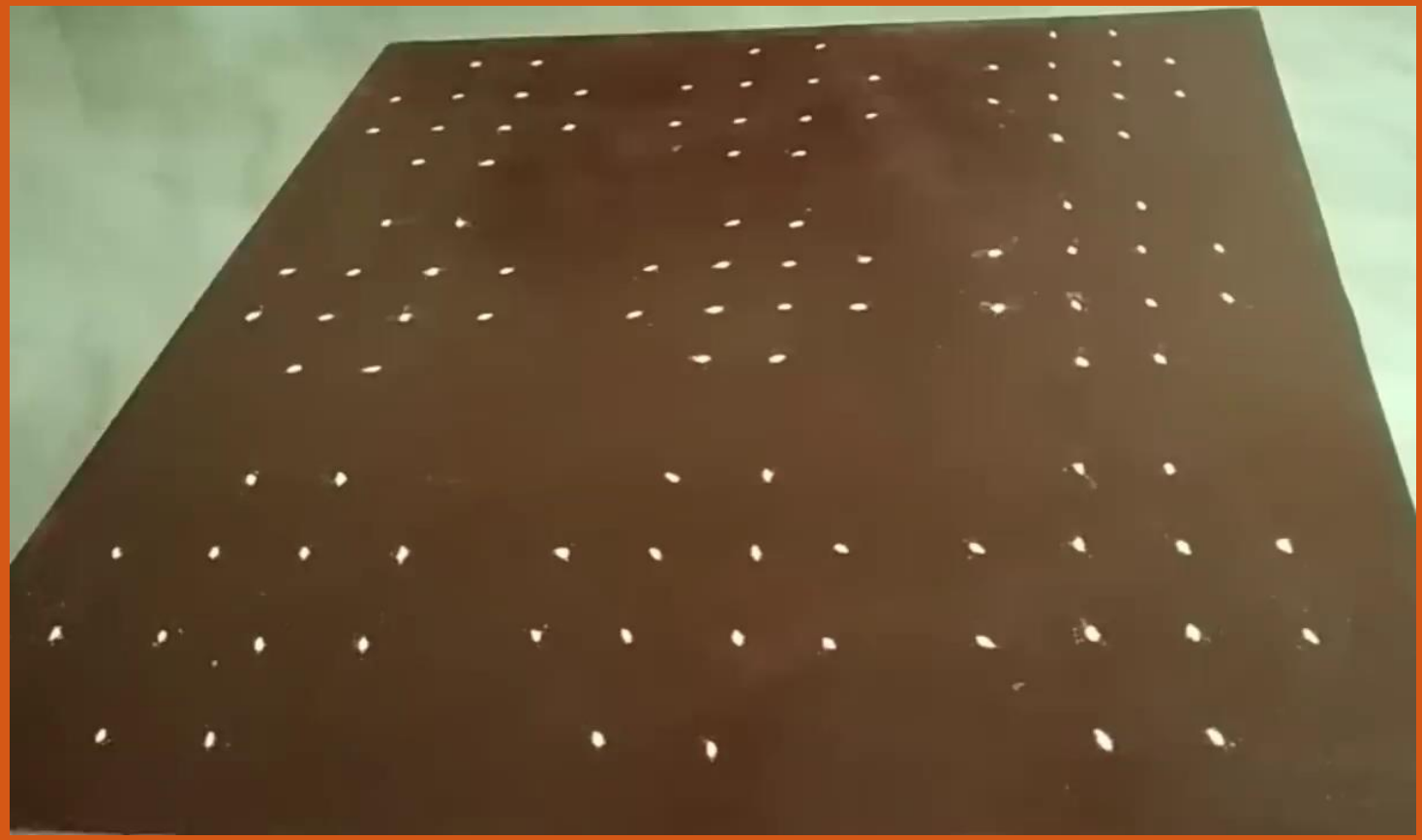
Zooming into Dot Rangoli





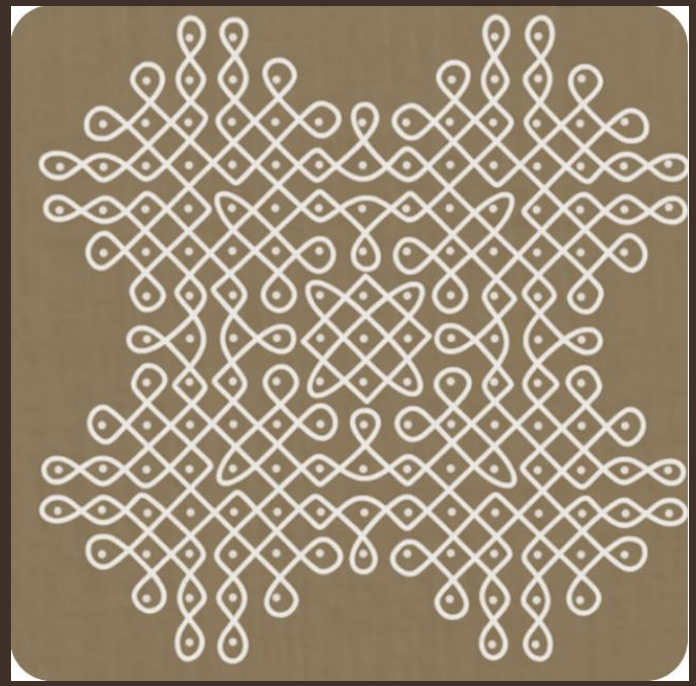


Traditional Dot Rangoli

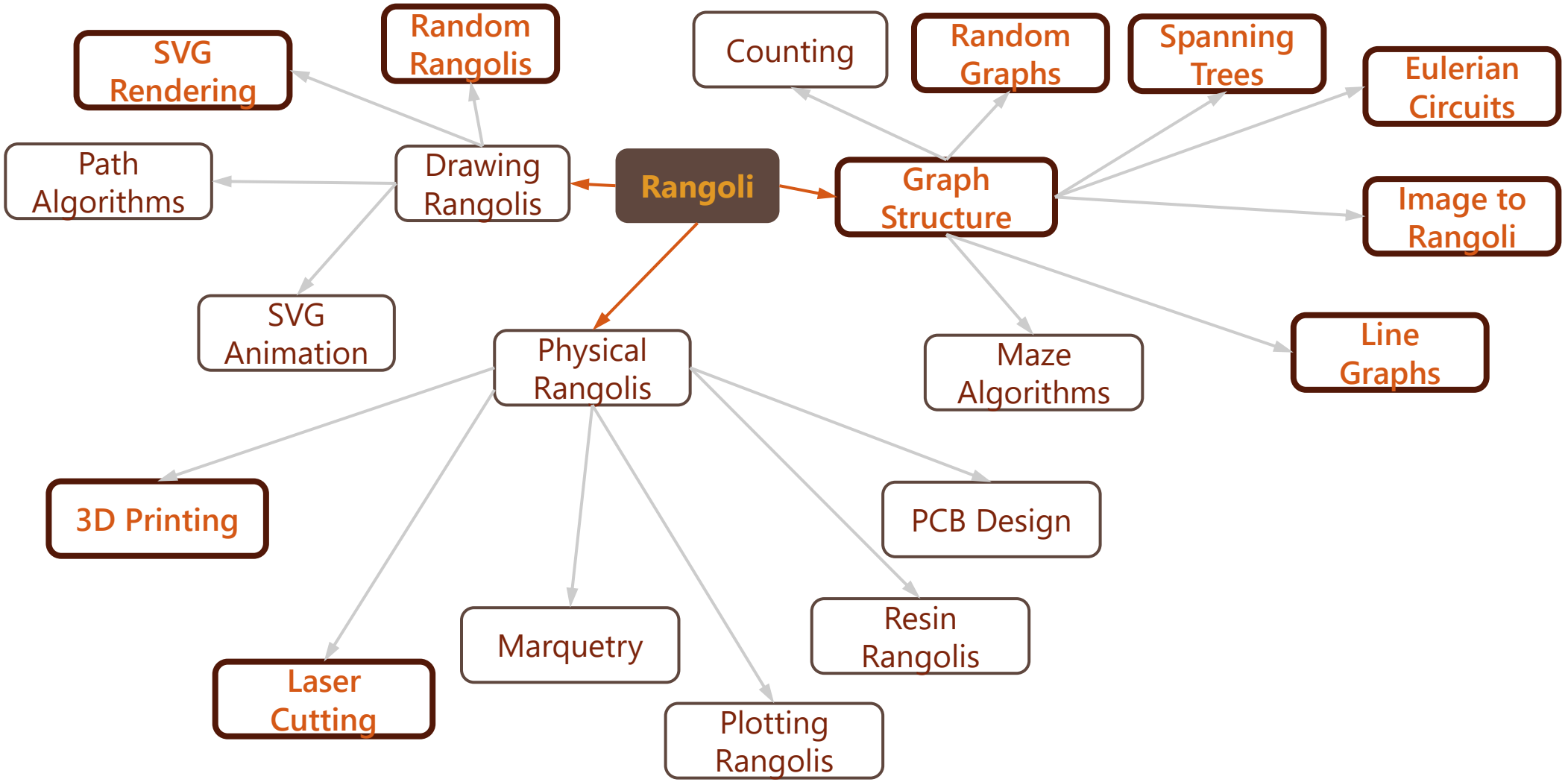


The question that started this journey

Is this a purely artistic endeavor or is there a mathematical structure behind these patterns?

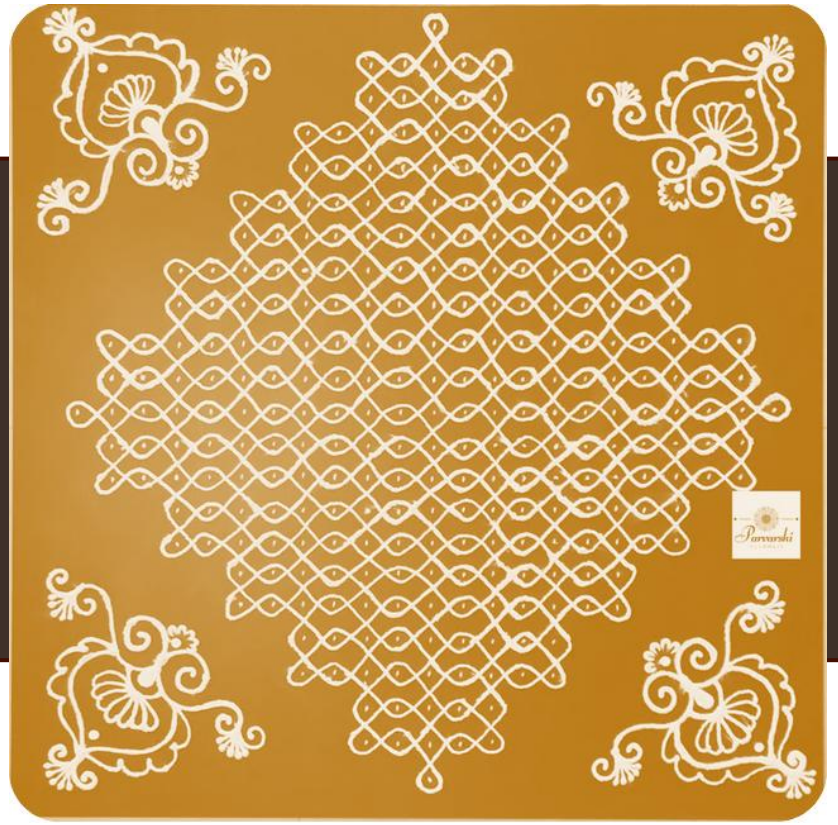


Our journey through exploration & play

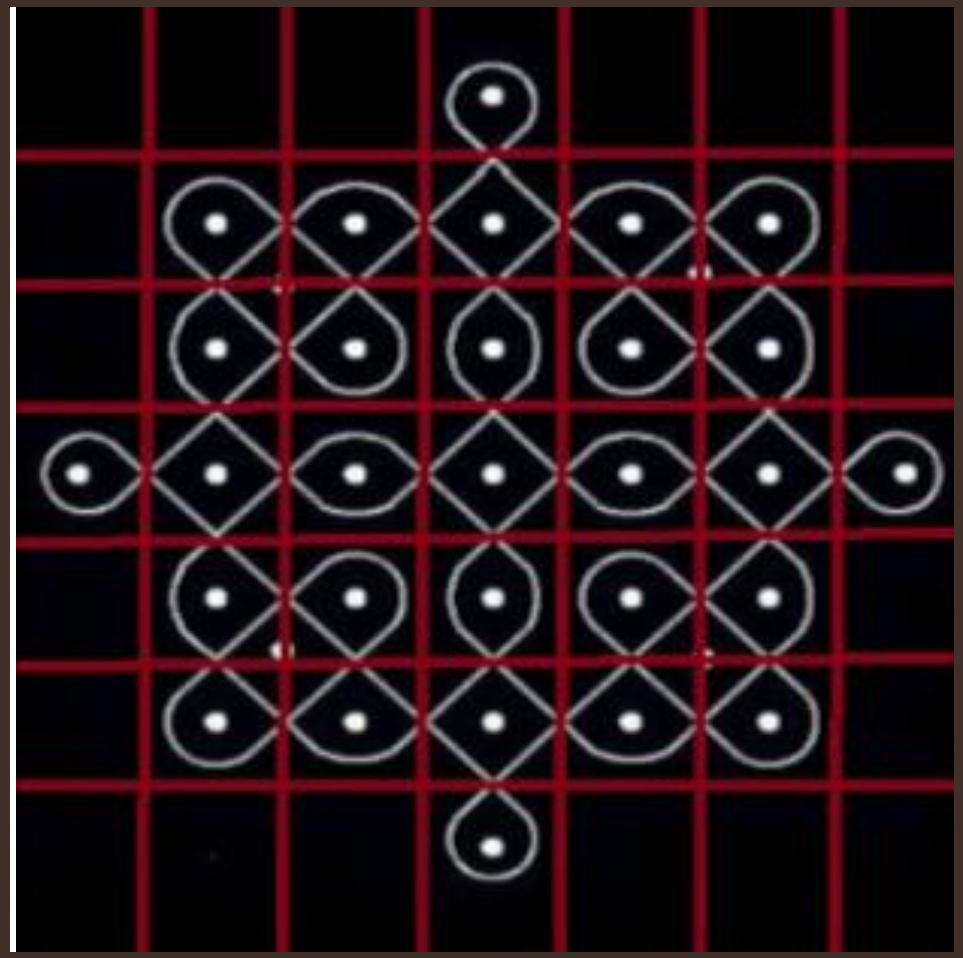


Deconstruction

The patterns in Dot Rangoli

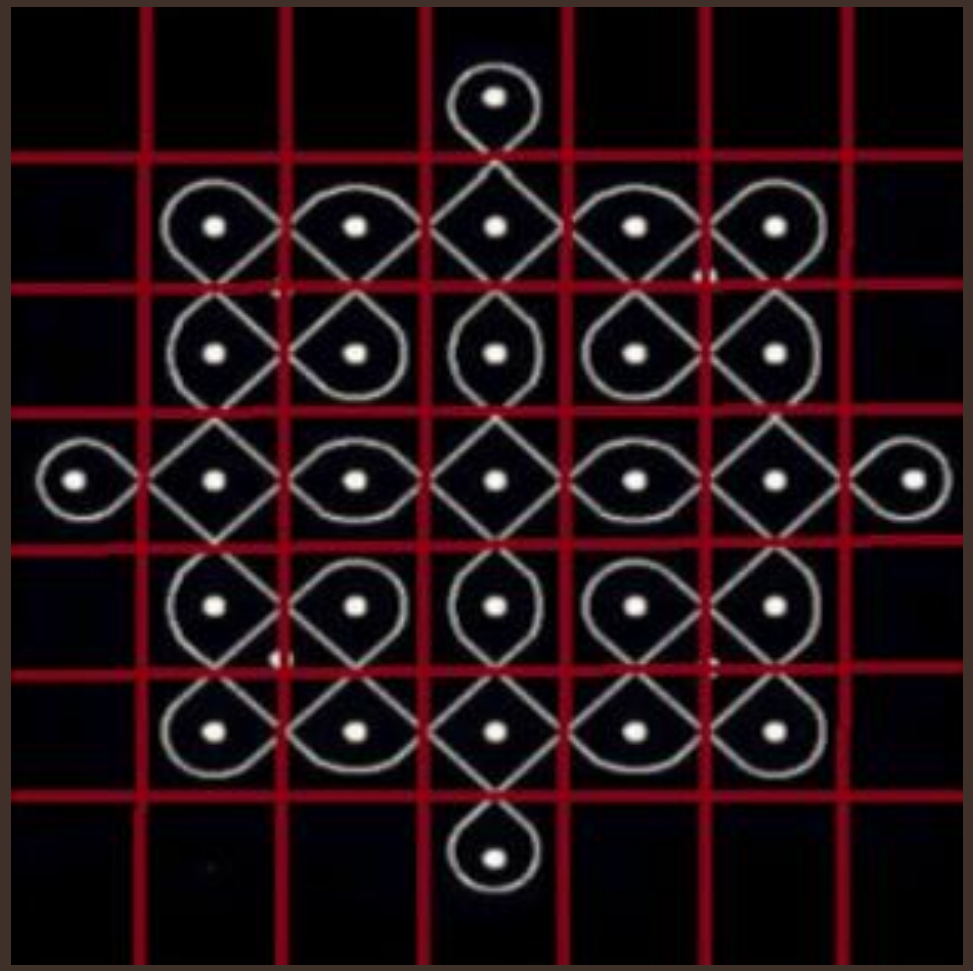


Overlay grid on sample rangoli



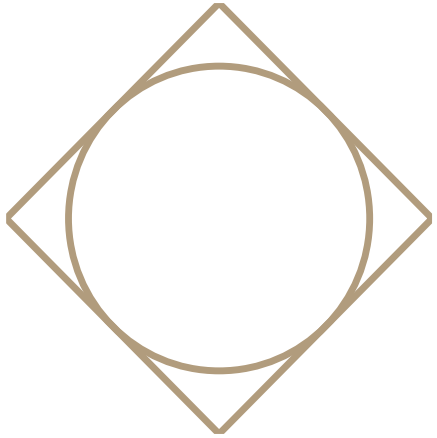
Converting a continuous curve to a set of discrete tiles was a process of trial and error over many days – exactly in the vein of a math circle!

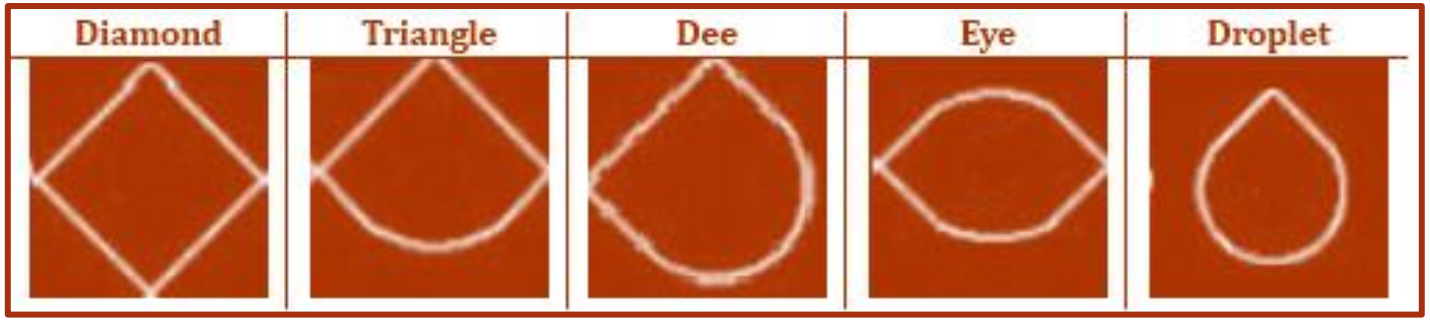
Constituent shapes

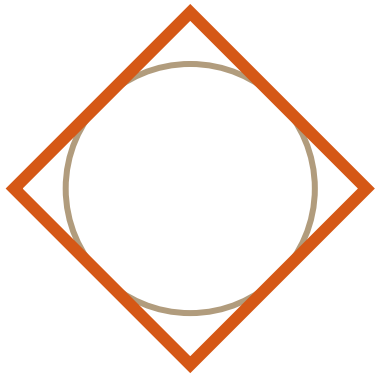
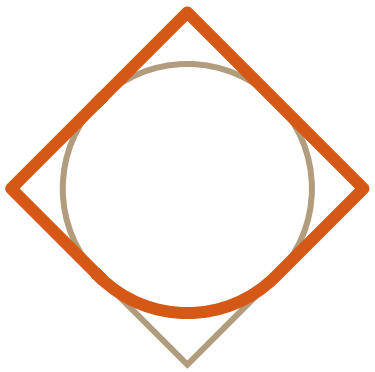
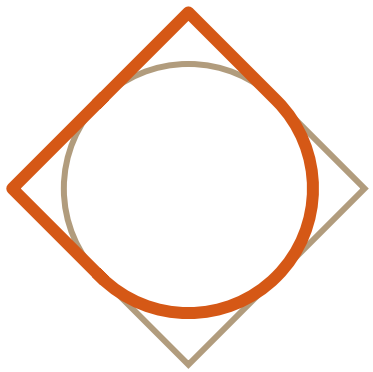
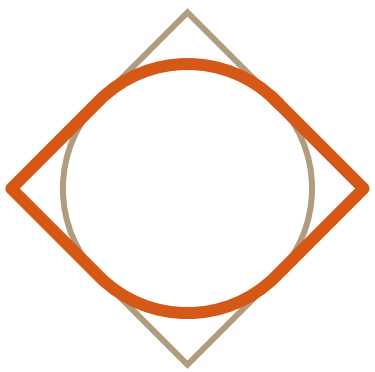
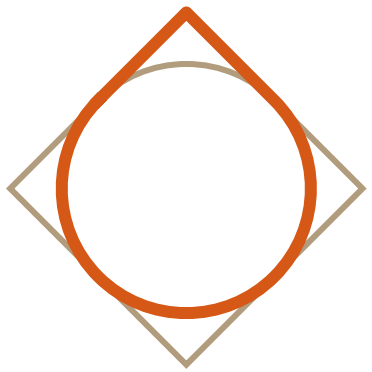


- Dot per cell – Each shape within the cell
- Adjacent shapes join at the mid point of the side → Four connection points per cell
- Five elementary shapes

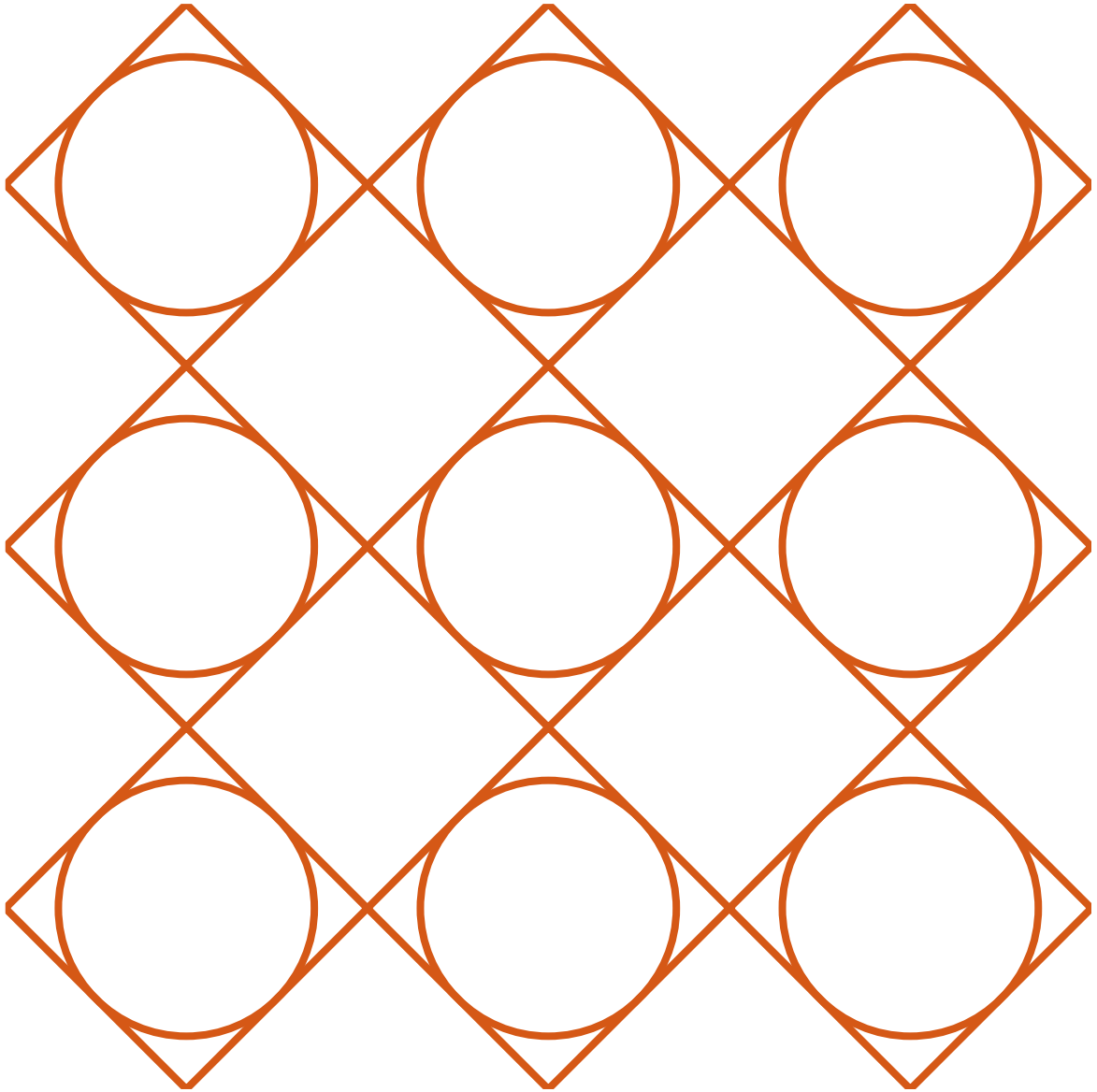
Template Shape





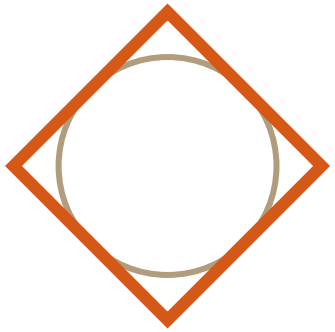
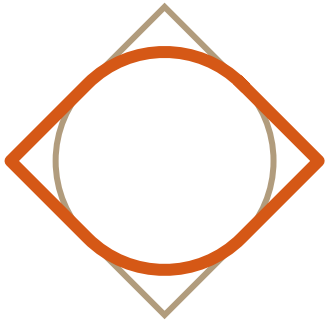
Diamond	Triangle	Dee	Eye	Droplet
				

Draw your own Rangoli



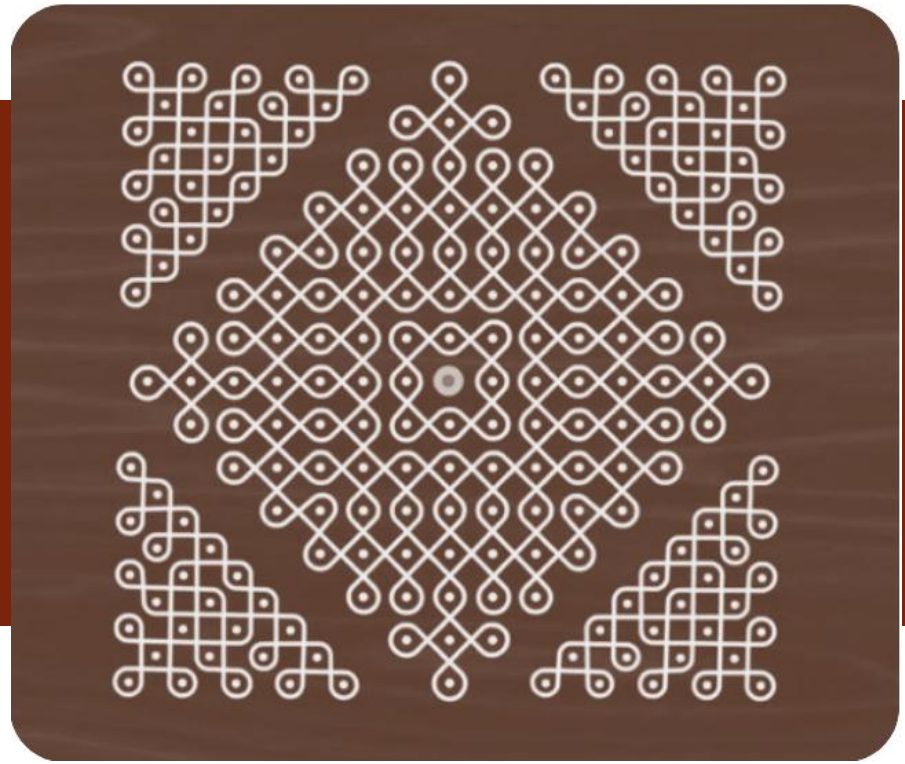
Exercise 1: Create a Rangoli

- Pick a random cell and draw a random shape
- Pick any shape for a cell without neighbors
- For others, pick a shape that has the same number of vertices as number of neighbors
- Draw the shape in the right orientation OR
- For each side of the cell, choose the red portion if it has a neighbor or the gray arc if not
- Add symmetry ☺

Diamond	Triangle	Dee	Eye	Droplet
				

Reconstruction

Math & Rangoli

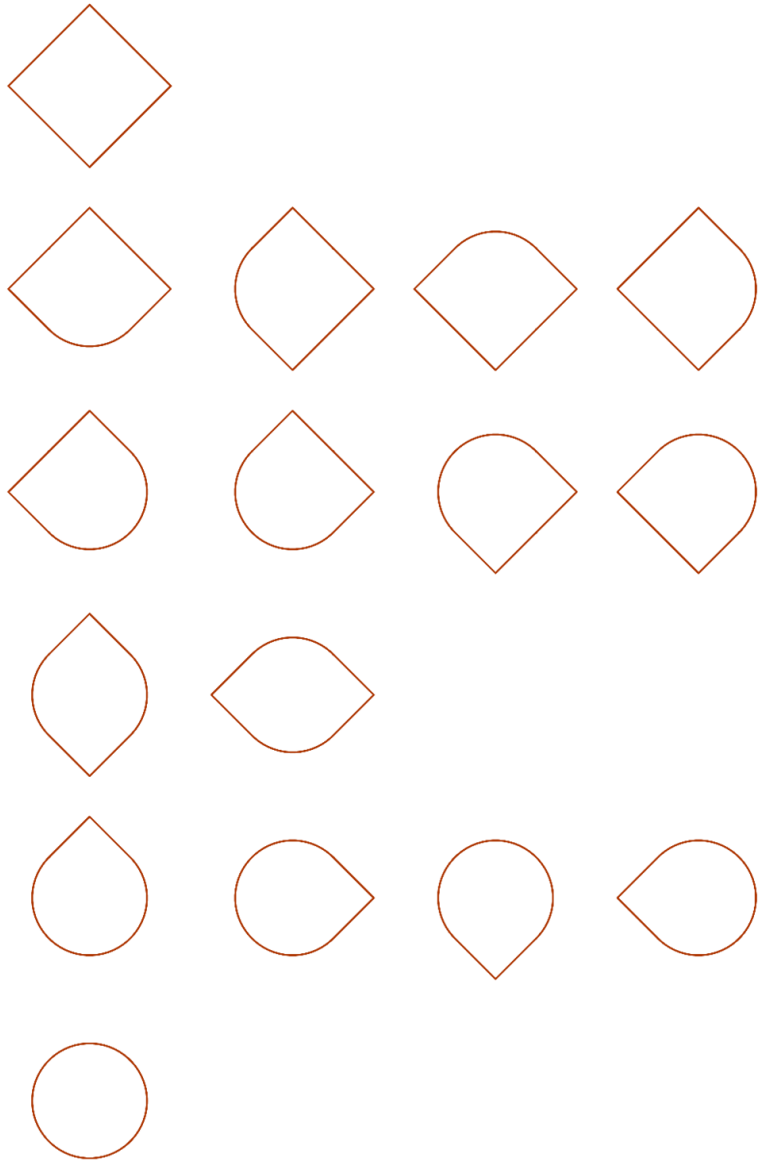


Math Circle Questions

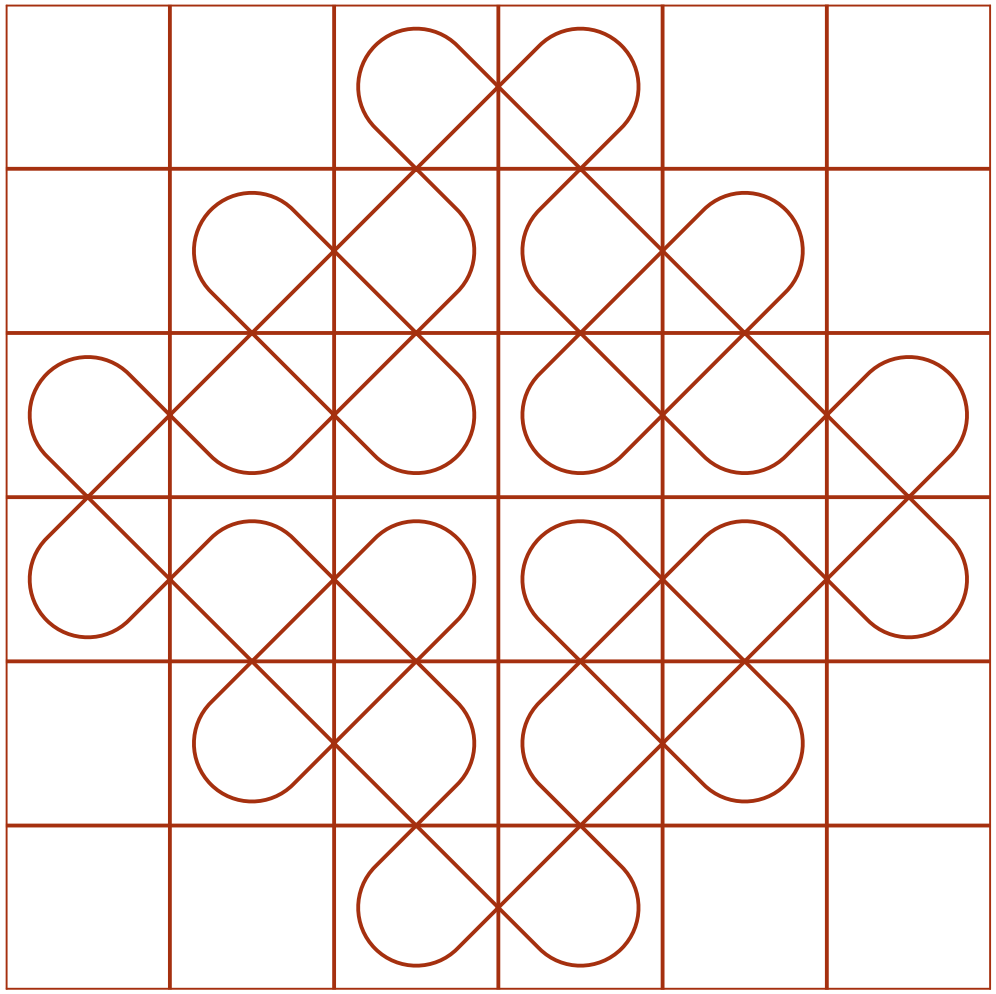
- Why five shapes?
- Why these shapes? Are others possible?
- Given a set of shapes, what are the rules to create new patterns?
- How many patterns can you create?
- How do you go from creating a pattern to drawing a pattern?
- What symmetries exist?
- How do you create patterns with a chosen symmetry?

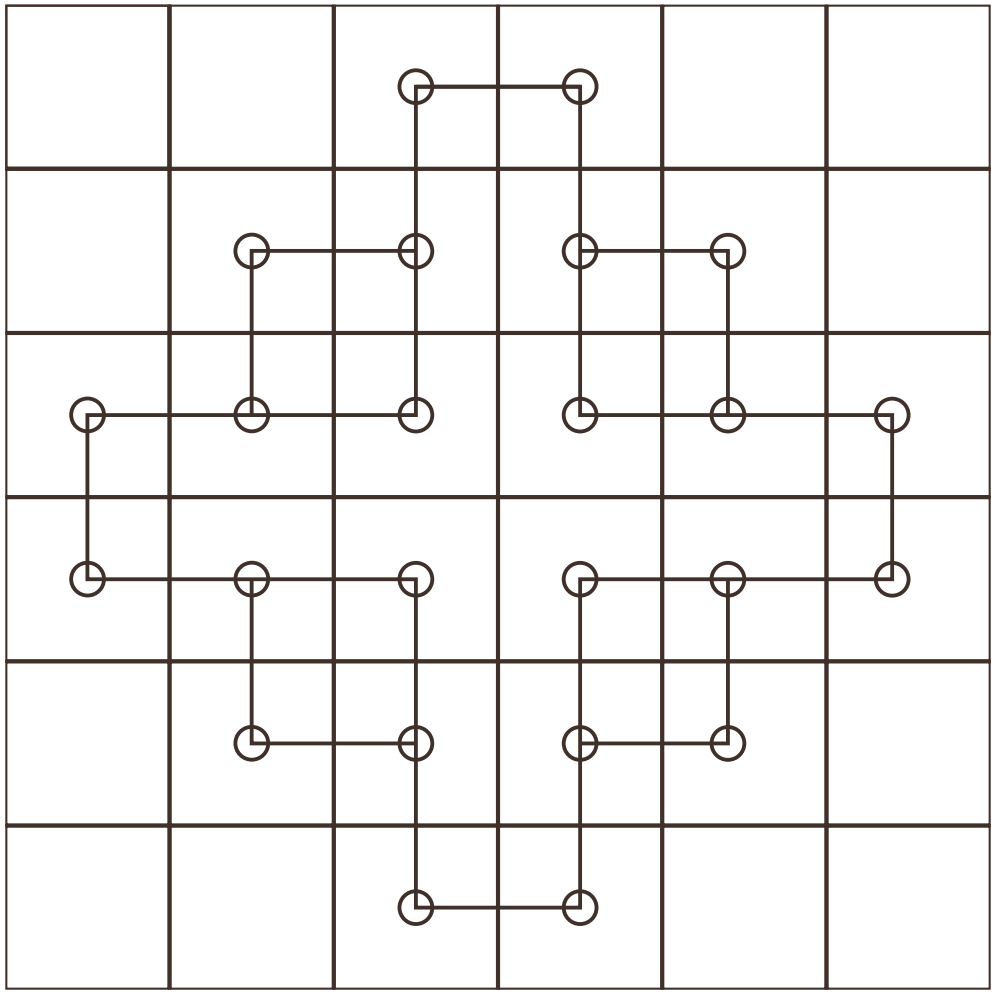
Why five shapes?

- Every cell has four potential neighbors
- Combinations of neighbor presence is $2^4 = 16$
- Eliminating rotations leaves 6 tiles
- No neighbors can be represented in two ways
  - A circle
  - A blank tile

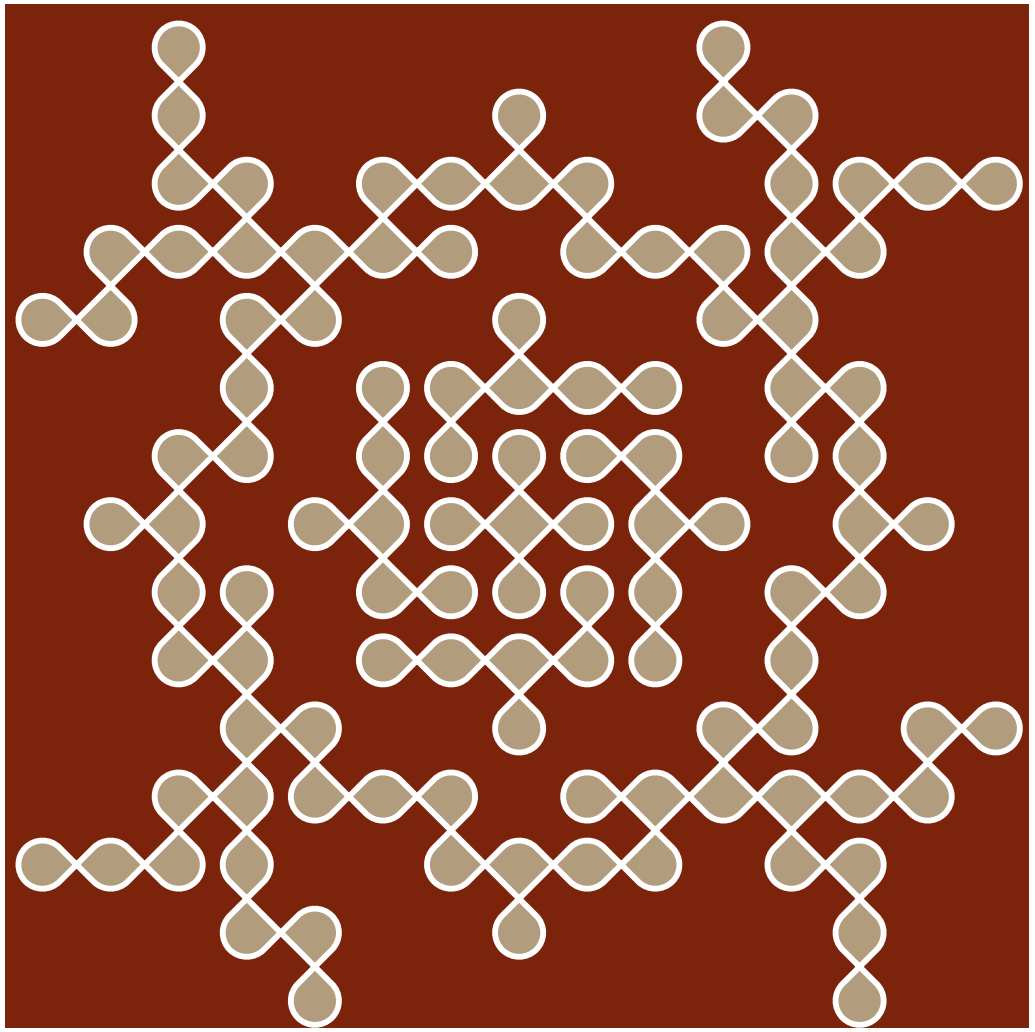


Graph Structure





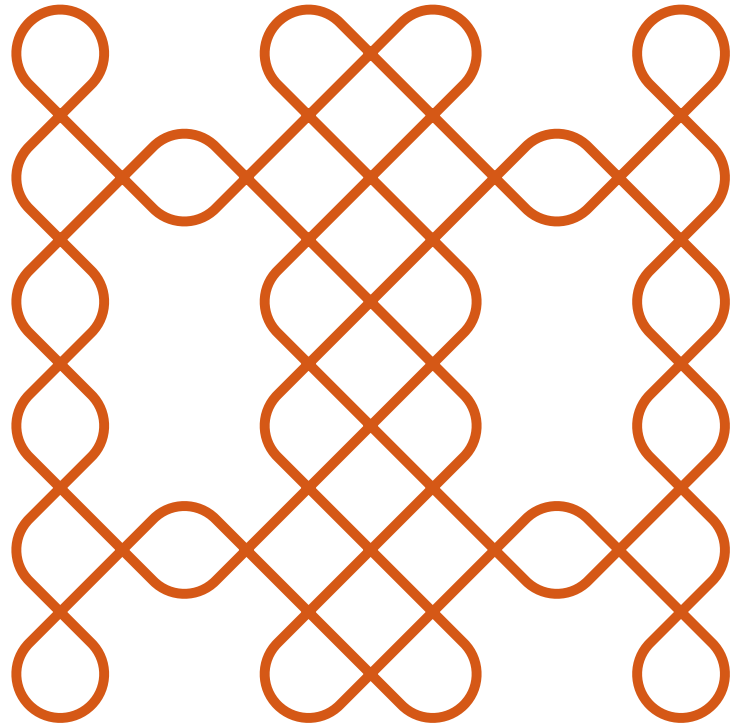
Random Rangolis



- Select a random number of random cells in a grid
- Remove all cells with no neighbors
- For each node:
  - Get the count and orientation of its edges
  - Assign a tile type for that cell given the count and orientation of its edges
  - Rotate the shape to match the location of its edges
- Draw the assigned tile in for each cell in the correct orientation

Eulerian Circuits

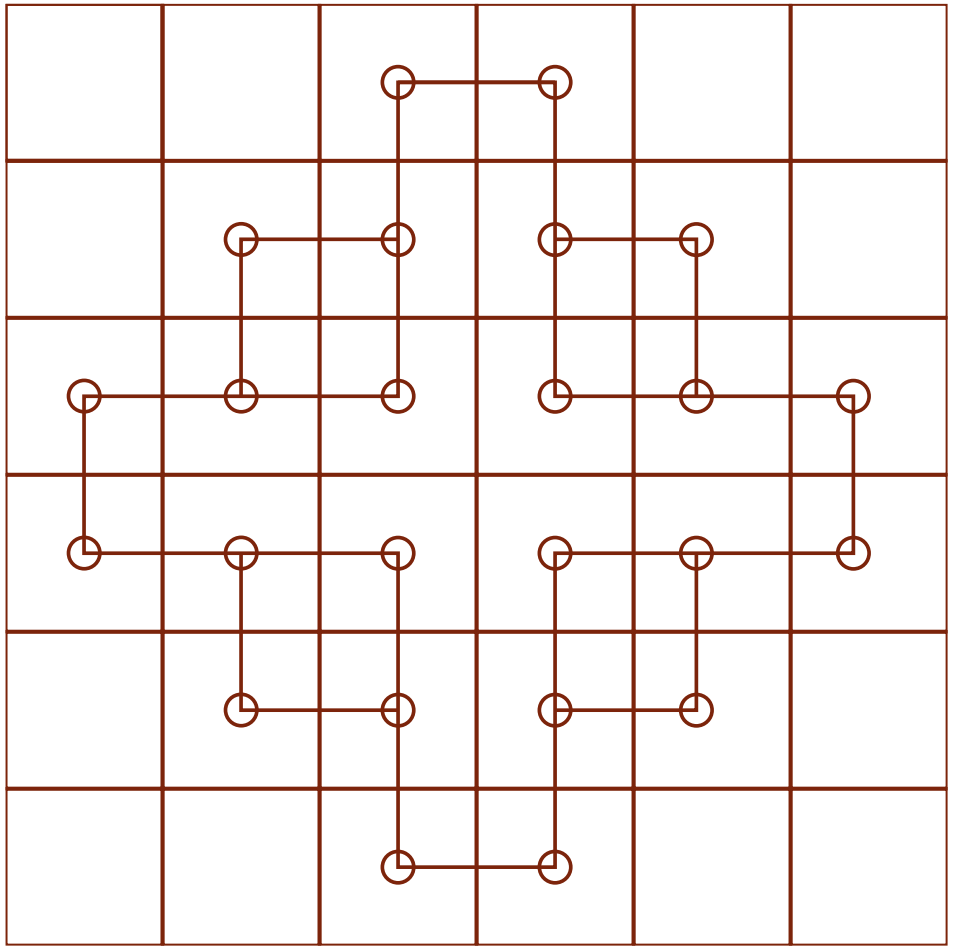
- No unconnected tiles in a valid rangoli
- Every intersection has an even number of lines
- Connected regions are Eulerian circuits



Exercise 2: Draw the rangoli you created as an *Eulerian circuit*

Counting Rangolis in an NxN grid

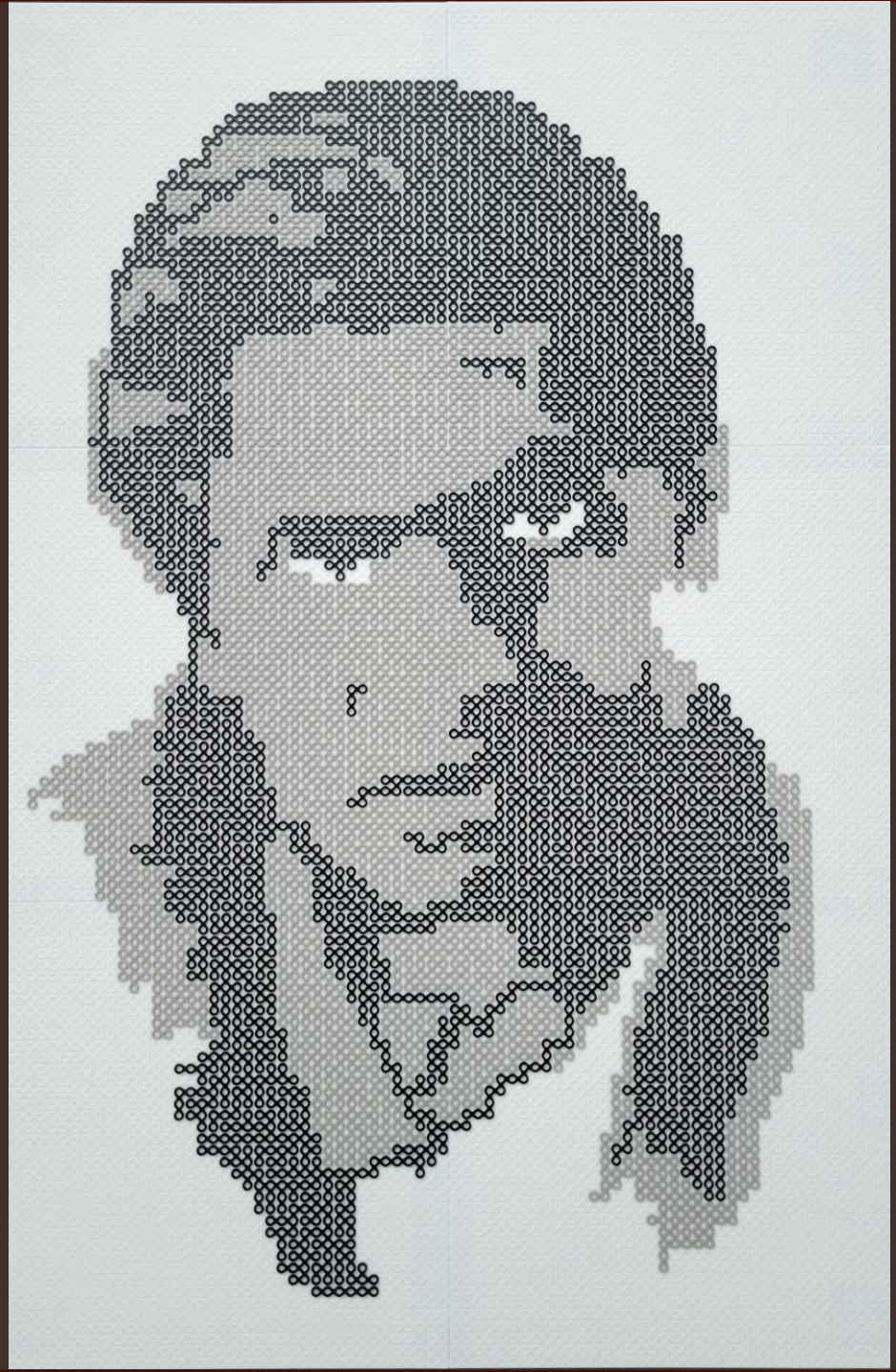
- Each cell is a node in a grid graph
  - N-1 horizontal edges for N rows
  - N-1 vertical edges for N columns
  - **= A total of $2N(N-1)$ edges**
- **All possible combinations: $2^{2n^2-2n} = 4^{n^2-n}$**
- $5.19 * 10^{33}$ rangolis in a 8x8 grid
- A053765 on OEIS



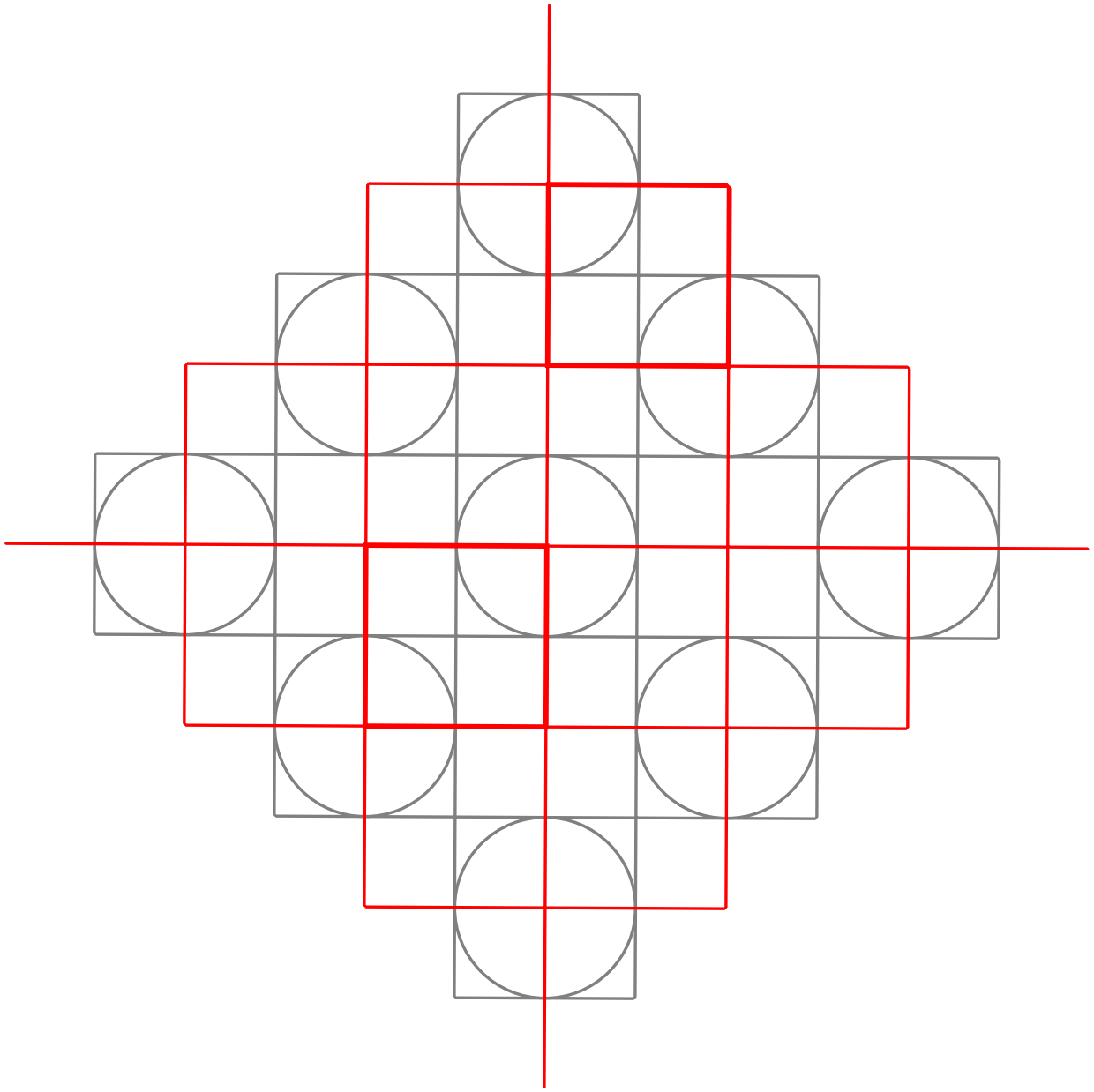
Homework1: Eliminate duplicates from symmetry

Rangoli from pictures

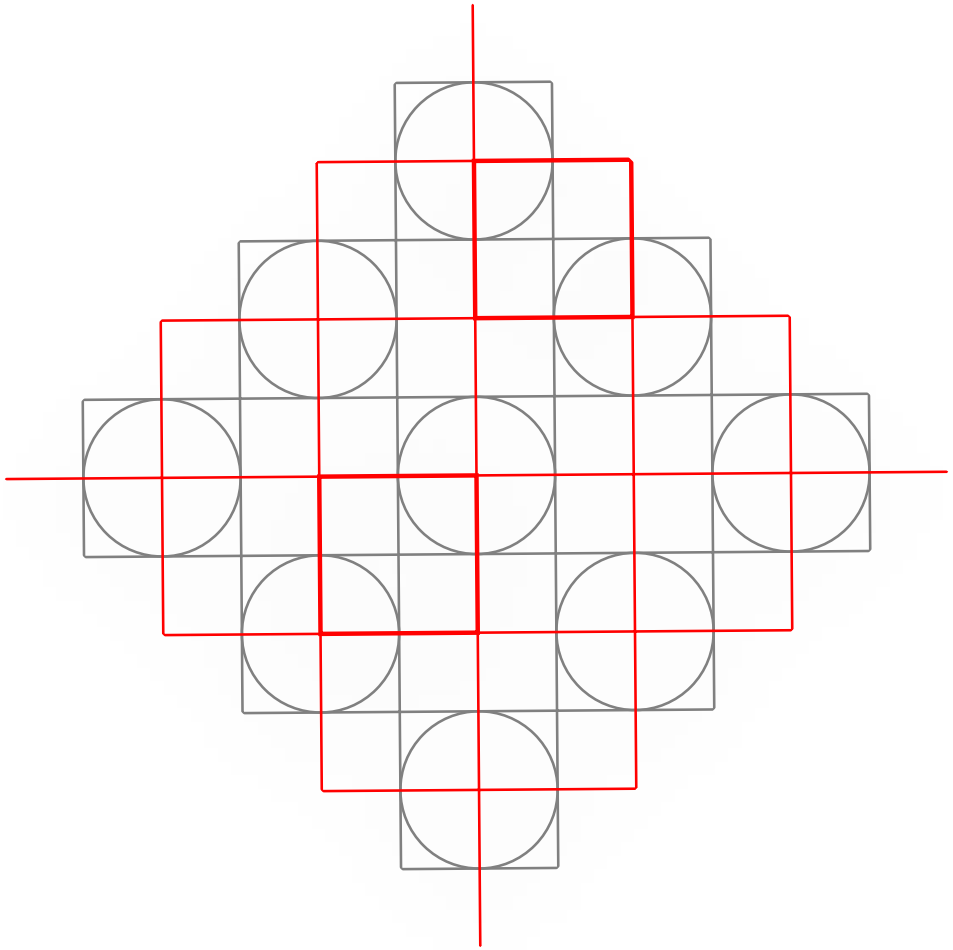
- Each pixel is cell
- Build a list of **connected components** – contiguous regions of the same colors are part of the same component
- For each connected component, create a **random spanning tree**
- For each cell assign a rangoli tile type based on the number and orientation of its neighbors
- Assign a color to the tile based on the pixel color
- Extrude and print

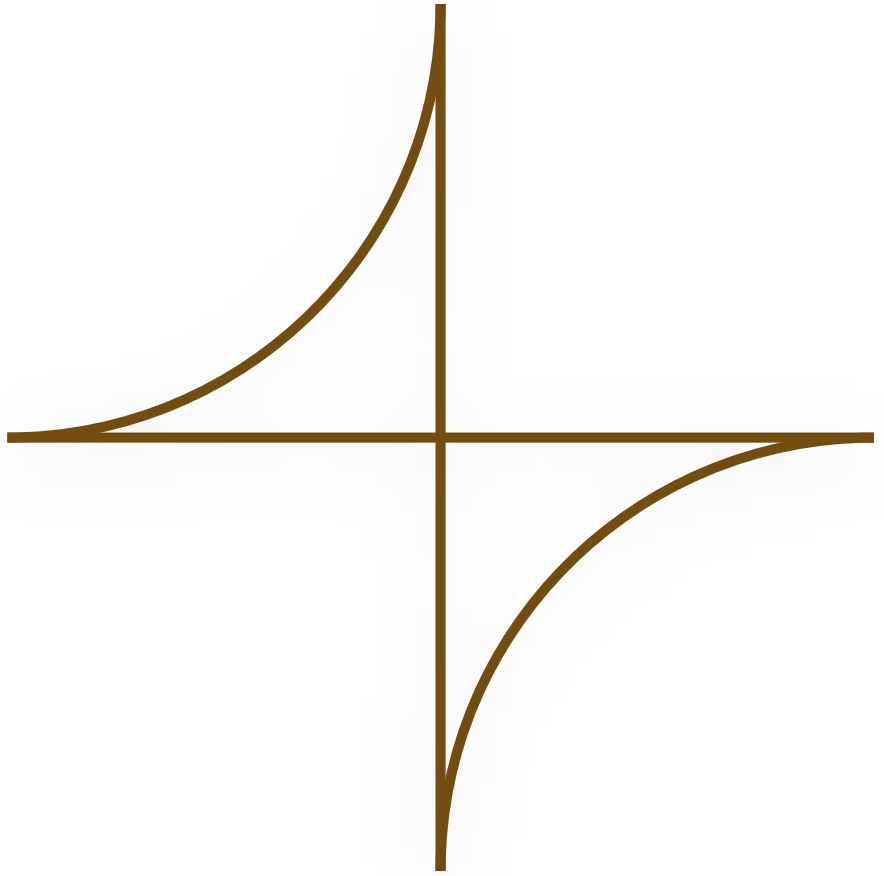


Rangolis from Line Graphs

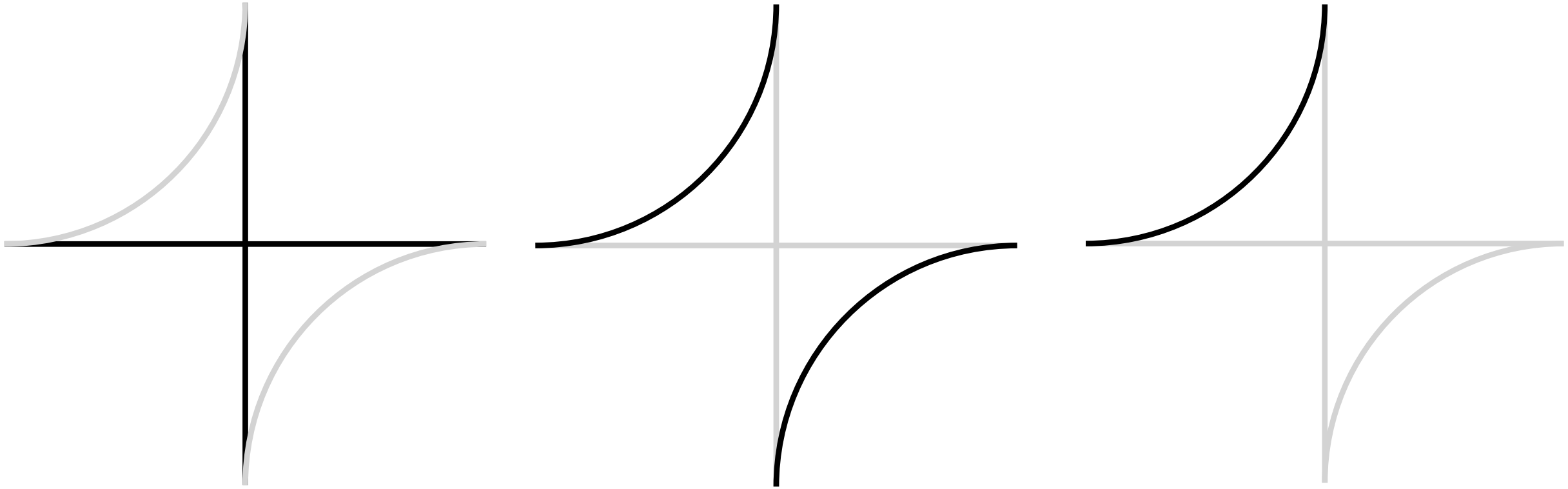


Template Shape for Line Graph





Elementary Shapes for Line Graph



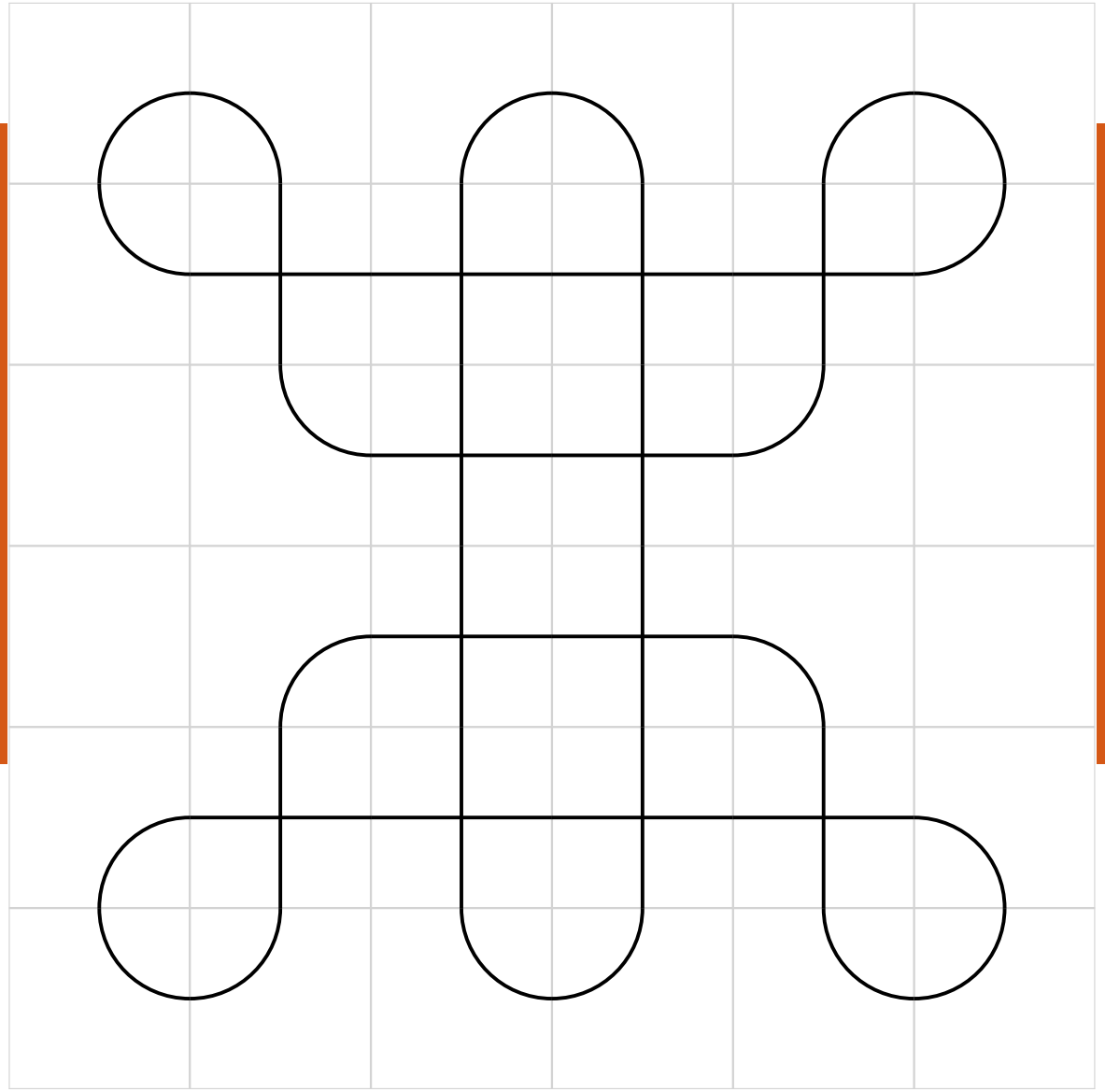
Serpentiles

4S2P-02

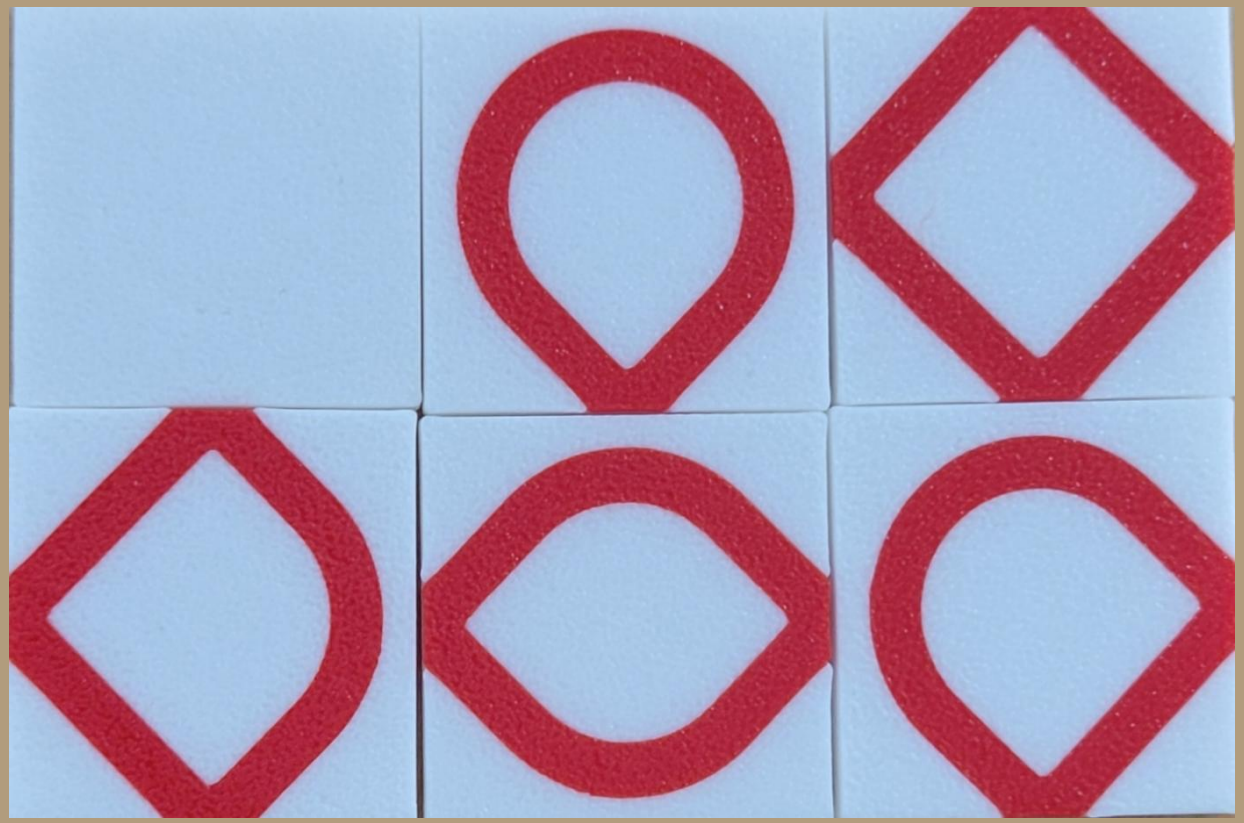
4S2P-20

4S1P-01

Rangoli from Serpentiles



Exercise 3: Play With Physical Tiles!



Please contact us if you have a color 3D printer and want to print your own tiles or want printed tiles for your math circles!!

Thank you!

harishsk@gmail.com
tanmayhk@uw.edu

https://rangoli.online