Let’s Have $\Sigma$ Fun!
Mathematical Games for Math Circles

Dr. Shelley Stahl

Joint Mathematics Meetings 2023

Wednesday January 4, 2023
Introduction Elementary School Middle School High School Math Circles and Outreach

Background

- Bard Math Circle
Introduction

Math Circles and Outreach

Background

- Bard Math Circle
• Middle and High School Math
Background

- Bard Math Circle

- Middle and High School Math
  - Bridge to Enter Advanced Math (BEAM) Summer Programs
Background

- Bard Math Circle
- Middle and High School Math
• Bridge to Enter Advanced Math (BEAM) Summer Programs

• ....and more

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Introduction Elementary School Middle School High School Why Games?

• Games are fun! ....and
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**Introduction**  
Elementary School  
Middle School  
High School  

**Why Games?**
• Games are fun! ....and
• Low floor, high ceiling
Why Games?

• Games are fun! ....and
• Low floor, high ceiling
• Adaptable
Introduction  
Elementary School  Middle School  High School

Why Games?

- Games are fun! ....and
- Low floor, high ceiling
• Adaptable
• Allows for multiple strategies/solutions

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Introduction Elementary School Middle School High School

Why Games?

• Games are fun! ....and
• Low floor, high ceiling
Adaptable
• Allows for multiple strategies/solutions
• Collaboration is encouraged

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Introduction Elementary School Middle School High School Strategy: Work Backwards

• Player 1 begins on the "Start" square
• Players alternate turns moving a single token either one square up, one square left, or one square diagonally up and left.
• The first to reach the "WIN" square is the winner!
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Introduction Elementary School Middle School High School

Strategy: Work Backwards

- Player 1 begins on the ”Start” square
- Players alternate turns moving a single token either one square up, one square left, or one square diagonally up and left.
- The first to reach the ”WIN” square is the winner!
What if we change the size of the game board? What if it is non-square; non-rectangular?

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Introduction Elementary School Middle School High School Strategy: Solve a Simpler the Problem
• Player 1 calls out any single digit number.
• Player 2 can then add any single digit number to the first one, and call out the result.
• The players continue to alternate, adding single digit numbers to the prior number.
• The first player to call out 100 wins
Problem

- Two players take turns popping up to 3 bubbles. Whoever pops the last bubble wins.
Introduction

Elementary School

Middle School

High School

Strategy: Solve a Simpler the

Problem

• Two players take turns popping up to 3 bubbles. Whoever pops the last bubble wins.

• Players take turns popping up to 4 bubbles. Whoever pops the last bubble loses.

• Players take turns popping any number of bubbles, as long as they
are in the same row. Whoever pops the last bubble wins/loses.

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Introduction

Elementary School Middle School High School

Strategy: Analyze a Problem

Systematically

• Player 1 selects a number from the board and circles it. • Player 2 puts a box around all of its factors remaining on the board, then chooses and boxes their own new number.

• Player 1 now circles all of the remaining factors of Player 2’s chosen number, before circling their own new number again.

• Play ends when there are no numbers left that have available factors. Each player adds the numbers in their circles/boxes, and the player with the highest sum wins.
The number 60 is written on a blackboard. Players take turns subtracting from the number on the blackboard any of its divisors (including 1 or the number itself), and replacing the original number with the result of this subtraction. The player who writes the number 0 loses.
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Introduction Elementary School Middle School High School Parity

Problems: Subtraction
The number 60 is written on a blackboard. Players take turns subtracting from the number on the blackboard any of its divisors (including 1 or the number itself), and replacing the original number with the result of this subtraction. The player who writes the number 0 loses.

- What if we started with a different number?
- What if we cannot subtract the same
Parity Problems: Coins

- Player 1 begins the game with some number of coins and Player 2 has none.
- Player 2 can take any (non-zero) number of coins from Player 1. Then Player 1 can take some (again, non-zero) number of coins back, but necessarily a different number.
- Then again Player 2 takes some from Player 1, but necessarily a number which did not occur before. And so on.
- Gameplay ends when someone cannot make a move. The player
• Player 1 begins the game with some number of coins and Player 2 has none.

• Player 2 can take any (non-zero) number of coins from Player 1. Then Player 1 can take some (again, non-zero) number of coins back, but necessarily a different number.

• Then again Player 2 takes some from Player 1, but necessarily a number which did not occur before. And so on.
Gameplay ends when someone cannot make a move. The player with the most coins wins.

What is the largest number of coins Player 2 can have at the end if:
- Player 1 had 13 coins at the beginning?
- Player 1 had 50 coins at the beginning?

Two players, Red and Blue, play on a rectangular grid. They will alternate turns choosing a box and filling it in with their color.

- Red wants to create a rectangle whose corners are all the same color
- Blue wants to prevent Red from doing so
Start with a small rectangle and build up. Is there a rectangle for which player Red is guaranteed to win?

Twenty points are marked on the circumference of a circle. Two players play the following game. On each turn, a player connects two of the 20 points with a segment, according to the following rules:

• a segment can only appear once during the game;
Shannon Switching Game

• Two players, Short and Cut, alternate turns on a graph with two designated vertices
• Short will choose an edge to protect
• Cut will delete an unprotected edge
• If Short is able to create a protected path between the designated vertices, she wins
• If Cut can disconnect these nodes first, he wins
• Cut will delete an unprotected edge
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Introduction Elementary School Middle School High School Graph Theory

Shannon Switching Game
• Two players, Short and Cut, alternate turns on a graph with two designated vertices
• Short will choose an edge to protect
Shannon Switching Game

- Two players, Short and Cut, alternate turns on a graph with two designated vertices
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Introduction Elementary School Middle School High School Graph

Theory

Shannon Switching Game
Two players, Short and Cut, alternate turns on a graph with two designated vertices.

Short will choose an edge to protect.

Cut will delete an unprotected edge.

If Short is able to create a protected path between the designated vertices, she wins.

If Cut can disconnect these nodes first, he wins.
Mastermind

- The codemaker chooses a sequence of 4 colored pegs (with or without repeated colors)
- The codebreaker takes guesses at what the code might be, one at a time
- After each guess, the codemaker indicates the number of correct colors in the correct spots, and the number of correct colors in incorrect spots
- If the codebreaker can guess the code, she wins! Otherwise,
the codemaker is the winner after 8-12 guesses

Set
- Players all compete at once to identify "sets" of 3 cards
- In a set, for each of the 4 characteristics on the cards (color, number, shape, and fill), the three cards must all match, or all differ
Geometry

Set
Players all compete at once to identify "sets" of 3 cards. In a set, for each of the 4 characteristics on the cards (color, number, shape, and fill), the three cards must all match, or all differ.

Each card can be considered as a "point" in 4-dimensional space.

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Spinpossible: A board of scrambled numbers 1-9 is displayed.

You can select any sub-rectangle within the square and spit it 180°. The goal is to return it to a standard position using only allowable spins.
Abstract

Algebra

Spinpossible: A board of scrambled numbers
1-9 is displayed.

You can select any sub-rectangle within the square and spit it $180^\circ$. The goal is to return it to a standard position using only allowable spins. • How many starting boards are possible?
• Can any board be solved?
• Does the order of the spins matter?
• Can you build some spins as a combination of other spins?

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Thank you!

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