



A GAME FOR 4 WITH LINEAR INDEPENDENCE & DETERMINANTS

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DETERMINANT OF A SQUARE MATRIX

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

$$\det \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix} = aqz + brx + cpy - (ary + bpz + cqx)$$

$\det = 0 \iff$ At least one row is a linear combo of the others
 \iff At least one column is a linear combo of the others

Each Summand is \pm products of entries
one from each row and column

determinant $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{pmatrix}$

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✕ =

 NATURAL LANGUAGE  MATH INPUT ★ √ ∂f (::)  $a\omega$...

VECTORS & MATRICES ✕

$[a,b]$ $[a,b,c]$ $[a,b,c,d]$ $\begin{bmatrix} a \\ b \end{bmatrix}$ $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ $\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$ $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ $\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}$

Input interpretation

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{vmatrix}$$

$|m|$ is the determinant

Result Step-by-step solution

0

Number line

10 RULES

(1) Start with a 4 by 4 matrix having 4 each of 1s, -1s, 0s, and 2s

(2) The initial determinant is non-zero so highly non-symmetric

(3) Each round involves the 4 players playing in a random order

(4) Moves consist of a capture/takeover or swapping yours with an adjacent

(5) Swapping can be diagonal BUT capture must be non-diagonal

(6) Capture requires two of same # on sides of victim

(7) No player suffers more than one elimination per round

(8) Game ends when $\det=0$ or a player is eliminated

(9) If $\det=0$ player(s) with smallest number win

(10) If player eliminated and $\det \neq 0$ Player(s) with largest number win

ROUND 1 ORDER

1 plays 1st
0 plays 2nd
-1 plays 3rd
2 plays 4th

1 captures zero!

ROUND 1

2	1	1	1
0	2	1	0
0	-1	2	0
-1	-1	-1	2

DET= 25

ROUND 1 ORDER

~~1 plays 1st~~

0 plays 2nd

-1 plays 3rd

2 plays 4th

ROUND 1

2	1	1	1
0	2	1	1
0	-1	2	0
-1	-1	-1	2

0 swaps right -1
Can't be captured again

DET= 28

ROUND 1 ORDER

~~1 plays 1st~~

~~0 plays 2nd~~

-1 plays 3rd

2 plays 4th

-1 diagonal
swaps 0

ROUND 1

2	1	1	1
0	2	1	1
-1	0	2	0
-1	-1	-1	2

DET = 25

ROUND 1 ORDER

~~1 plays 1st~~

~~0 plays 2nd~~

~~-1 plays 3rd~~

2 plays 4th

2 captures upper 1

ROUND 1

2	1	1	1
0	2	1	1
-1	-1	2	0
0	-1	-1	2

DET= 29

ROUND 2 ORDER

-1 plays 1st

2 plays 2nd

0 plays 3rd

1 plays 4th

-1 captures 0

ROUND 2

2	2	1	1
0	2	1	1
-1	-1	2	0
0	-1	-1	2

DET = 26

To start 2 has the most and zero the least!

ROUND 2 ORDER

~~-1 plays 1st~~

2 plays 2nd

0 plays 3rd

1 plays 4th

2 captures 1

ROUND 2

2	2	1	1
0	2	1	1
-1	-1	2	0
-1	-1	-1	2

DET = 26

2 cannot capture zero

ROUND 2 ORDER

~~-1 plays 1st~~

~~2 plays 2nd~~

0 plays 3rd

1 plays 4th

0 swaps up 2

ROUND 2

2	2	1	1
0	2	2	1
-1	-1	2	0
-1	-1	-1	2

DET = 26

ROUND 2 ORDER

~~-1 plays 1st~~

~~2 plays 2nd~~

~~0 plays 3rd~~

1 plays 4th

1 captures down 2

ROUND 2

0	2	1	1
2	2	2	1
-1	-1	2	0
-1	-1	-1	2

DET = -30

ROUND 3 ORDER

0 plays 1st

1 plays 2nd

2 plays 3rd

-1 plays 4th

ROUND 3

0	2	1	1
2	2	1	1
-1	-1	2	0
-1	-1	-1	2

0 swaps diagonal 2

DET = -26

To start 2 & -1 have the most and zero the least

ROUND 3 ORDER

~~0 plays 1st~~
1 plays 2nd
2 plays 3rd
-1 plays 4th

1 swaps down zero

ROUND 3

2	2	1	1
2	0	1	1
-1	-1	2	0
-1	-1	-1	2

DET = -26

ROUND 3 ORDER

~~0 plays 1st~~

~~1 plays 2nd~~

2 plays 3rd

-1 plays 4th

2 captures 0

ROUND 3

2	2	1	1
2	0	1	0
-1	-1	2	1
-1	-1	-1	2

DET = -28

ROUND 3 ORDER

- ~~0 plays 1st~~
- ~~1 plays 2nd~~
- ~~2 plays 3rd~~
- 1 plays 4th

ZERO WINS!

ROUND 3

2	2	1	1
2	2	1	0
-1	-1	2	1
-1	-1	-1	2

DET=0

11A	22	22	11	11
-14B	-28	-28	-14	0
-5D	5	5	5	-10
TOTAL Add C	-1	-1	2	1

A

B

C

D

2	2	1	1
2	2	1	0
-1	-1	2	1
-1	-1	-1	2

These
columns
are equal

ORDER

1

-1

2

0

DET= 3

2	0	0	-1
1	2	-1	0
1	-1	2	0
-1	1	1	2

1st ORDER

2

0

-1

1

DET= 2

2	2	-1	-1
2	0	2	-1
0	1	-1	1
0	0	1	1