

Explorations in Rigidity

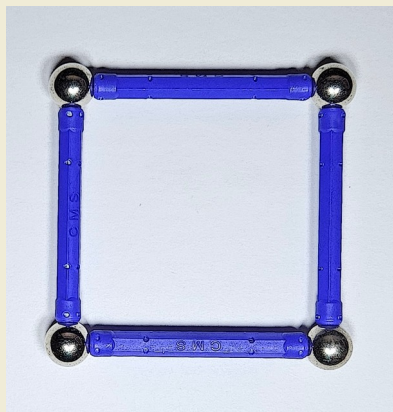


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Seattle Universal Math Museum

Premise

Students build with rods (edges) and hinge joints (vertices), experimentally discovering rules of combinatorial rigidity.



Flexible



Rigid

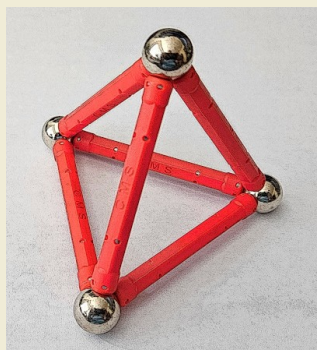
Basic Questions

- Which familiar structures are rigid? Which are flexible?
 - Cube? Prisms? Tetrahedron? Pyramids? Platonic solids?
- If a structure is flexible, can you add edges to make it rigid?
- Is there a relationship between the number of vertices and edges in a rigid structure?
- Given 4 vertices, how many edges do you need to make a rigid structure? 5 vertices? 6 vertices?

Rigidity by Vertices

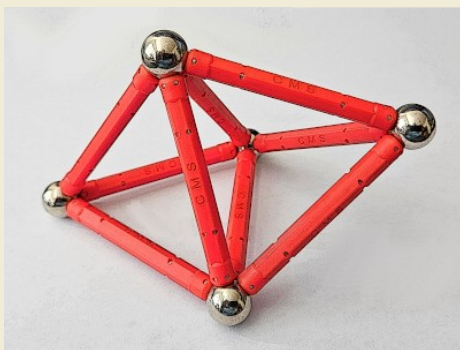
If all edges have the same length:

$$v = 4$$



$$e = 6$$

$$v = 5$$

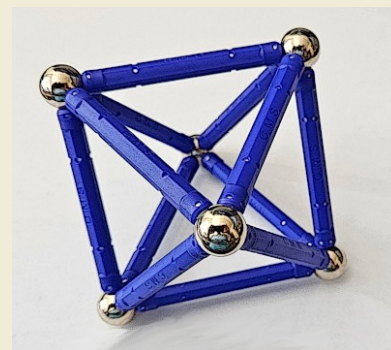


$$e = 9$$

$$v = 6$$



$$e = 12$$

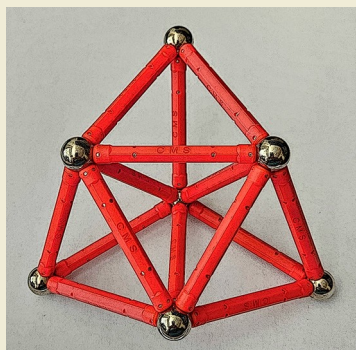


$$e = 12$$

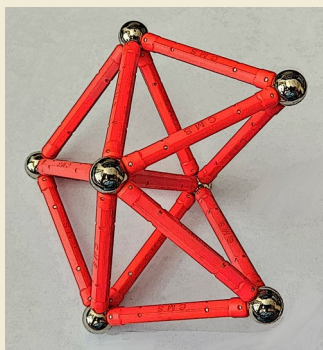
Rigidity by Vertices

If all edges have the same length:

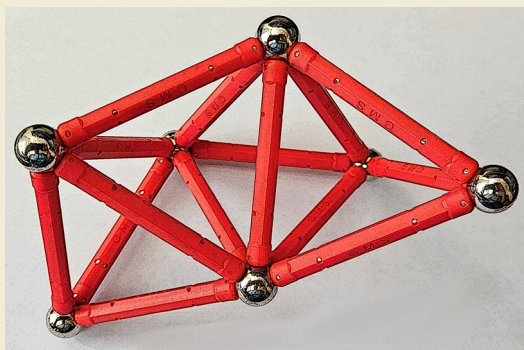
$$v = 7$$



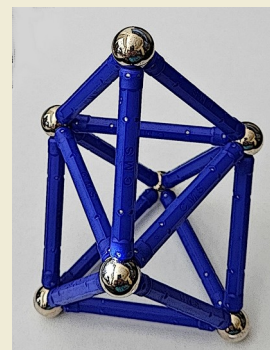
$$e = 15$$



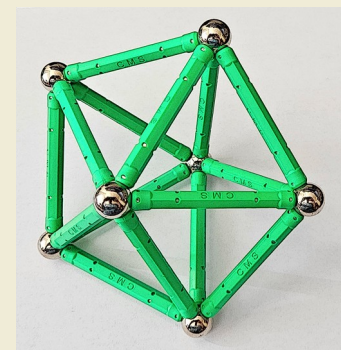
$$e = 15$$



$$e = 15$$



$$e = 15$$

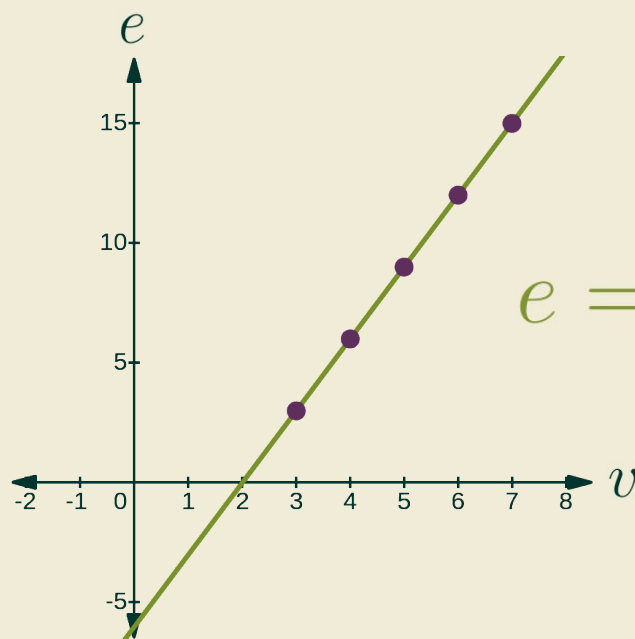


$$e = 15$$

Rigidity by Vertices

A conjecture emerges:

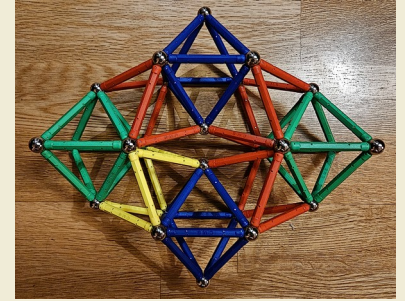
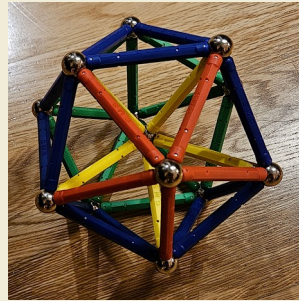
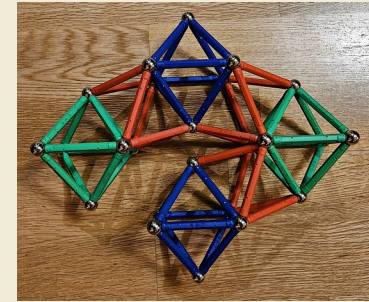
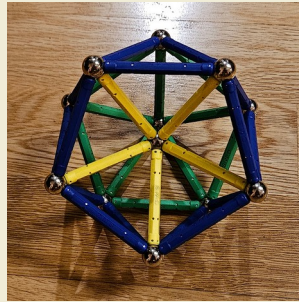
v	e
3	3
4	6
5	9
6	12
7	15



$$e = 3v - 6$$

Rigidity by Vertices

Some issues with that conjecture:



Rigidity by Vertices

Maxwell's Criterion:

For a rigid graph in 3 dimensions,

$$e \geq 3v - 6$$

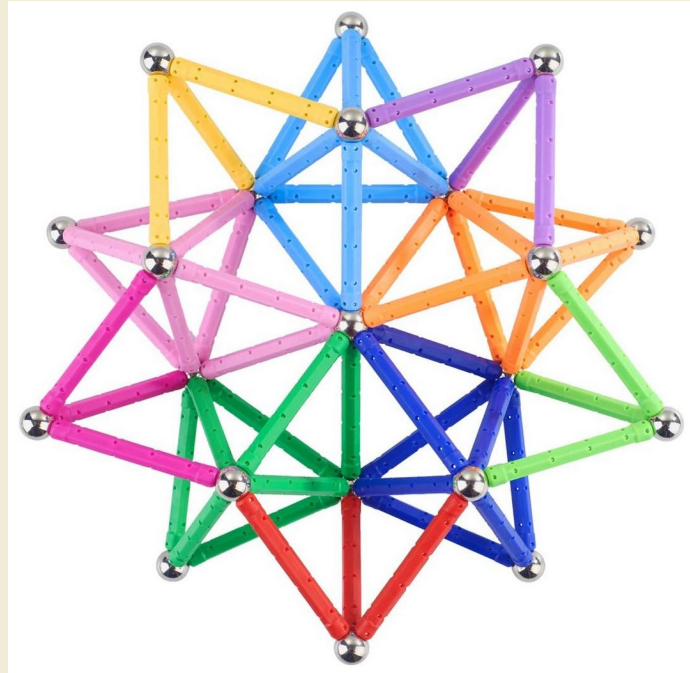
Some Algebra

For structures satisfying Euler's formula (spherical polyhedra) without redundant edges:

$$\begin{cases} e = 3v - 6 \\ v - e + f = 2 \end{cases} \Rightarrow 2e = 3f$$

You can double-count edges by adding the edge counts for all faces, so the formula above indicates that the average edge count per face is 3.

Some Material Options



magnetic stick and ball building toy

Some Material Options



toothpicks



marshmallows
or spice drops

Some Material Options



dowels with end caps



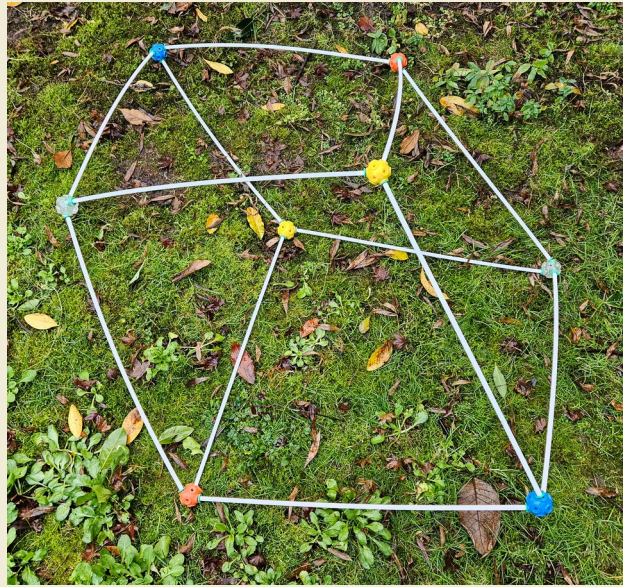
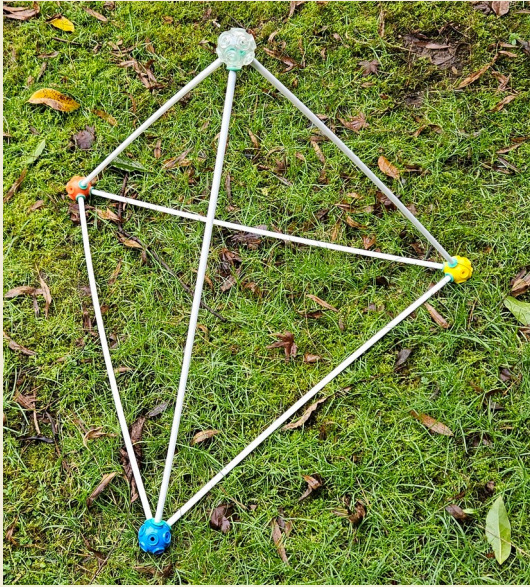
zip ties

Some Material Options



OgoSport OgoBild Pod

Some Material Options



OgoSport OgoBild Pod

Some Material Options

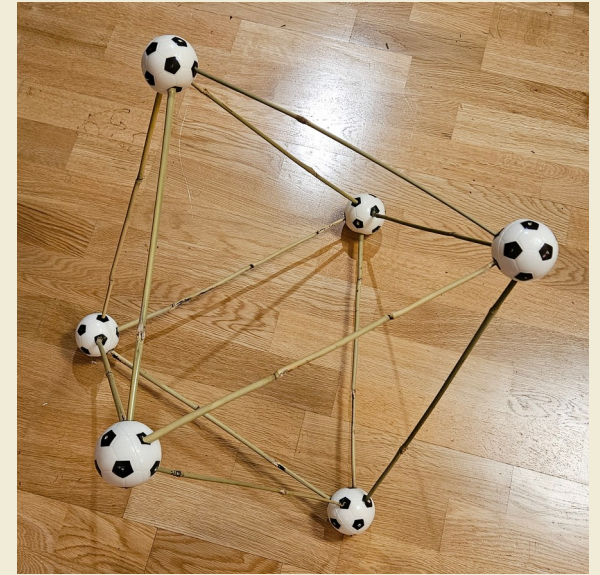
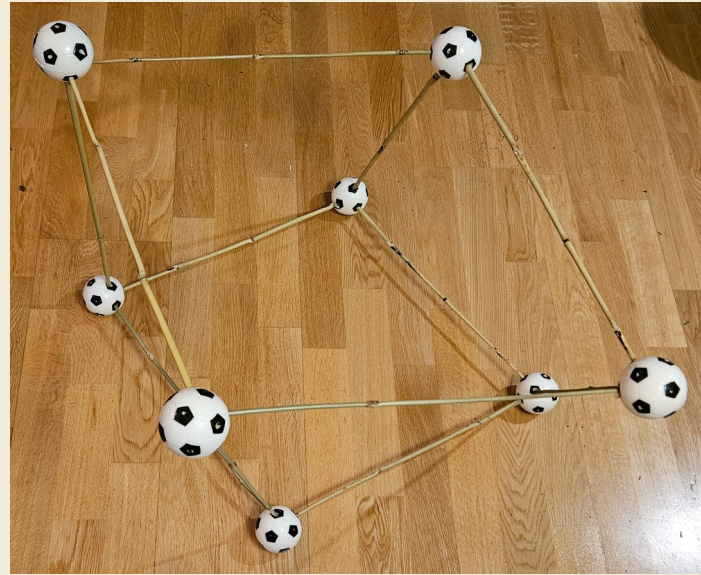


bamboo
plant stakes



(drilled)
foam balls

Some Material Options



bamboo stakes and foam balls

Please clap

Thank you!