On Art-Math-Education Lessons in Polynomiography (POLY–NOMI–OGRAPHY)

Bahman Kalantari

Kalantari@cs.rutgers.edu
Emeritus Professor of Computer Science, Rutgers University

AIM Special Session on Math Circles for Makers, Creators, and Artists, January 8, 2025

Abstract:

Abstract:

Around the year 2000, I introduced the term polynomiography, representing the algorithmic visualization of polynomial equations and resulting in images called polynomiographs.

Abstract:

Around the year 2000, I introduced the term polynomiography, representing the algorithmic visualization of polynomial equations and resulting in images called polynomiographs.

This was a bold move, given the preexisting popularity of the Mandelbrot set, fractals, and complex dynamics, perhaps leading some to believe that a new term was unnecessary.

Abstract:

Around the year 2000, I introduced the term polynomiography, representing the algorithmic visualization of polynomial equations and resulting in images called polynomiographs.

This was a bold move, given the preexisting popularity of the Mandelbrot set, fractals, and complex dynamics, perhaps leading some to believe that a new term was unnecessary.

However, polynomiography offers a groundbreaking approach, distinct from these earlier visualizations. Unlike fractals, polynomiographs are not bound to a specific visual pattern, and even when they exhibit fractal properties, the images remain deeply unique and meaningful. This distinction arises from the mathematical foundations of polynomiography, combined with novel techniques that give users control over the rendering process.

Overview:

- The first part of the presentation is based on the article:

Art and Math via Cubic Polynomials, Polynomiography and Modulus Visualization, B.K., LASER Journal, Volume 2, Issue 1 (2024).

In particular, this suggests that the study of cubic polynomials provides a rich source of art-math activities at high school and college level courses, allowing to introduce many deep mathematical topics, as well as techniques for producing artistic images, fashionable items, jewelry designs, etc. It also opens the way to extension of these to general degree polynomials.

Overview:

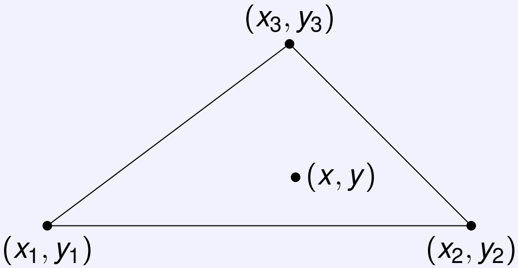
- The first part of the presentation is based on the article:

Art and Math via Cubic Polynomials, Polynomiography and Modulus Visualization, B.K., LASER Journal, Volume 2, Issue 1 (2024).

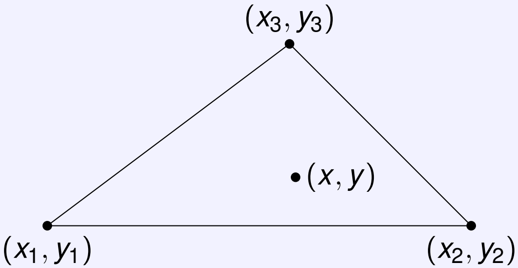
In particular, this suggests that the study of cubic polynomials provides a rich source of art-math activities at high school and college level courses, allowing to introduce many deep mathematical topics, as well as techniques for producing artistic images, fashionable items, jewelry designs, etc. It also opens the way to extension of these to general degree polynomials.

- The second part of the presentation shows many images based general polynomials and shares experiences with students and teachers.

The Algebraic Art Gallery Problem

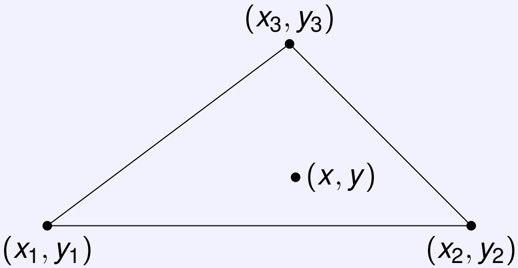


The Algebraic Art Gallery Problem



Where to place (x, y) (a security camera) so that product of its distances to the vertices (precious diamonds) is maximized?

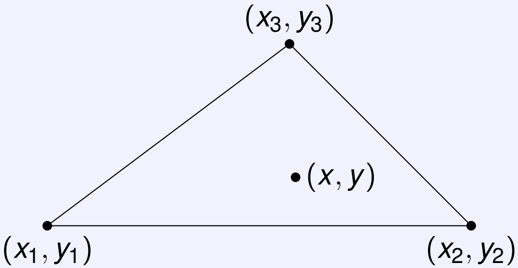
The Algebraic Art Gallery Problem



Where to place (x, y) (a security camera) so that product of its distances to the vertices (precious diamonds) is maximized?

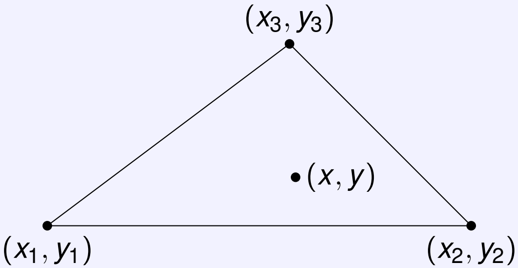
$$F(x,y) = \prod_{j=1}^{3} \sqrt{(x - x_j)^2 + (y - y_j)^2}.$$

The Algebraic Art Gallery Problem



For $j = 1, 2, 3$, set $z_j = x_j + iy_j$, where $i = \sqrt{-1}$ and $z = x + iy$.

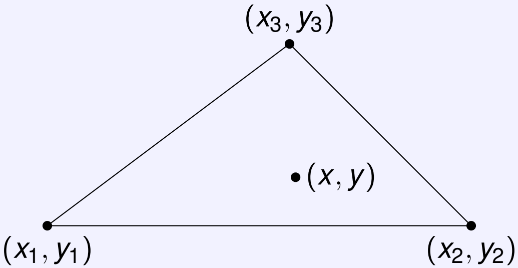
The Algebraic Art Gallery Problem



For $j = 1, 2, 3$, set $z_j = x_j + iy_j$, where $i = \sqrt{-1}$ and $z = x + iy$.

The product of distances from (x, y) happens to be $|p(z)|$, the modulus of the polynomial:

The Algebraic Art Gallery Problem

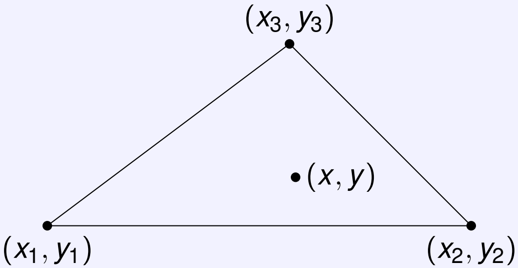


For $j = 1, 2, 3$, set $z_j = x_j + iy_j$, where $i = \sqrt{-1}$ and $z = x + iy$.

The product of distances from (x, y) happens to be $|p(z)|$, the modulus of the polynomial:

$$p(z) = (z - z_1)(z - z_2)(z - z_3), \quad \left(|z| = \sqrt{x^2 + y^2}\right).$$

The Algebraic Art Gallery Problem



For $j = 1, 2, 3$, set $z_j = x_j + iy_j$, where $i = \sqrt{-1}$ and $z = x + iy$.

The product of distances from (x, y) happens to be $|p(z)|$, the modulus of the polynomial:

$$p(z) = (z - z_1)(z - z_2)(z - z_3), \quad \left(|z| = \sqrt{x^2 + y^2}\right).$$

Claim: Maximizing point is on a side!

The Modulus Surface

The Modulus Surface

$$F(x, y) = |p(z)|.$$

The Modulus Surface

$$F(x, y) = |p(z)|.$$

The surface of $F(x, y)$ touches the xy-plane at the roots of $p(z)$.

The Modulus Surface

$$F(x, y) = |p(z)|.$$

The surface of $F(x, y)$ touches the xy-plane at the roots of $p(z)$.

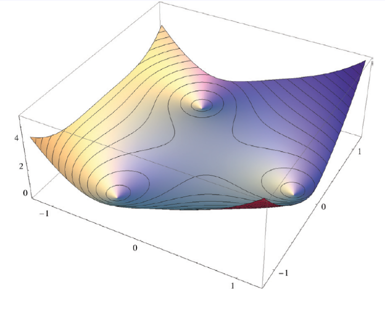


Figure: Graph of $F(x, y) = |z^3 - 1| = \sqrt{(x^3 - 3xy^2 - 1)^2 + (3x^2y - y^3)^2}$.

Ascent-Descent Direction for Modulus of a Polynomial

Ascent-Descent Direction for Modulus of a Polynomial

Write

$$F(z) = F(x, y) = |p(z)|.$$

Ascent-Descent Direction for Modulus of a Polynomial

Write

$$F(z) = F(x, y) = |p(z)|.$$

Given z_0 that is not a root, a *direction of descent* for $F(z)$ at z_0, is a complex number $u \in \mathbb{C}$ such that for some positive real number α_*

$$F(z_0 + \alpha u) < F(z_0), \quad \forall \alpha \in (0, \alpha_*).$$

Ascent-Descent Direction for Modulus of a Polynomial

Write

$$F(z) = F(x, y) = |p(z)|.$$

Given z_0 that is not a root, a *direction of descent* for $F(z)$ at z_0, is a complex number $u \in \mathbb{C}$ such that for some positive real number α_*

$$F(z_0 + \alpha u) < F(z_0), \quad \forall \alpha \in (0, \alpha_*).$$

The "cone of descent" at z_0 is the set of all descent directions. Likewise, the "cone of ascent" can be defined.

The Geometric Modulus Principle (GMP)

Theorem

(Geometric Modulus Principle)

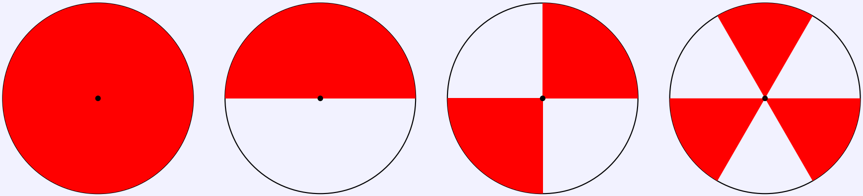


Figure: From left to right: Sectors of ascent (red) and descent (white) at z_0 for $p(z_0) = 0$; $p'(z_0) \neq 0$; $p'(z_0) = 0$ but $p''(z_0) \neq 0$; $p'(z_0) = p''(z_0) = 0$ but $p'''(z_0) \neq 0$.

A Geometric Modulus Principle for Polynomials, Monthly, 2011, B.K.

GMP Visualization at Critical Point of $z^3 - 1$

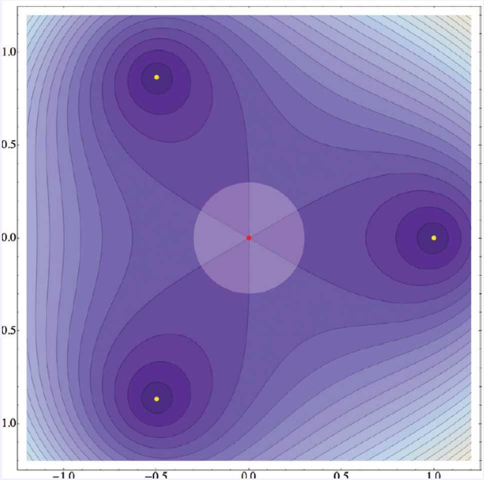


Figure: Ascent-Decent direction for $z^3 - 1$ at origin.

GMP as Design

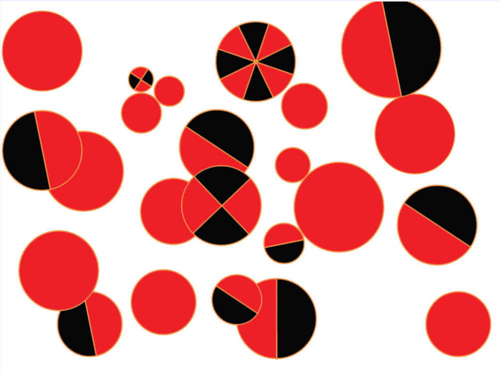


Figure: There is a polynomial whose modulus plot conforms to the ascent and descent sectors of these disks!

GMP as Design

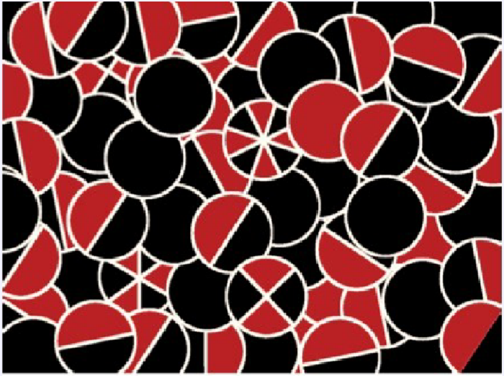


Figure: There is a polynomial whose modulus plot conforms to the ascent and descent sectors of these disks!

GMP as Fashion

Mathematical Applications of GMP

Theorem

(FTA) *A nonconstant polynomial* $p(z)$ *must have a root.*

Mathematical Applications of GMP

Theorem

(FTA) *A nonconstant polynomial $p(z)$ must have a root.*

One Line Proof: Use GMP and that minimum of $F(x, y)$ is attained.

Mathematical Applications of GMP

Theorem

(FTA) *A nonconstant polynomial $p(z)$ must have a root.*

One Line Proof: Use GMP and that minimum of $F(x, y)$ is attained.

Also **(Gauss-Lucas Theorem)** and **(Maximum Modulus Theorem)**.

Mathematical Applications of GMP

Theorem

(FTA) *A nonconstant polynomial $p(z)$ must have a root.*

One Line Proof: Use GMP and that minimum of $F(x, y)$ is attained.

Also **(Gauss-Lucas Theorem)** and **(Maximum Modulus Theorem)**.

A non-technical description of GMP is given in:

Mathematical Applications of GMP

Theorem

(FTA) *A nonconstant polynomial $p(z)$ must have a root.*

One Line Proof: Use GMP and that minimum of $F(x, y)$ is attained.

Also **(Gauss-Lucas Theorem)** and **(Maximum Modulus Theorem)**.

A non-technical description of GMP is given in:

The fundamental theorem of algebra for artists, "Math Horizons", 2013, B.K. and B. Torrence.

Mathematical Applications of GMP

Theorem

(FTA) *A nonconstant polynomial $p(z)$ must have a root.*

One Line Proof: Use GMP and that minimum of $F(x, y)$ is attained.

Also **(Gauss-Lucas Theorem)** and **(Maximum Modulus Theorem)**.

A non-technical description of GMP is given in:

The fundamental theorem of algebra for artists, “Math Horizons”, 2013, B.K. and B. Torrence.

The Best Writing in Mathematics, Princeton University Press, 2014.

GMP as Art and Design (Jewelry)

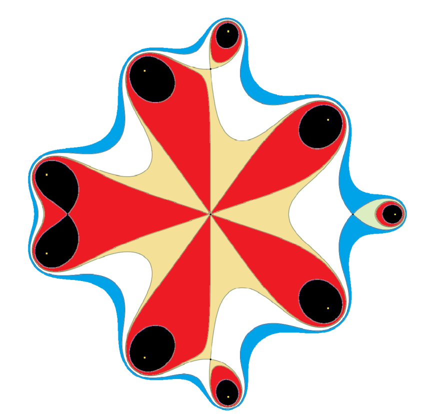


Figure: Actual Modulus plot of $z^9 - z^5 - 1$.

Solving a Real Cubic Equation by Cardano's Formula

$$P(x) = x^3 + px + q = 0$$

Solving a Real Cubic Equation by Cardano's Formula

$$P(x) = x^3 + px + q = 0$$

$$\theta = \sqrt[3]{-\frac{q}{2} + \sqrt{-\Delta}} + \sqrt[3]{-\frac{q}{2} - \sqrt{-\Delta}}, \quad \Delta = -(\frac{q^2}{4} + \frac{p^3}{27})$$

Solving a Real Cubic Equation by Cardano's Formula

$$P(x) = x^3 + px + q = 0$$

$$\theta = \sqrt[3]{-\frac{q}{2} + \sqrt{-\Delta}} + \sqrt[3]{-\frac{q}{2} - \sqrt{-\Delta}}, \quad \Delta = -(\frac{q^2}{4} + \frac{p^3}{27})$$

When $\Delta < 0$, all roots are real nevertheless the formula expresses them in terms of complex numbers and these in turn require approximation.

Solving a Real Cubic Equation by Cardano's Formula

$$P(x) = x^3 + px + q = 0$$

$$\theta = \sqrt[3]{-\frac{q}{2} + \sqrt{-\Delta}} + \sqrt[3]{-\frac{q}{2} - \sqrt{-\Delta}}, \quad \Delta = -(\frac{q^2}{4} + \frac{p^3}{27})$$

When $\Delta < 0$, all roots are real nevertheless the formula expresses them in terms of complex numbers and these in turn require approximation.

Much is written on cubic equations and their fascinating history.

Solving a Real Cubic Equation by Cardano's Formula

$$P(x) = x^3 + px + q = 0$$

$$\theta = \sqrt[3]{-\frac{q}{2} + \sqrt{-\Delta}} + \sqrt[3]{-\frac{q}{2} - \sqrt{-\Delta}}, \quad \Delta = -(\frac{q^2}{4} + \frac{p^3}{27})$$

When $\Delta < 0$, all roots are real nevertheless the formula expresses them in terms of complex numbers and these in turn require approximation.

Much is written on cubic equations and their fascinating history.

"On Tusi's classification of cubic equations and its connections to Cardano's formula and Khayyam's geometric solution", Palestine Journal of Mathematics. **11**(4), 7–22 (2022), B. K. and R. Zaare-Nahandi.

Solution of Real Cubic Equation without Cardano's

"Solution of Real Cubic Equations without Cardano's Formula", arxiv (2023), B.K.:

Building on Tusi's classification together with Smale's *point estimation*:

• First, reduce any cubic equation into one of four canonical forms with 0, ± 1 coefficients, except the constant term $\pm q$, $q \geq 0$.

• Next, compute ρ_q, any approximation to $\sqrt[3]{q}$ to within a relative error of five percent.

• Finally, in terms of ρ_q a *seed* x_0 can be defined so that in t Newton iterations:

$$|x_t - \theta_q| \leq \sqrt[3]{q} \cdot 2^{-2^t}, \quad \theta_q \quad \text{a real root.}$$

(essentially 5-6 Newton iterations are enough!)

Solution of Real Cubic Equation without Cardano's

"Solution of Real Cubic Equations without Cardano's Formula", arxiv (2023), B.K.:

Building on Tusi's classification together with Smale's *point estimation*:

• First, reduce any cubic equation into one of four canonical forms with 0, ± 1 coefficients, except the constant term $\pm q$, $q \geq 0$.

• Next, compute ρ_q, any approximation to $\sqrt[3]{q}$ to within a relative error of five percent.

• Finally, in terms of ρ_q a *seed* x_0 can be defined so that in t Newton iterations:

$$|x_t - \theta_q| \leq \sqrt[3]{q} \cdot 2^{-2^t}, \quad \theta_q \quad \text{a real root.}$$

(essentially 5-6 Newton iterations are enough!)
Solving real cubic equations has applications in computer graphics.

Newton's Iteration

Given a polynomial $p(z)$ and a *seed* $z_0 \in \mathbb{C}$,

$$z_{j+1} = N_p(z_j) = z_j - \frac{p(z_j)}{p'(z_j)}, \quad j \geq 1.$$

Newton's Iteration

Given a polynomial $p(z)$ and a *seed* $z_0 \in \mathbb{C}$,

$$z_{j+1} = N_p(z_j) = z_j - \frac{p(z_j)}{p'(z_j)}, \quad j \geq 1.$$

The *orbit* of z_0, denoted by $O^+(z)$, is the sequence $z_0, z_1, z_2, \ldots,$

Newton's Iteration

Given a polynomial $p(z)$ and a *seed* $z_0 \in \mathbb{C}$,

$$z_{j+1} = N_p(z_j) = z_j - \frac{p(z_j)}{p'(z_j)}, \quad j \geq 1.$$

The *orbit* of z_0, denoted by $O^+(z)$, is the sequence $z_0, z_1, z_2, \ldots,$

Basin of attraction of a root θ of $p(z)$ is the set of all seeds z_0 where $O^+(z_0)$ converges to θ. It is an open set.

Newton's Iteration

Given a polynomial $p(z)$ and a *seed* $z_0 \in \mathbb{C}$,

$$z_{j+1} = N_p(z_j) = z_j - \frac{p(z_j)}{p'(z_j)}, \quad j \geq 1.$$

The *orbit* of z_0, denoted by $O^+(z)$, is the sequence $z_0, z_1, z_2, \dots,$

Basin of attraction of a root θ of $p(z)$ is the set of all seeds z_0 where $O^+(z_0)$ converges to θ. It is an open set.

Julia set is the boundary of any basin of attraction, often a fractal set.

Newton's Iteration

Given a polynomial $p(z)$ and a *seed* $z_0 \in \mathbb{C}$,

$$z_{j+1} = N_p(z_j) = z_j - \frac{p(z_j)}{p'(z_j)}, \quad j \geq 1.$$

The *orbit* of z_0, denoted by $O^+(z)$, is the sequence $z_0, z_1, z_2, \dots,$

Basin of attraction of a root θ of $p(z)$ is the set of all seeds z_0 where $O^+(z_0)$ converges to θ. It is an open set.

Julia set is the boundary of any basin of attraction, often a fractal set.

Fatou set is the complement of Julia set.

Newton's Iteration

Given a polynomial $p(z)$ and a *seed* $z_0 \in \mathbb{C}$,

$$z_{j+1} = N_p(z_j) = z_j - \frac{p(z_j)}{p'(z_j)}, \quad j \geq 1.$$

The *orbit* of z_0, denoted by $O^+(z)$, is the sequence $z_0, z_1, z_2, \dots,$

Basin of attraction of a root θ of $p(z)$ is the set of all seeds z_0 where $O^+(z_0)$ converges to θ. It is an open set.

Julia set is the boundary of any basin of attraction, often a fractal set.

Fatou set is the complement of Julia set.

Newton's function is one of infinitely many iteration functions. In particular, there is an infinite family, called Basic Family, used individually or collectively. Basic Family goes with other names but we discovered many novel and useful properties of the family for polynomiography.

Polynomiography

Visualization of polynomial root-finding via iterative methods, individually or collectively.

Polynomiography

Visualization of polynomial root-finding via iterative methods, individually or collectively.

It leads to images called polynomiographs.

Polynomiography

Visualization of polynomial root-finding via iterative methods, individually or collectively.

It leads to images called polynomiographs.

Polynomiographs could be fractal or non-fractal.

Polynomiography

Visualization of polynomial root-finding via iterative methods, individually or collectively.

It leads to images called polynomiographs.

Polynomiographs could be fractal or non-fractal.

Polynomiography can serve as a tool for mathematicians, scientists, educators, students, artists and amateurs.

Polynomiography

Visualization of polynomial root-finding via iterative methods, individually or collectively.

It leads to images called polynomiographs.

Polynomiographs could be fractal or non-fractal.

Polynomiography can serve as a tool for mathematicians, scientists, educators, students, artists and amateurs.

Polynomiography can serve as an effective tool for teaching and learning deep topics about math and algorithms.

Polynomiography

Visualization of polynomial root-finding via iterative methods, individually or collectively.

It leads to images called polynomiographs.

Polynomiographs could be fractal or non-fractal.

Polynomiography can serve as a tool for mathematicians, scientists, educators, students, artists and amateurs.

Polynomiography can serve as an effective tool for teaching and learning deep topics about math and algorithms.

Several polynomiography article are in publication and some online.

Polynomiography

Visualization of polynomial root-finding via iterative methods, individually or collectively.

It leads to images called polynomiographs.

Polynomiographs could be fractal or non-fractal.

Polynomiography can serve as a tool for mathematicians, scientists, educators, students, artists and amateurs.

Polynomiography can serve as an effective tool for teaching and learning deep topics about math and algorithms.

Several polynomiography article are in publication and some online.

"Polynomial Root-Finding and Polynomiography," World Scientific, 2008, B.K.

Newton Polynomiograph of $z^3 - 1$

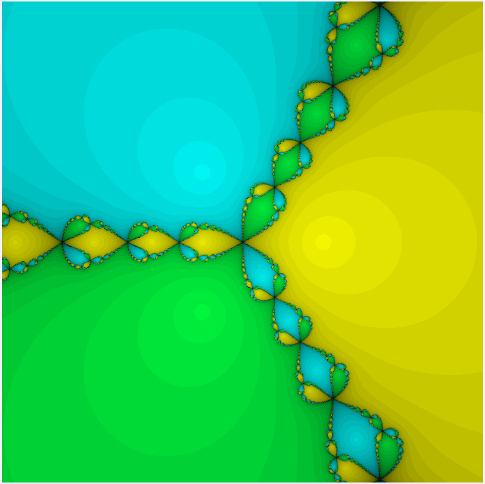


Figure: Cayley (1897) thought basins of attraction would be Voronoi regions.

Approximate Voronoi Region (Cayley almost right)

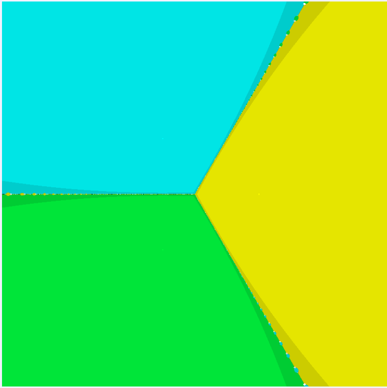


Figure: Polynomiograph of $z^3 - 1$ but via a complicated iteration function.

Approximate Voronoi Region (Cayley almost right)

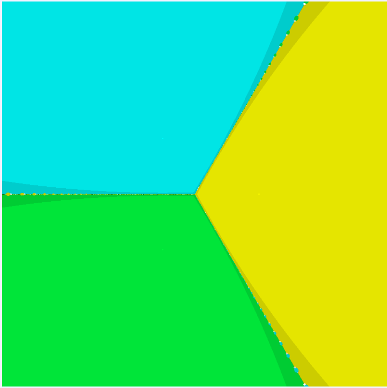


Figure: Polynomiograph of $z^3 - 1$ but via a complicated iteration function.

Polynomial root-finding methods whose basins of attractions approximate Voronoi diagrams, Discrete and Comput. Geometry, 2011, B. K.

Polynomiographs Are Not Necessarily Fracral

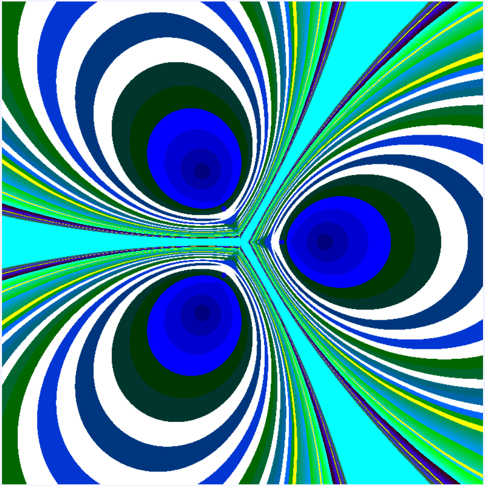


Figure: A polynomiograph of $z^3 - 1$ via a family of iteration functions.

Some Cubic Polynomiographs

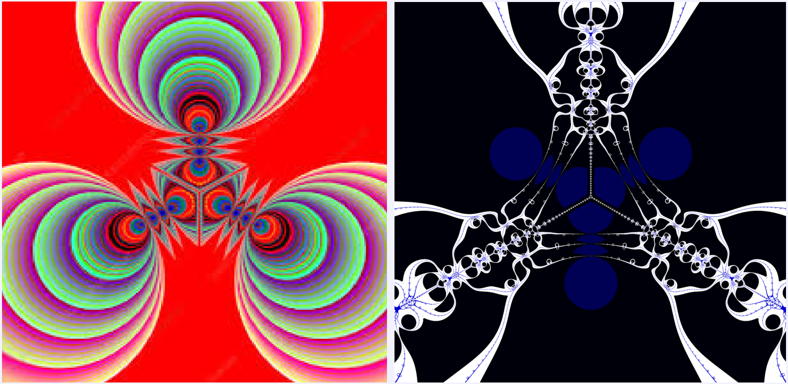
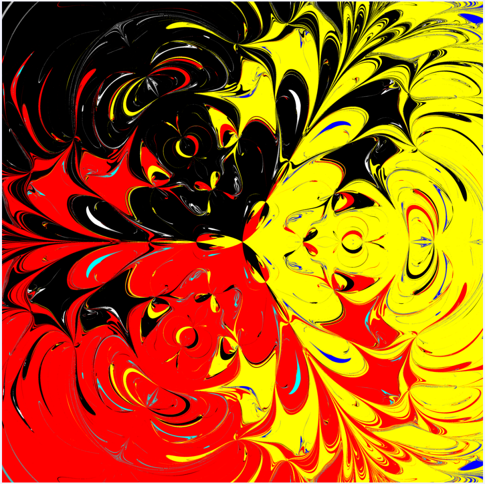


Figure: Two polynomiographs of the same cubic (Life and Death).

A T-Shirt Design

Newton-Ellipsoid Polynomiograph of $z^3 - 1$



Newton-Ellipsoid Polynomiography” in *Journal of Mathematics and the Arts*, 2019, B.K. and E. Lee.

Another Non-Fractal Polynomiograph of $z^3 - 1$

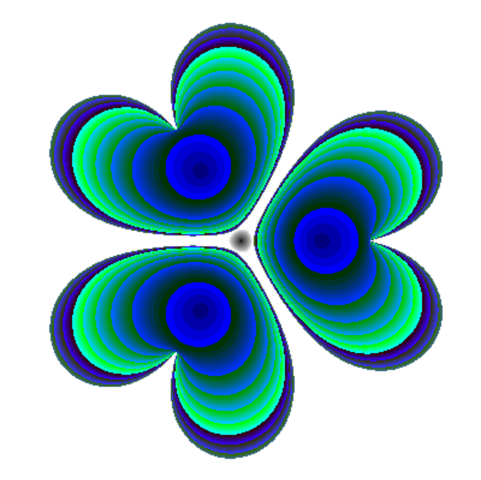


Figure: Based on a Family of Iteration Functions

Polynomiograph of a Quartic

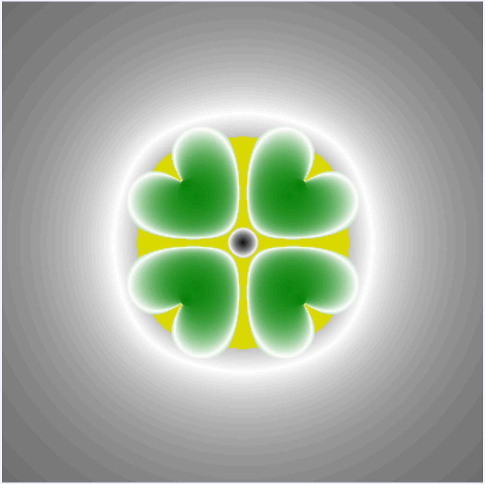


Figure: Clover Leaf - A familiar jewelry design?

Playing with Cubic Polynomials

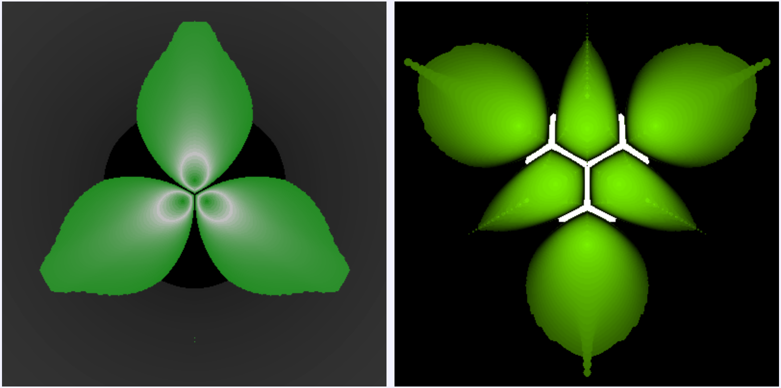


Figure: Polynomigraphs of a cubic (left) and product of two cubics.

Cubic Polynomiographs

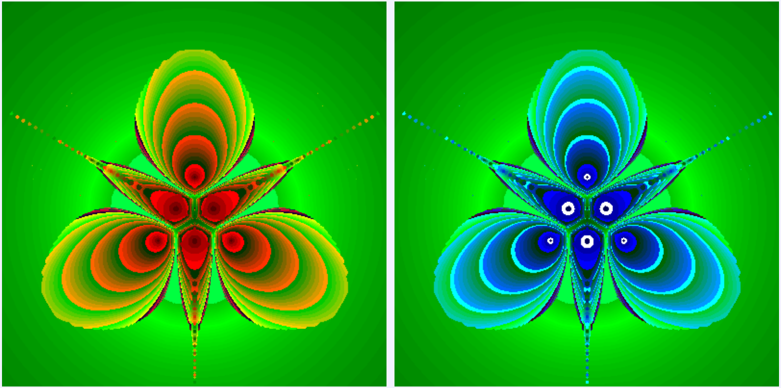


Figure: Polynomiographs from products of cubics.

Cactus and Cactus Polynomiograph

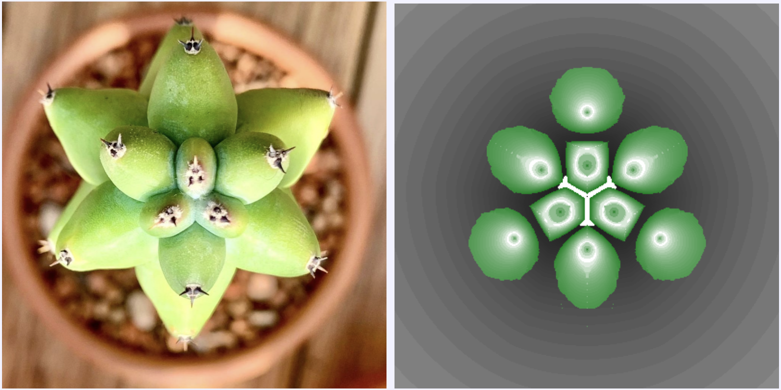


Figure: Actual Cactus and a Polynomiograph from product of three cubics.

Some Quadratic Polynomographs

Figure: Polynomiographs of $z^2 - 1$ (left) and $z^2 + 1$ via Newton's.

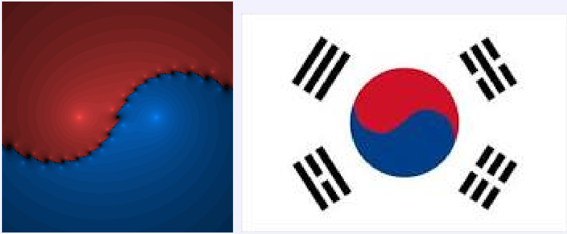
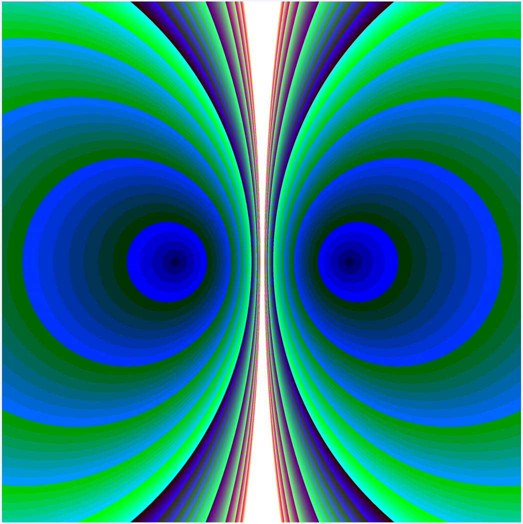


Figure: $z^2 - 1$ under $z_{j+1} = z_j - \alpha p(z_j)/p'(z_j)$, α a complex number.

Presented to over 200 South Korean middle schoolers.

In the image $\alpha = .3 - .3i$, found by an IIT graduate student, during polynomiography presentation with a demo software in India.

Quadratic Polynomiograph on Cover of SIGGRAPH



Polynomial & Polynomiography = Math&Art

- Polynomiography is a game of hide-and-seek with a bunch of dots on a painting canvas. Hide with polynomial equations, seek with iterative methods.

Polynomial & Polynomiography = Math&Art

- Polynomiography is a game of hide-and-seek with a bunch of dots on a painting canvas. Hide with polynomial equations, seek with iterative methods.

- Working with polynomiography software is like to working with a camera, a musical instrument or any artistic tool.

Polynomial & Polynomiography = Math&Art

- Polynomiography is a game of hide-and-seek with a bunch of dots on a painting canvas. Hide with polynomial equations, seek with iterative methods.
- Working with polynomiography software is like to working with a camera, a musical instrument or any artistic tool.
- Through practice, one can learn to produce exquisite and complex designs analogous to the most sophisticated human designs, abstract art and more.

Polynomial & Polynomiography = Math&Art

• Polynomiography is a game of hide-and-seek with a bunch of dots on a painting canvas. Hide with polynomial equations, seek with iterative methods.

• Working with polynomiography software is like to working with a camera, a musical instrument or any artistic tool.

• Through practice, one can learn to produce exquisite and complex designs analogous to the most sophisticated human designs, abstract art and more.

• Polynomiographic designs could be reminiscent of intricate patterning of Islamic art, Oriental and Persian carpets, or design of French fabrics.

Polynomial & Polynomiography = Math&Art

- Polynomiography is a game of hide-and-seek with a bunch of dots on a painting canvas. Hide with polynomial equations, seek with iterative methods.
- Working with polynomiography software is like to working with a camera, a musical instrument or any artistic tool.
- Through practice, one can learn to produce exquisite and complex designs analogous to the most sophisticated human designs, abstract art and more.
- Polynomiographic designs could be reminiscent of intricate patterning of Islamic art, Oriental and Persian carpets, or design of French fabrics.
- Polynomiographic designs could also be reminiscent of the work of artists associated with abstract expressionism and minimalism.

Polynomial & Polynomiography = Math&Art

• Polynomiography is a game of hide-and-seek with a bunch of dots on a painting canvas. Hide with polynomial equations, seek with iterative methods.

• Working with polynomiography software is like to working with a camera, a musical instrument or any artistic tool.

• Through practice, one can learn to produce exquisite and complex designs analogous to the most sophisticated human designs, abstract art and more.

• Polynomiographic designs could be reminiscent of intricate patterning of Islamic art, Oriental and Persian carpets, or design of French fabrics.

• Polynomiographic designs could also be reminiscent of the work of artists associated with abstract expressionism and minimalism.

• Polynomiography could be used in classrooms for teaching art or mathematics at every level, as well as in professional settings.

Polynomial & Polynomiography = Math&Art

• Polynomiography is a game of hide-and-seek with a bunch of dots on a painting canvas. Hide with polynomial equations, seek with iterative methods.

• Working with polynomiography software is like to working with a camera, a musical instrument or any artistic tool.

• Through practice, one can learn to produce exquisite and complex designs analogous to the most sophisticated human designs, abstract art and more.

• Polynomiographic designs could be reminiscent of intricate patterning of Islamic art, Oriental and Persian carpets, or design of French fabrics.

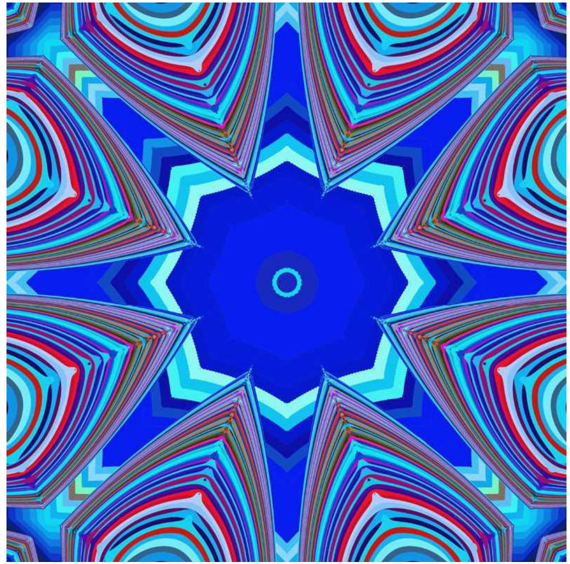
• Polynomiographic designs could also be reminiscent of the work of artists associated with abstract expressionism and minimalism.

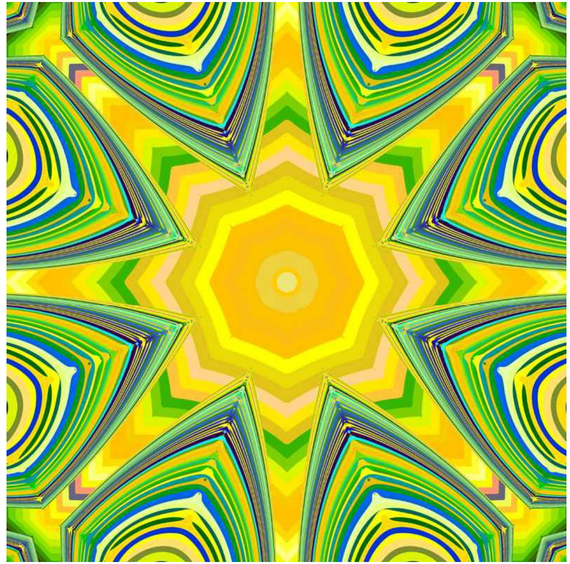
• Polynomiography could be used in classrooms for teaching art or mathematics at every level, as well as in professional settings.

• With the rise of AI, polynomials & polynomiographs, even when restricted to small degrees, provide a huge source of fantastic training images.

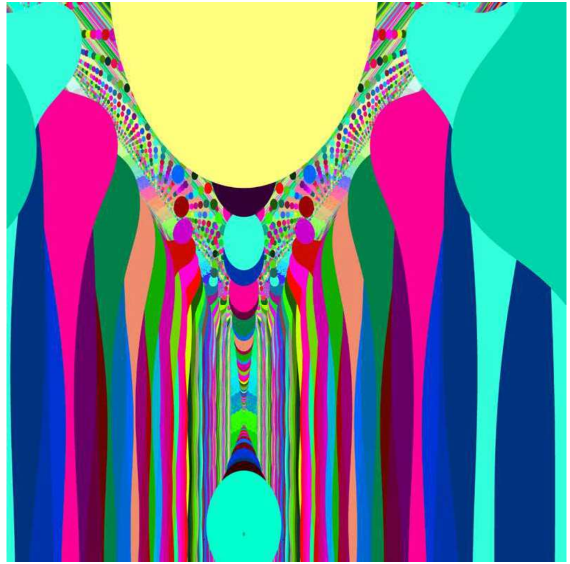
Butterfly

Symmetry





Cathedral



Some Quotes

"*Lose your fear of math with computer graphics that displays the beauty and symmetry hidden within algebraic equations.*"

DISCOVER Magazine ... *on polynomiography*

"Over the centuries, mathematicians have developed a variety of methods of solving equations. Bahman Kalantari of Rutgers University has developed visualization software that brings the process of finding the roots of a polynomial equation into the realm of design and art."

Ivars Peterson *SCIENCE NEWS*

"*Professor Kalantari's work combines in a very striking way mathematics and visual arts. His work on 'polynomiography' is very original and pretty.*"

Cumrun Vafa *Professor of Physics, Harvard*

“Bahman Kalantari’s work on Polynomiography is visually striking and provides profound insight into root finding algorithms.

In future generations, I expect that visualization of mathematical algorithms will become an expected part of mathematical research.
Bahman Kalantari’s skills are here now and we can enjoy the beautiful results as he has applied them to Polynomiography.”

Cliff Reiter *Professor of Mathematics Lafayette College, Pennsylvania*

“The visual results are often elegant. This method has led [Kalantari] to develop a new and powerful method of artistic creation, ..., a playful and instructive technique where mathematics helps art, which gratefully, comes to support mathematics.”

Claude Bruter *Professor of Mathematics, U. of Paris*

“*Polynomiography ... has an enormous and fruitful field of applications in visual arts, education and scientific research...*”

Vera W. de Spinadel *President of International Mathematics & Design Association, Argentina*

“POLYNOMIOGRAPHY!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
I lOVE IT!!!
It was so incredibly cool.
I just want to take it home and play on it the rest of the summer!”

Alexandria Munger *(age 14, a middle schooler at Girls Plus Math Camp., Illinois)*

“I didn’t know math could make such beautiful images.”

A 9 years old boy *(Rutgers Day, April 25th, 2009)*

What is a Polynomial Equation and What is it Good For?

Polynomial equation is “**solving for** x,” a problem with life-long usage, intellectually and otherwise.

What is 17 percent of 3574?

If I know the length of the two sides of a right triangle, can I **measure** the length of the hypothenuse?

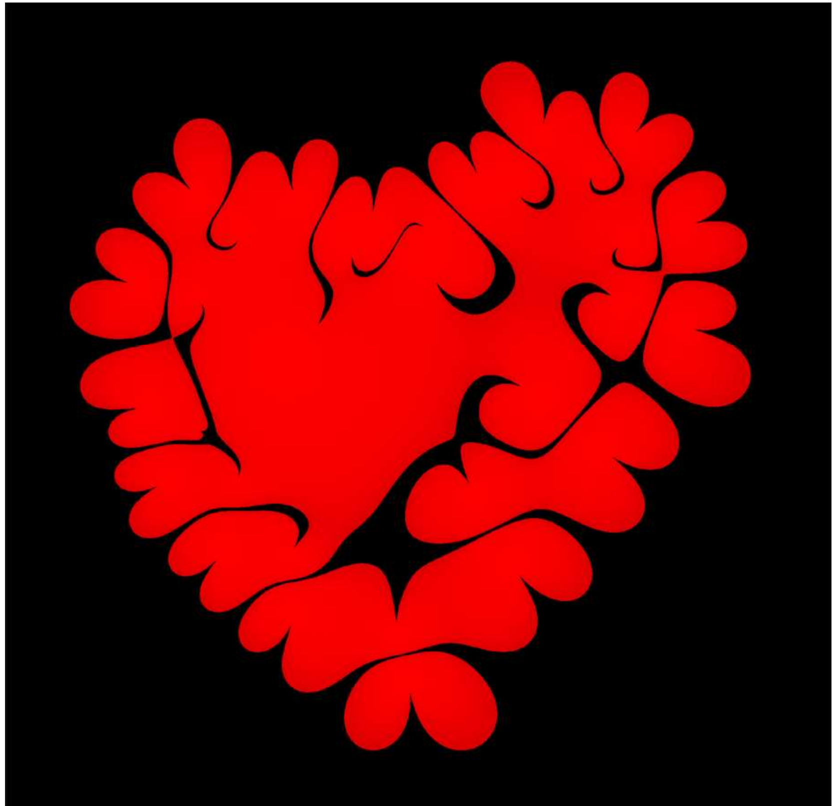
What is the square-root of two? And how do I **compute** it?

Why Is Solving Polynomial Equations Important?

The very ideas of abstract thinking and using mathematical notation are largely due to the study of polynomial equations.

Furthermore, solving polynomial equations has historically motivated the introduction of some fundamental concepts of mathematics ...

Victor Pan, an internationally recognized leader in the field of computer science



Hearts

$$x^{17}+(-0.99+2.865\sqrt{-1})x^{16}+(-4.0289-3.7579\sqrt{-1})x^{15}+(7.6906-8.1716\sqrt{-1})x^{14}+(10.1661+10.81\sqrt{-1})x^{13}+(-14.9238+14.8406\sqrt{-1})x^{12}+(-17.6515-21.7828\sqrt{-1})x^{11}+(19.445-19.7326\sqrt{-1})x^{10}+(26.861+26.7445\sqrt{-1})x^{9}+(-4.5597+36.2215\sqrt{-1})x^{8}+(-23.0202+22.955\sqrt{-1})x^{7}+(-103.1901+18.1539\sqrt{-1})x^{6}+(-56.6536+20.3531\sqrt{-1})x^{5}+(-47.8313+228.2366\sqrt{-1})x^{4}+(-208.2068-229.4483\sqrt{-1})x^{3}+(-117.5589-248.0972\sqrt{-1})x^{2}+(-1092.4738-7.5769\sqrt{-1})x+(103.3231+536.4582\sqrt{-1})$$

How Do I Select a Nice Polynomial?

Take any number, say 387624730. You can convert it into a polynomial in many ways.

Take any number, say 387624730. You can convert it into a polynomial in many ways.

$$3x^8 + 8x^7 + 7x^6 + 6x^5 + 2x^4 + 4x^3 + 7x^2 + 3x + 0$$

Take any number, say 387624730. You can convert it into a polynomial in many ways.

$$3x^8 + 8x^7 + 7x^6 + 6x^5 + 2x^4 + 4x^3 + 7x^2 + 3x + 0$$

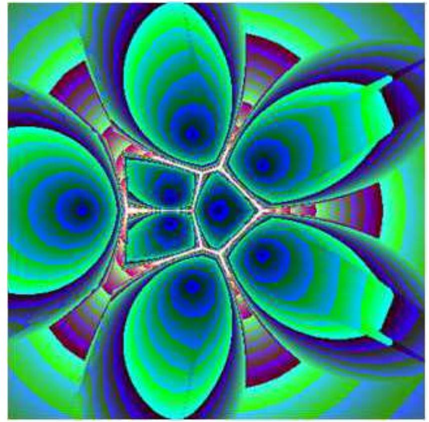
One can capture infinitely many polynomiographs of this single equation.

Take any number, say 387624730. You can convert it into a polynomial in many ways.

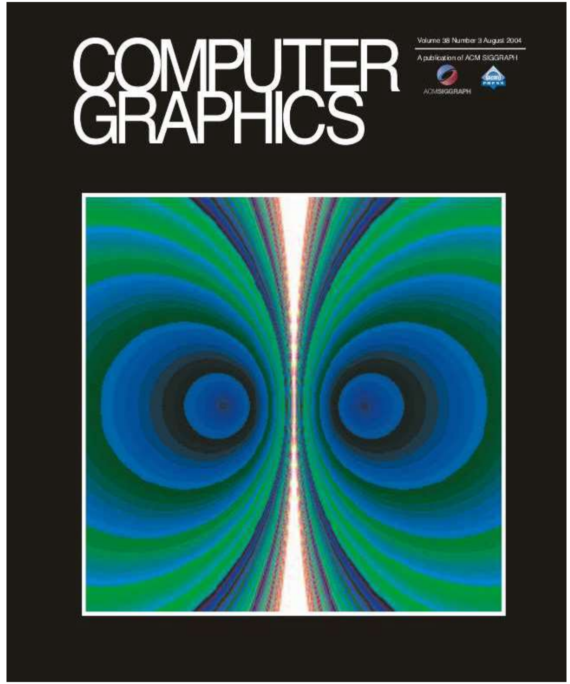
$$3x^8 + 8x^7 + 7x^6 + 6x^5 + 2x^4 + 4x^3 + 7x^2 + 3x + 0$$

One can capture infinitely many polynomiographs of this single equation.

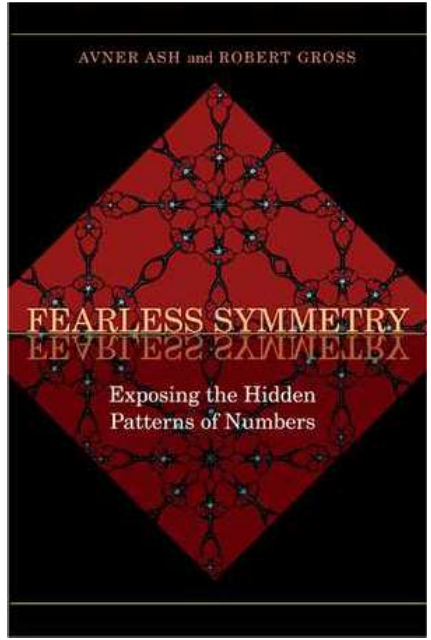
Here is one:



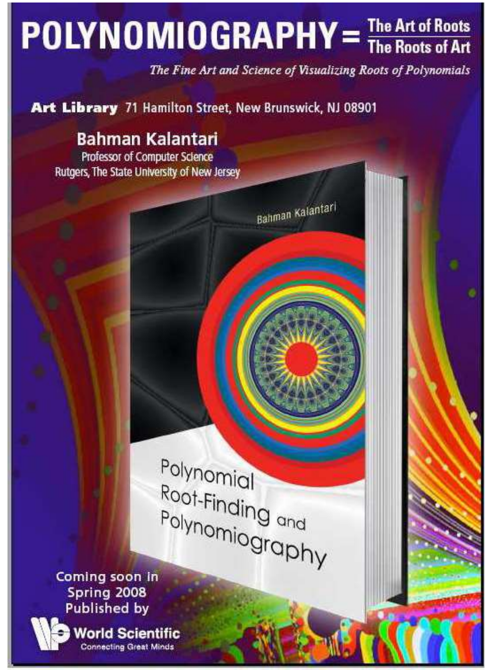
Polynomiography In Media



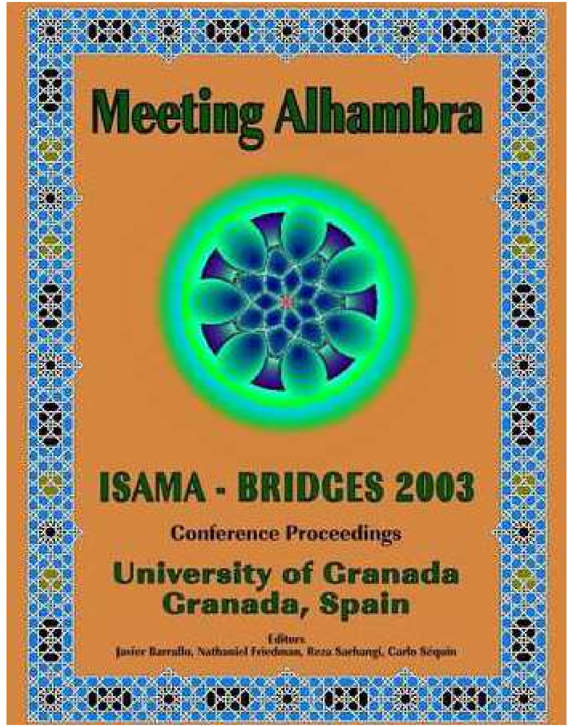
SIGGRAPH Quarterly (cover)



Princeton University Mathematics (cover)



World Scientific, “POLYNOMIAL ROOT-FINDING and POLYNOMIOGRAPHY”



Art-Math Proceedings (cover)

physicsworld.com

BEAUTIFUL MATTER

Symmetry is central to modern physics. Source: Bahman Kalantari/Science Photo Library

Back to article

Physicsworld

OBSERVATORIO

Un visor infrarrojo facilita la conducción

Nuevos ojos en la noche

El mismísimo coche fantástico envidiaría a los nuevos automóviles equipados con visores infrarrojos que facilitan la conducción nocturna. Estos ojos de lince, que cuestan 3.514,80 € y sólo pueden adquirirse en combinación con los faros bixenón, alumbran la carretera 200 metros. La cámara se sitúa en uno de los faros y capta la imagen, que aparece en la pantalla del salpicadero. Con este sistema se logra una visión similar a la obtenida con las luces largas, pero con la ventaja de que no deslumbran al resto de conductores, ya que la longitud de onda de los infrarrojos no es perceptible a simple vista. El automovilista, al ver el obstáculo en la carretera con mayor antelación, puede reaccionar antes frenando o desviándose.

Los ingenieros de Mercedes Benz son de los primeros que han instalado el aparato en sus vehículos, más concretamente en sus berlinas de lujo y los automóviles de la clase S, pero otros fabricantes no se han quedado atrás y también están equipando sus modelos estrella con este útil invento que puede evitar accidentes. ■

Con la luz de los faros no se ve bien el cervatillo que está a la izquierda de la carretera. Sin embargo, con la nueva cámara –arriba– que se coloca en una de las luces, el animal aparece claramente en la pantalla del salpicadero –derecha–. El conductor entonces puede hacer la maniobra pertinente.

■ Matemáticas

Arte con los números

Bienvenidos a la polinomiografía, término inventado por el profesor de informática de la Universidad Rutgers, EE UU, Bahman Kalantari para describir cómo se puede hacer arte usando un ordenador y determinados polinomios, las funciones matemáticas más famosas de nuestros tiempos de escuela. En esencia, lo que hace Kalantari es buscar los ceros de determinados polinomios, esto es, buscar para qué valores la función polinómica se hace cero. Una vez identificados, se programa la función y se "adereza" según el sentido artístico del matemático, esto es, con las estructuras y colores que ha seleccionado. Así, las soluciones numéricas se convierten en bellas imágenes.

Estas imágenes se usan resolviendo polinomios, unas funciones matemáticas con formas como esta: $10x^{45} - 11x^{14} + 23x^{5} - 2$

FOTOS: BAHMAN KALANTARI/SPL

INSÓLITO... PERO CIERTO

¿Un porrito? Las 10.000 toneladas de cannabis recolectadas cada año en EE UU convierten este cultivo en el principal producto agrícola del país, según su valor en el mercado.

La señal del pez. El primer símbolo de los cristianos no fue la cruz. Antes se identificaban con un pez, que en griego se dice ixthus. Con la palabra formaron el acróstico Iesous Xhristos Theou Hyios Soter (Jesucristo, de Dios el Hijo, Salvador).

Un baño que pela. La temperatura media de los océanos hace 3.500 millones de años era de 80 °C. Hoy es de 17 °C en la superficie.

Calmante supersalivado. Científicos del Instituto Pasteur, en París, han descubierto en la saliva humana un potente analgésico, la opiorfina. Es tres veces más eficaz contra el dolor que la morfina, según se ha demostrado en ratones.

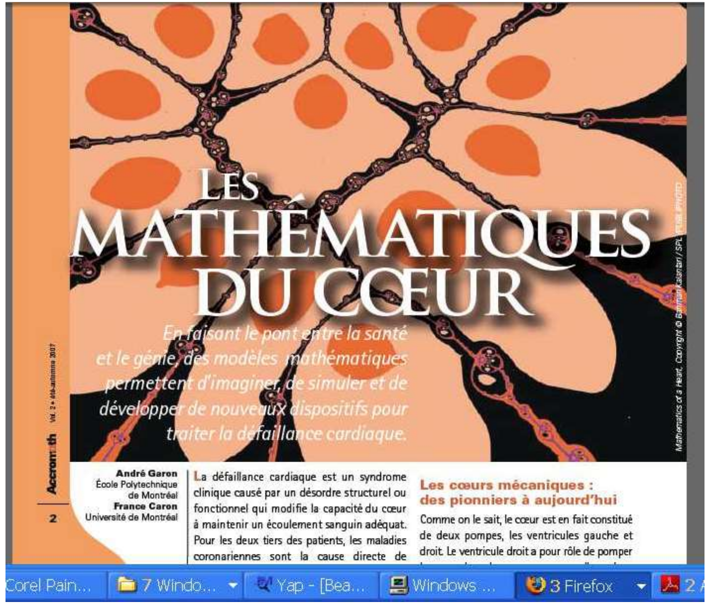
Que viene el pájaro... Además de su afición a la cirugía estética, Cher y Michael Jackson comparten su ornitofobia. Así se llama al pánico a las aves.

¡Guau, pásame el tumor! El tumor genital, que sólo afecta a perros, es el único cáncer que se contagia. Las células malignas pasan de un animal a otro durante el acto sexual.

32 muy

Muy-Interesante (popular science magazine of Spain)

Tiede (popular science magazine of Finland)

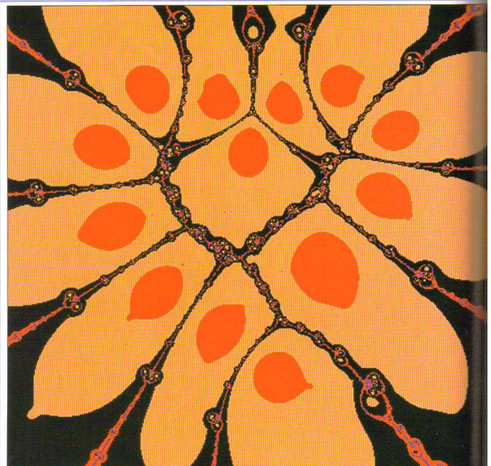


Accromath (U. of Montreal University magazine)

SPRING 2003

BY LEE LUSARDI CONNOR

PEOPLE & PLACES



BEAUTY BY THE NUMBERS

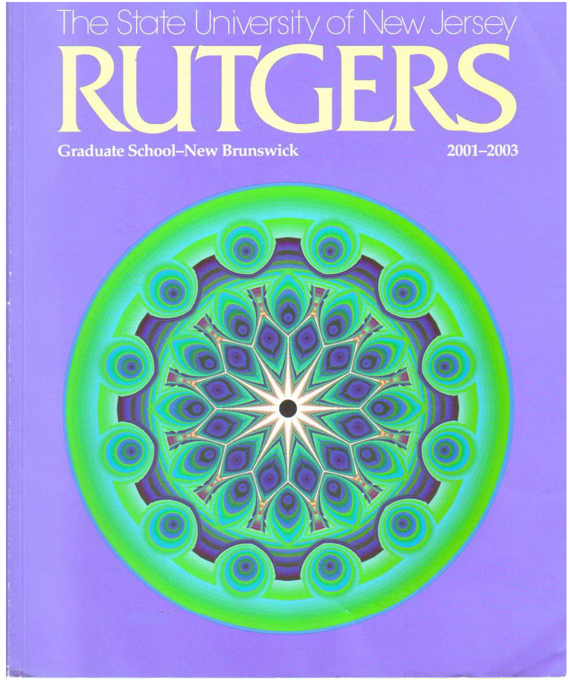
What if you could use a computer to turn equations into dazzling, colorful designs? That's the kind of question only a computer scientist—a particularly *creative* computer scientist—would ask.

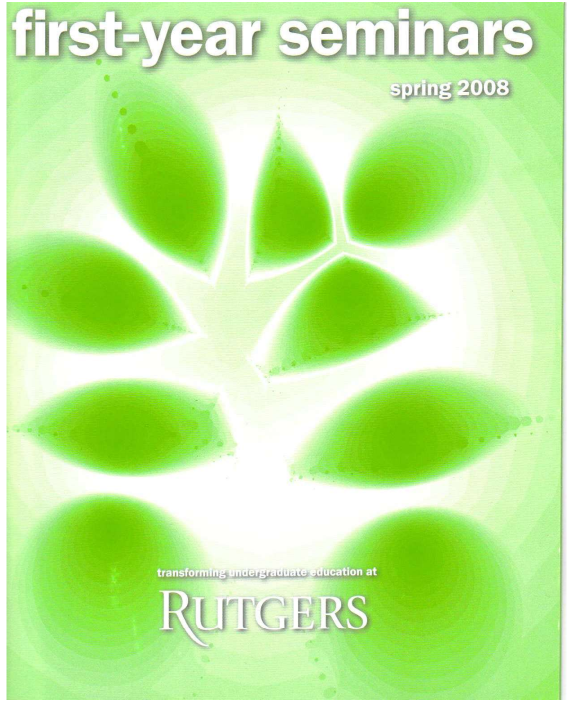
Enter Bahman Kalantari, an associate professor of computer science at Rutgers University in New Brunswick. His answer: "polynomiography"—a computer art form created by turning polynomials, a fundamental algebraic function, into patterns. (Polynomials are defined as "linear combinations of integral powers of a variable," such as x-1.) "We can 'shoot pictures' of polynomials and then color them using our own personal artistry," says Kalantari. "Just as with photography and painting, with practice one gets to be better and better at it."

Shown above is Kalantari's "Mathematics of a Heart." The possibilities are limitless, he says. "You can design images that would look wonderful as abstract painting, greeting cards, upholstery or any kind of decorative fabric."

Patents are now pending for software that will make polynomiography available to the public. In the meantime, check it out at www.polynomiography.com.

New Jersey Savvy-Living


The State University of New Jersey
RUTGERS
Graduate School–New Brunswick
2001–2003


first-year seminars
spring 2008
transforming undergraduate education at
RUTGERS

Also featured in New Jersey media and more

Polynomiography In Schools

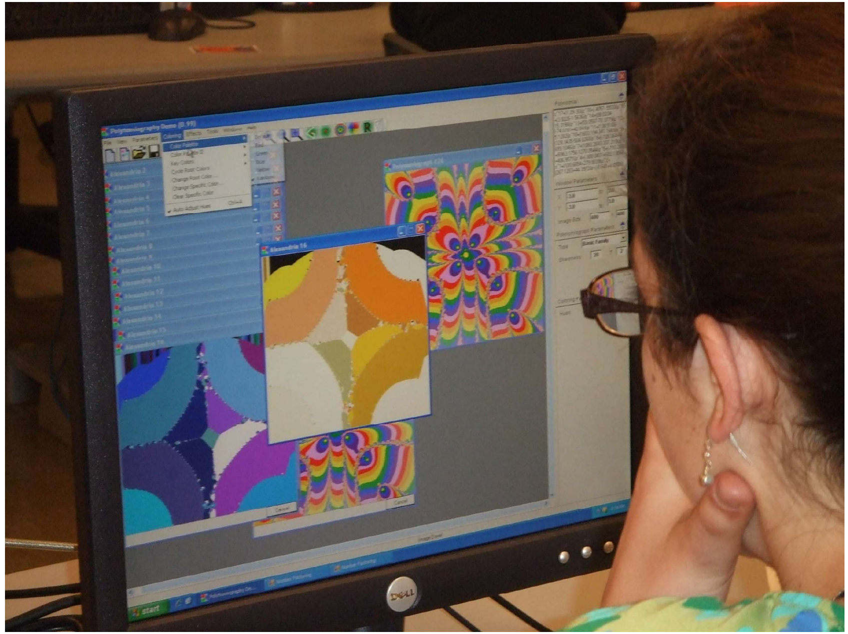
First-Year Seminar Polynomiography students (and their cake)

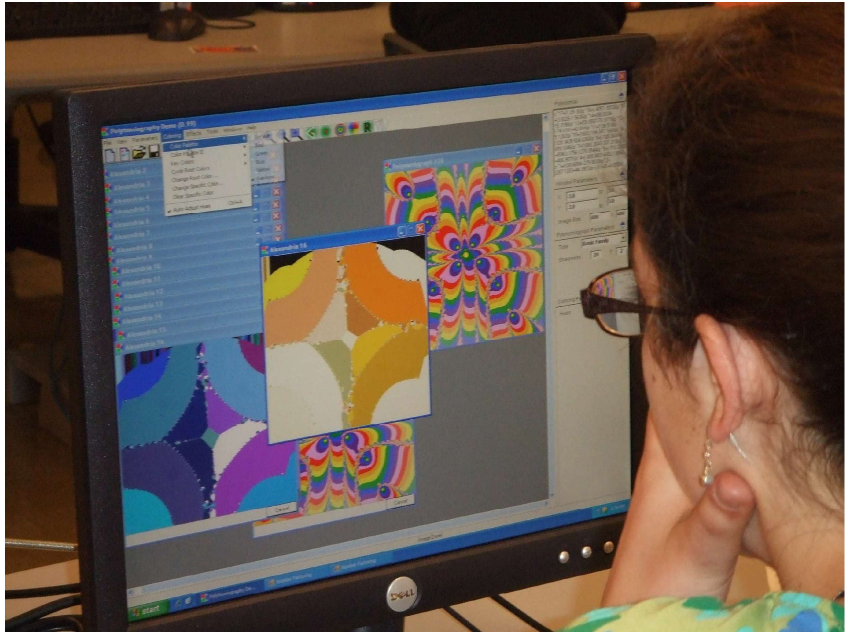

Welcome to Randolph Middle School Dr. Kalantari!

With New Jersey Randolph Middle School Students


POLYNOMIOGRAPHY

Girls Plus Math Camp 6-8th graders (Western Illinois University, Macomb IL)





Young polynomiographer at work, discovering math and art.

Alexandia returns to the camp for second time. Her request last year was to raise the camper age limit to 14, otherwise she could not attend. First time camper are as enthusiastic.


07/22/2008

A happy camper smiles as she has discovered much beauty behind math and its potentials...

What Are Kids Saying?

What Are Kids Saying?

"You see a totally different side of something you see everyday."

What Are Kids Saying?

"You see a totally different side of something you see everyday."

"I have never seen anything like it before and it's very fun to do."

What Are Kids Saying?

"You see a totally different side of something you see everyday."

"I have never seen anything like it before and it's very fun to do."

"It was interesting to see how the numbers and exponents affected the polynomials."

What Are Kids Saying?

“You see a totally different side of something you see everyday.”

“I have never seen anything like it before and it’s very fun to do.”

“It was interesting to see how the numbers and exponents affected the polynomials.”

“It was fun, they looked cool, and I learned a lot and it’s useful.”

What Are Kids Saying?

“You see a totally different side of something you see everyday.”

“I have never seen anything like it before and it’s very fun to do.”

“It was interesting to see how the numbers and exponents affected the polynomials.”

“It was fun, they looked cool, and I learned a lot and it’s useful.”

“Learning about polynomials was much more fun than I thought and I love learning.”

What Are Kids Saying?

"You see a totally different side of something you see everyday."

"I have never seen anything like it before and it's very fun to do."

"It was interesting to see how the numbers and exponents affected the polynomials."

"It was fun, they looked cool, and I learned a lot and it's useful."

"Learning about polynomials was much more fun than I thought and I love learning."

"I want to know how all those numbers could make such cool pictures. It seems more interesting now."

What Are Teachers Saying?

What Are Teachers Saying?

"I learned new ways to motivate my student."

What Are Teachers Saying?

"I learned new ways to motivate my student."

"I hope to find a use for these ideas and activities in the museum (exhibits, labs, etc.) It was a wonderful workshop! I hope to incorporate it in my classroom."

What Are Teachers Saying?

"I learned new ways to motivate my student."

"I hope to find a use for these ideas and activities in the museum (exhibits, labs, etc.) It was a wonderful workshop! I hope to incorporate it in my classroom."

"I need now to spend more time trying to develop meaningful lessons utilizing the software."

What Are Teachers Saying?

"I learned new ways to motivate my student."

"I hope to find a use for these ideas and activities in the museum (exhibits, labs, etc.) It was a wonderful workshop! I hope to incorporate it in my classroom."

"I need now to spend more time trying to develop meaningful lessons utilizing the software."

"It directly applies to the subjects I teach."

What Are Teachers Saying?

"I learned new ways to motivate my student."

"I hope to find a use for these ideas and activities in the museum (exhibits, labs, etc.) It was a wonderful workshop! I hope to incorporate it in my classroom."

"I need now to spend more time trying to develop meaningful lessons utilizing the software."

"It directly applies to the subjects I teach."

"A product like this would be a good tool to motivate students."

What Are Teachers Saying?

"I learned new ways to motivate my student."

"I hope to find a use for these ideas and activities in the museum (exhibits, labs, etc.) It was a wonderful workshop! I hope to incorporate it in my classroom."

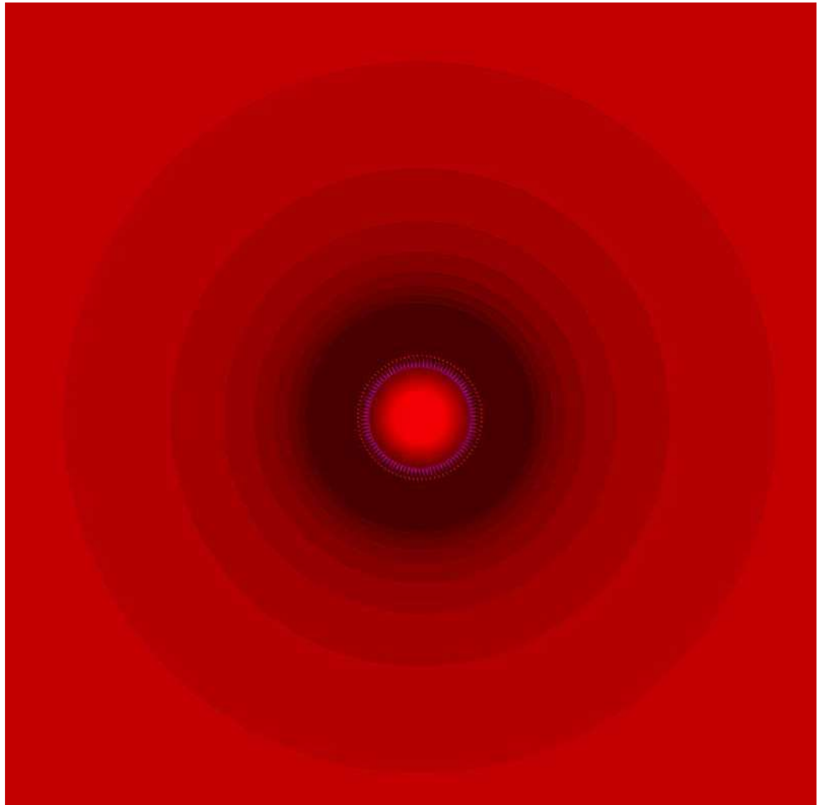
"I need now to spend more time trying to develop meaningful lessons utilizing the software."

"It directly applies to the subjects I teach."

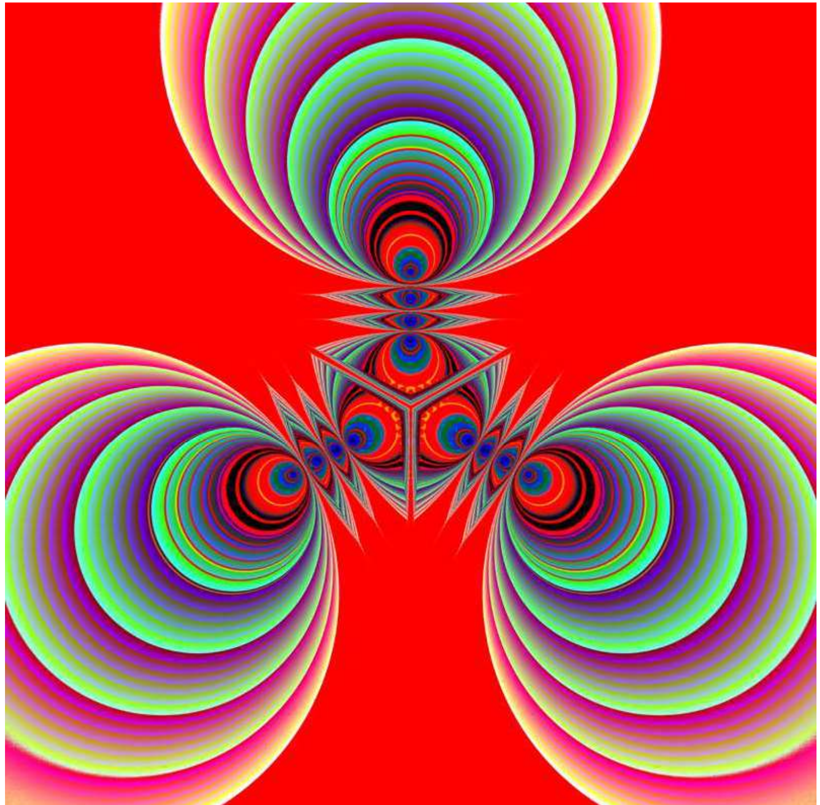
"A product like this would be a good tool to motivate students."

"Gives me ideas to pursue for Discrete Math Curriculum!"

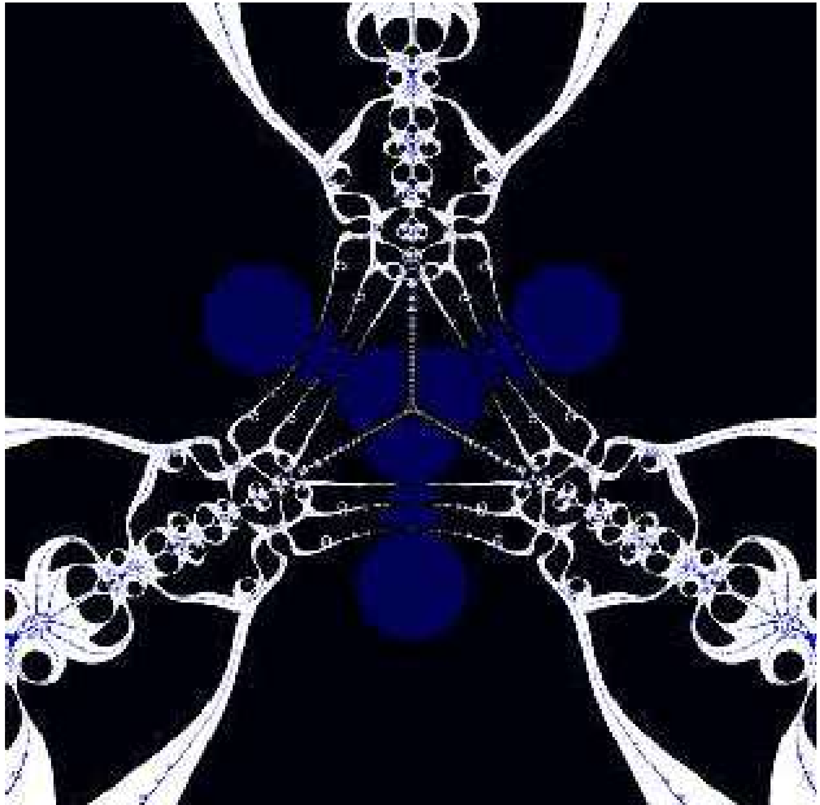
Polynomiography In Art



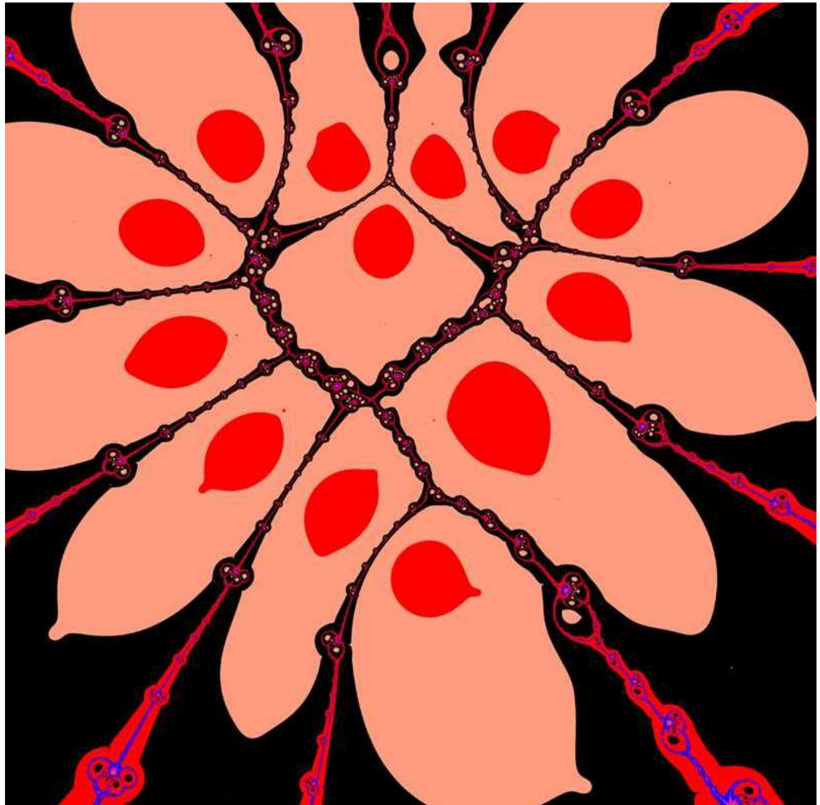
Hal



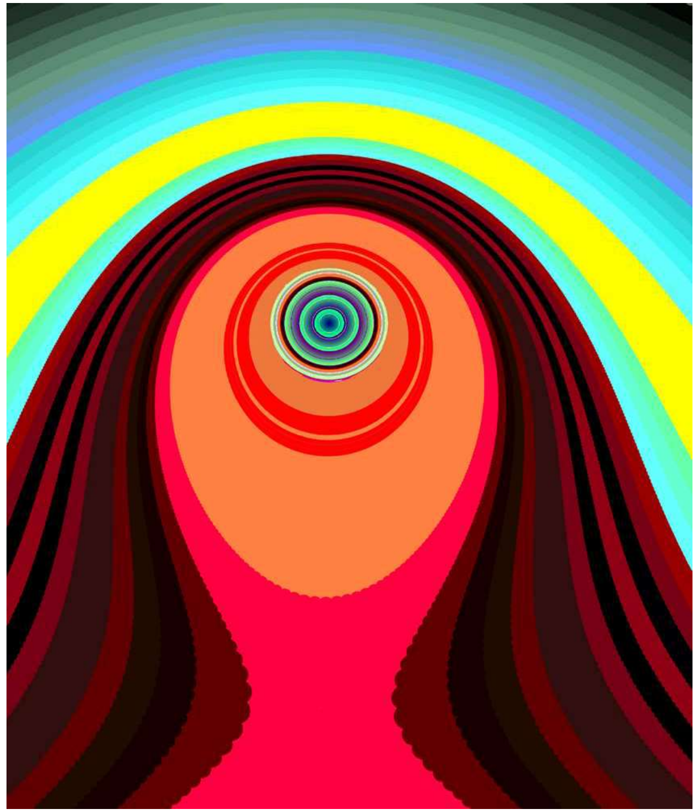
Life



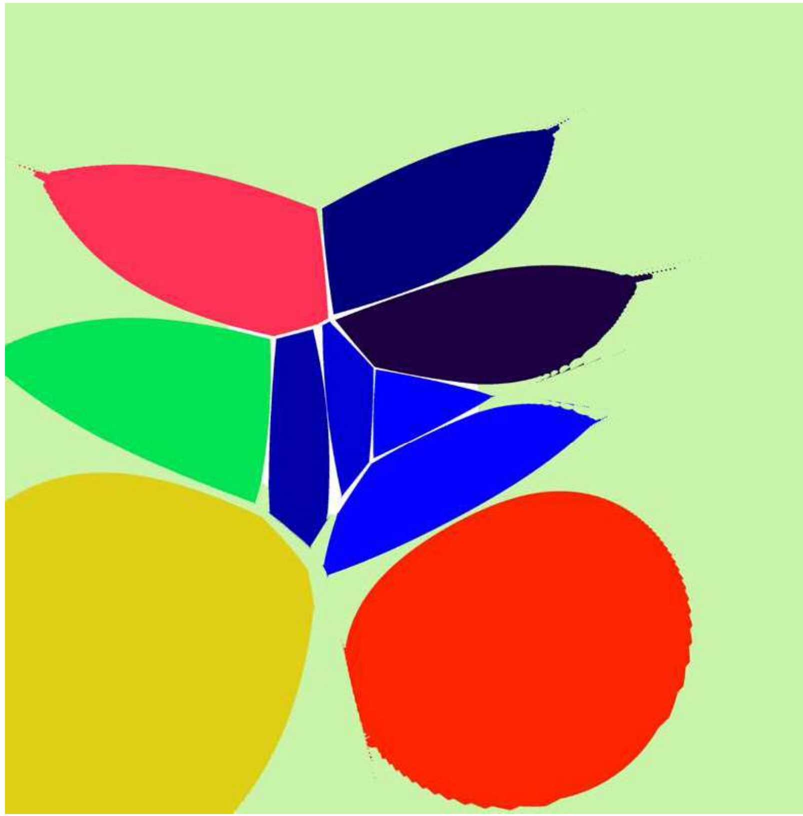
Death



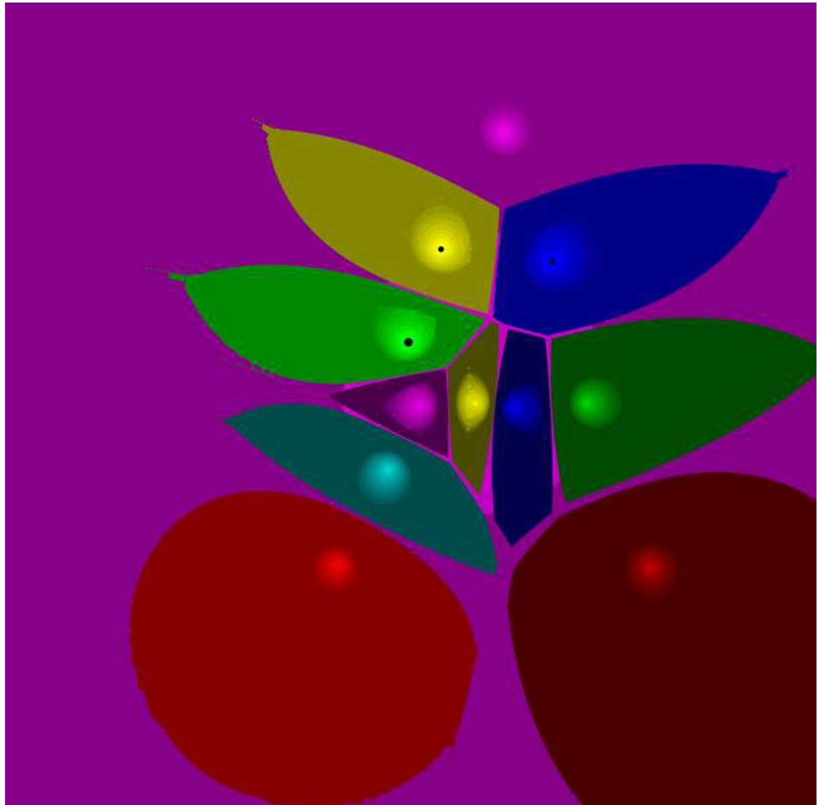
Mathematics of a Heart

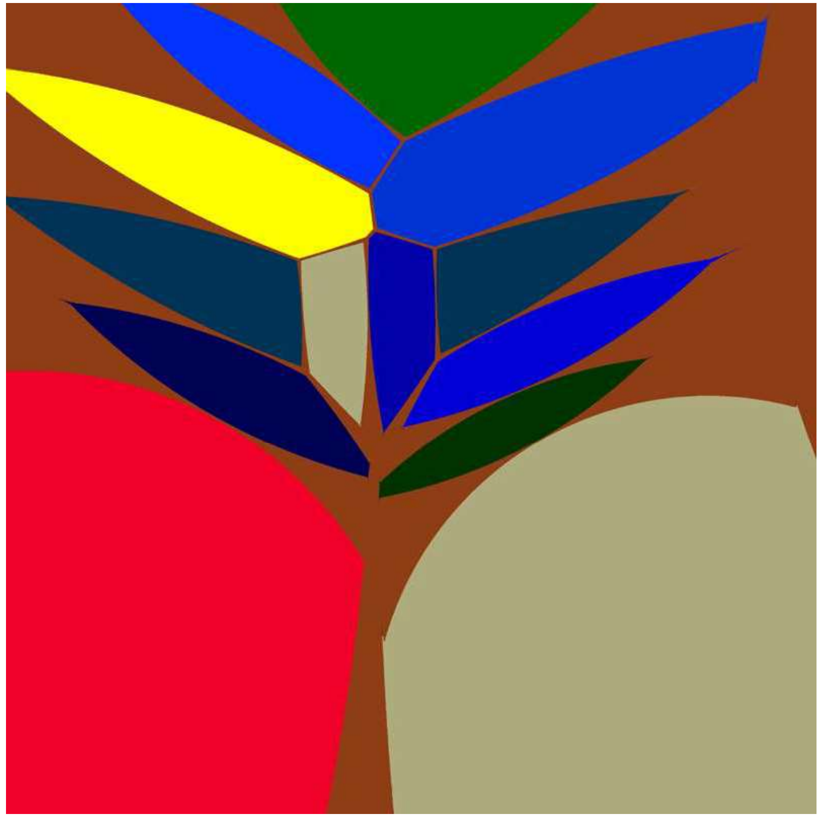


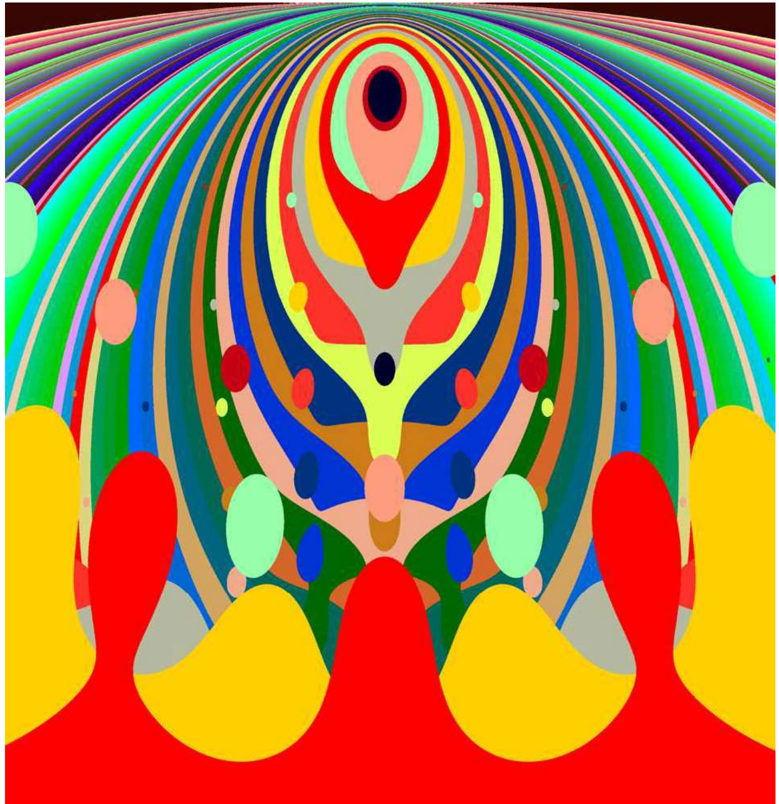
Mona Lisa in 2001



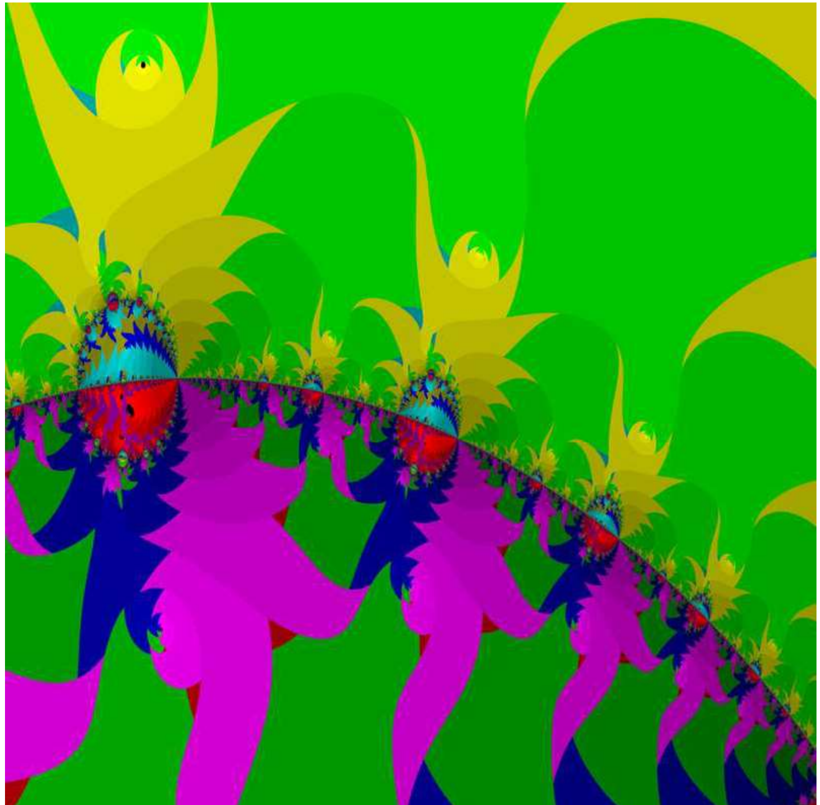
Summer



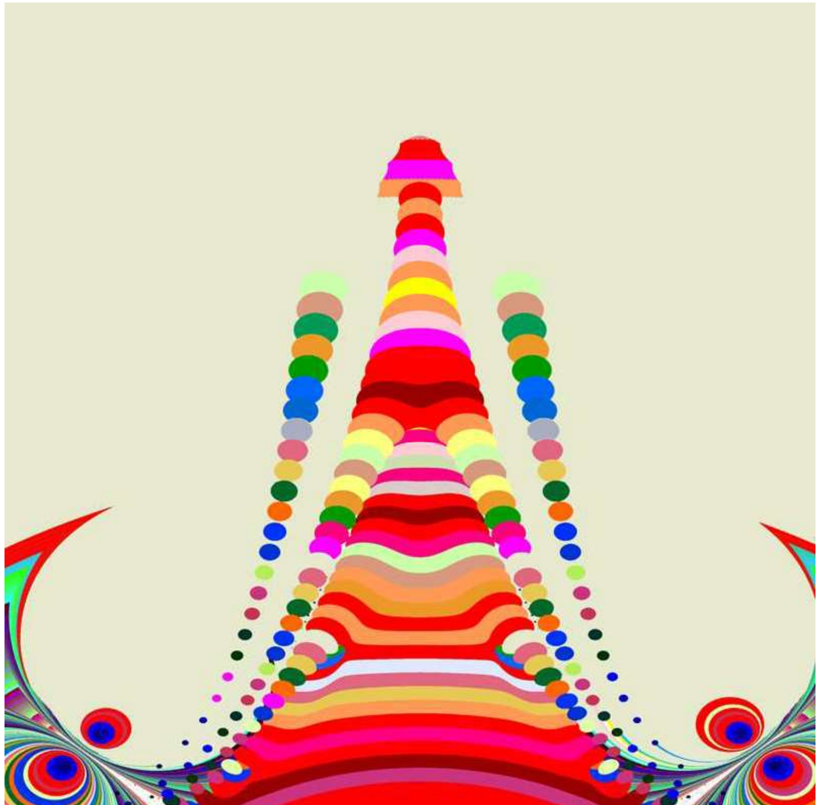




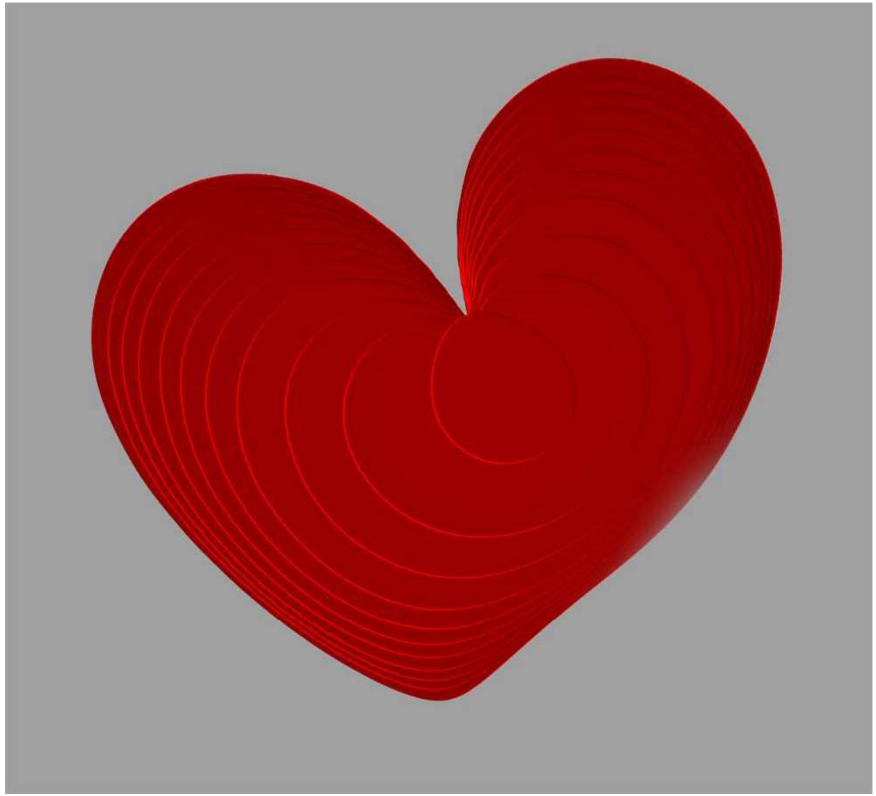
Symphony



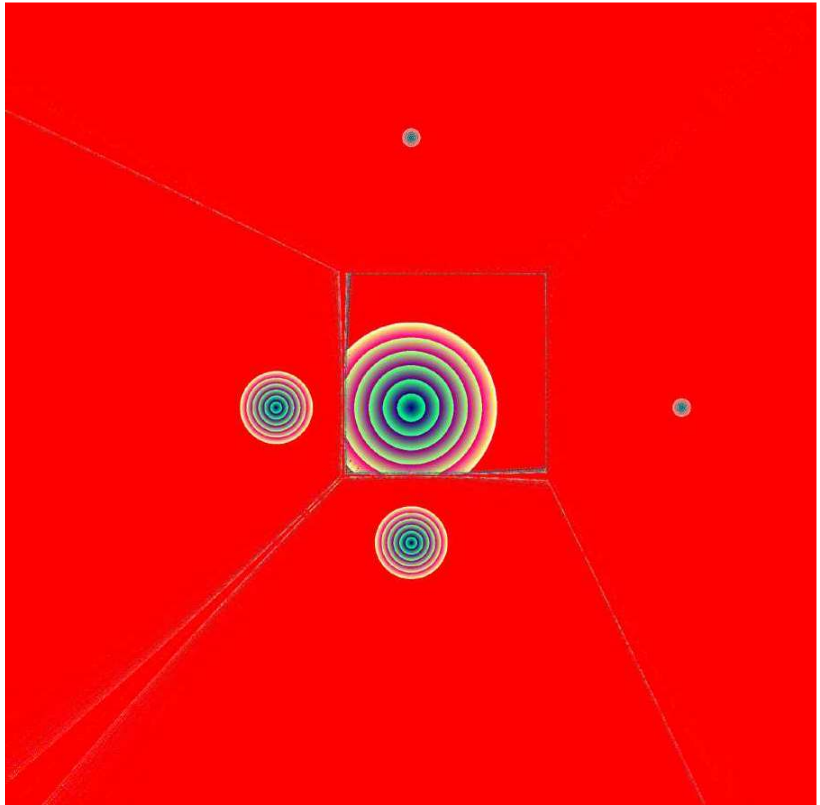
Waltz



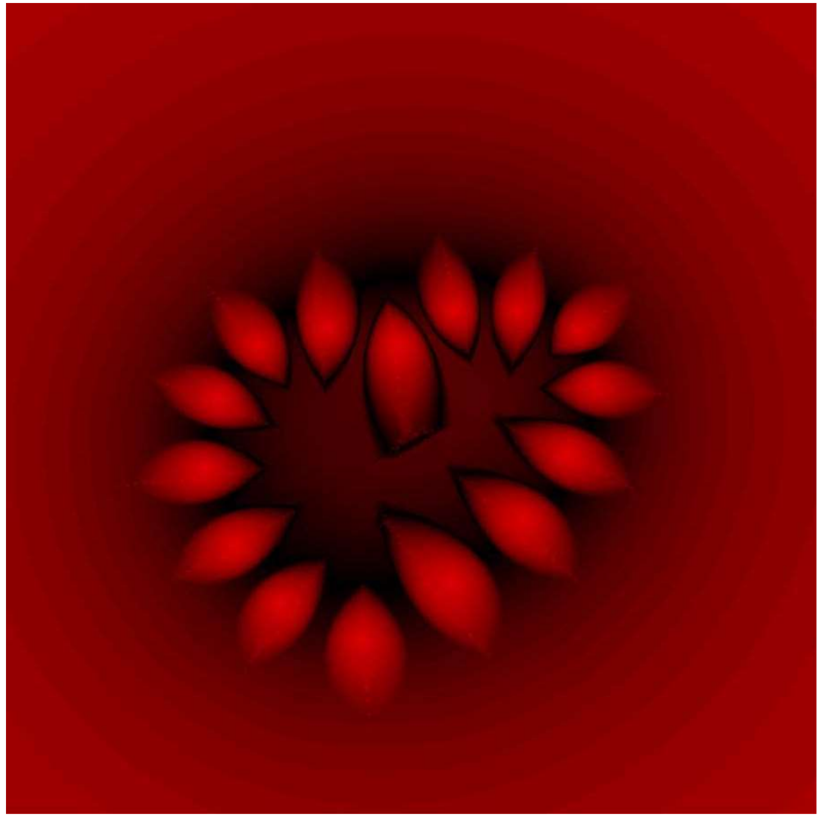
Eiffel Tower



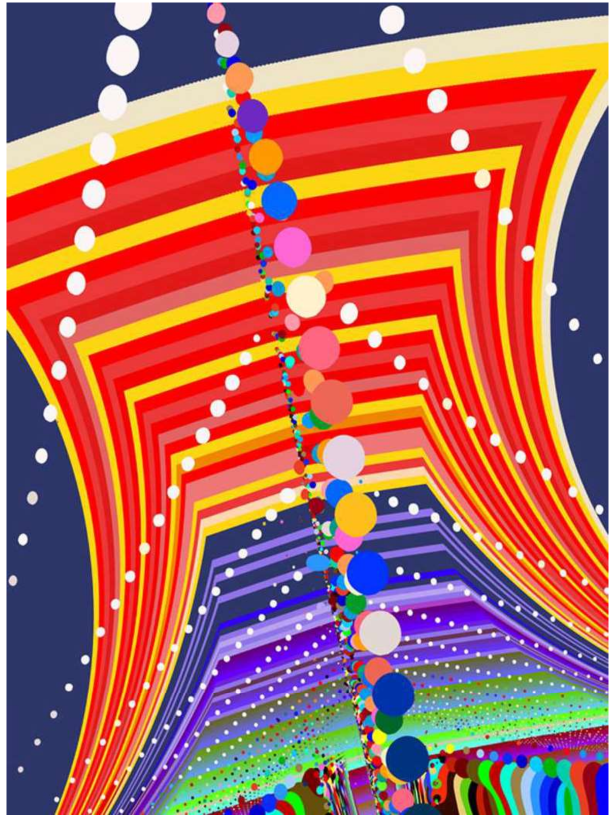
Valentine



SquaringTheCircle

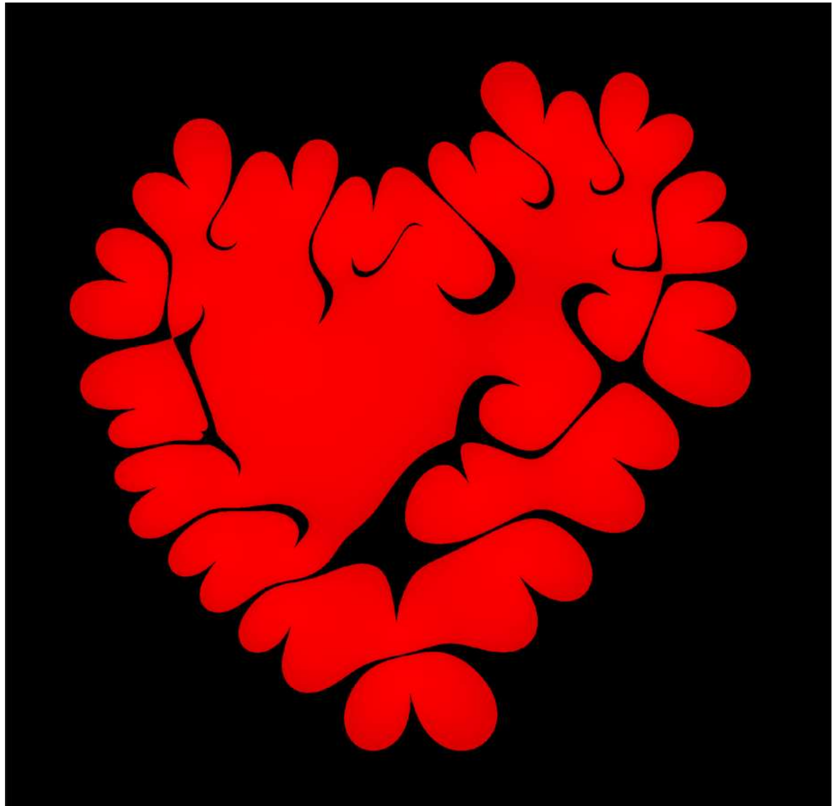


Shaping a Heart

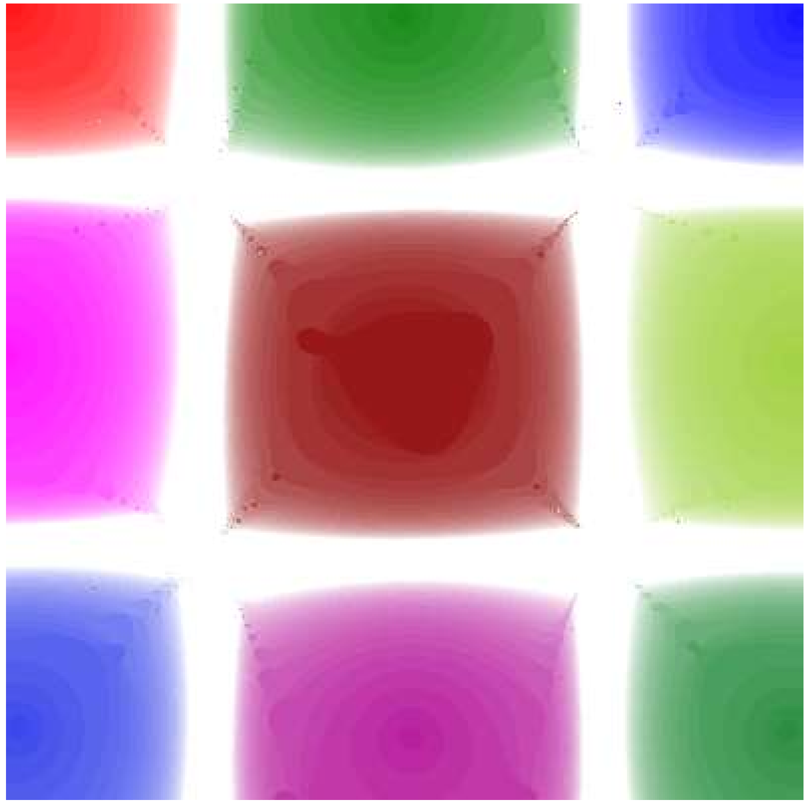


Party on the Brooklyn Bridge

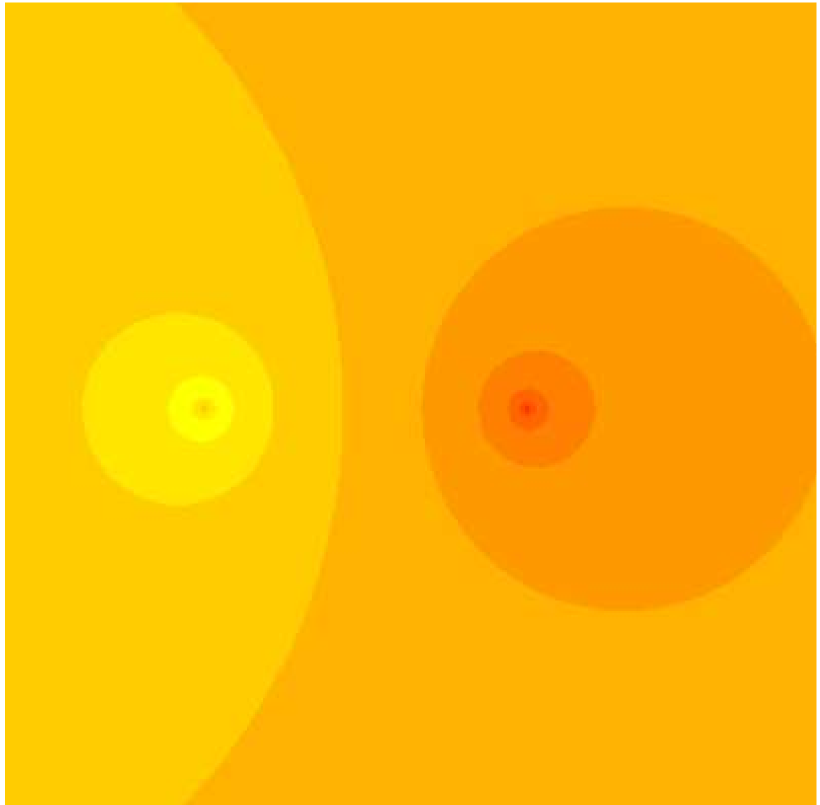
Masked Queen



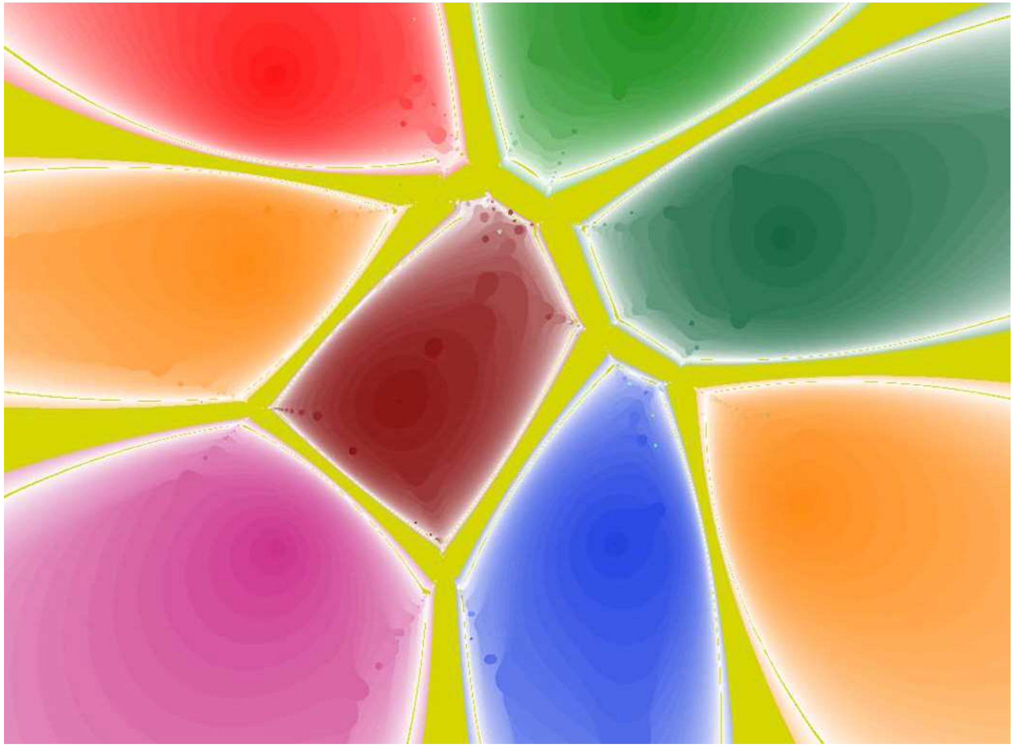
Hearts

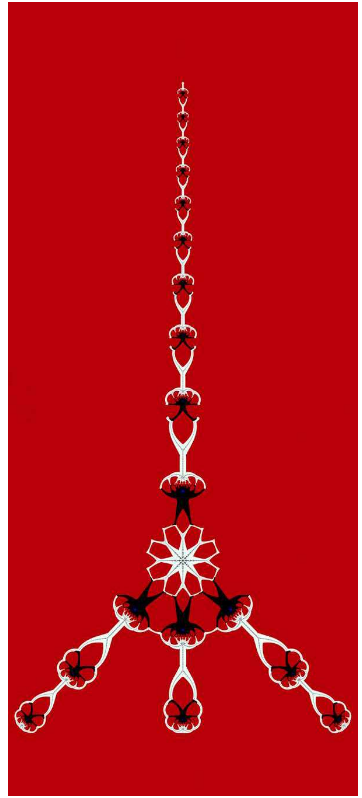


Squares

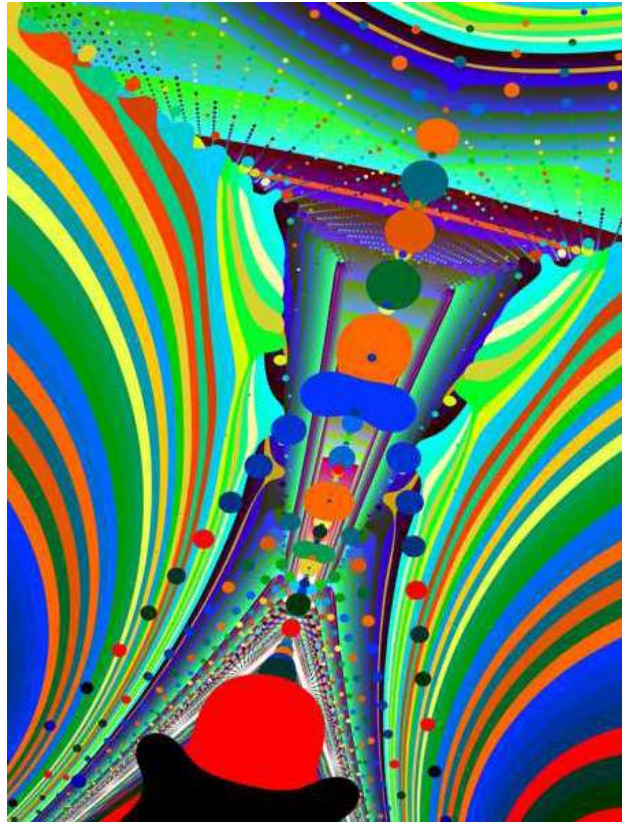


Circles





Acrobats in Paris



Circus

Times Square

Polynomiography In Exhibitions

Exhibition at Rutgers Art Library

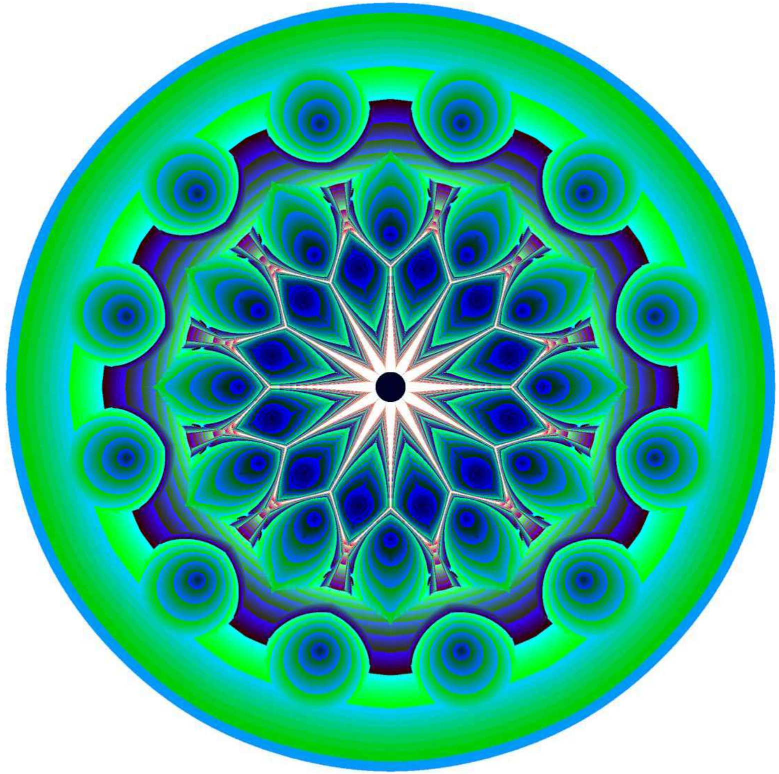
Exhibition can also be viewed with 3D glasses

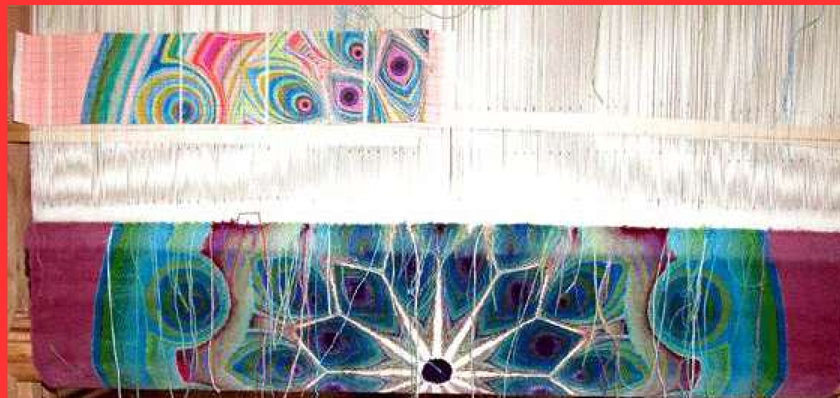
Polynomiography Artwork of Montgomery High School Students, Using a Demo Software

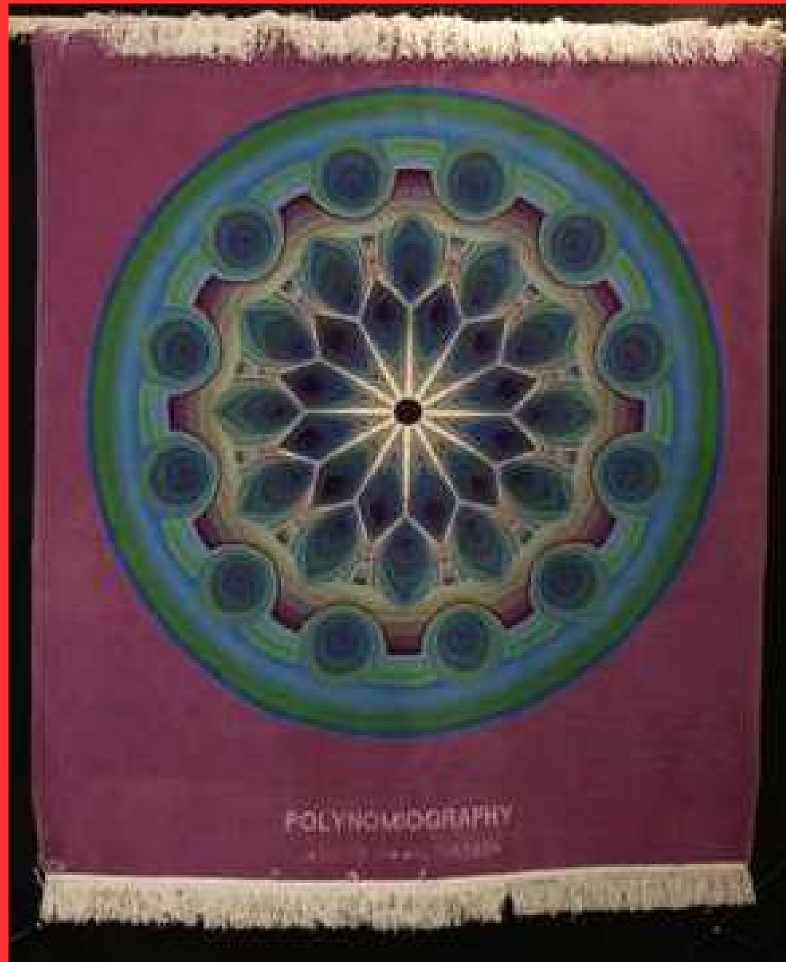
Polynomiography In Design

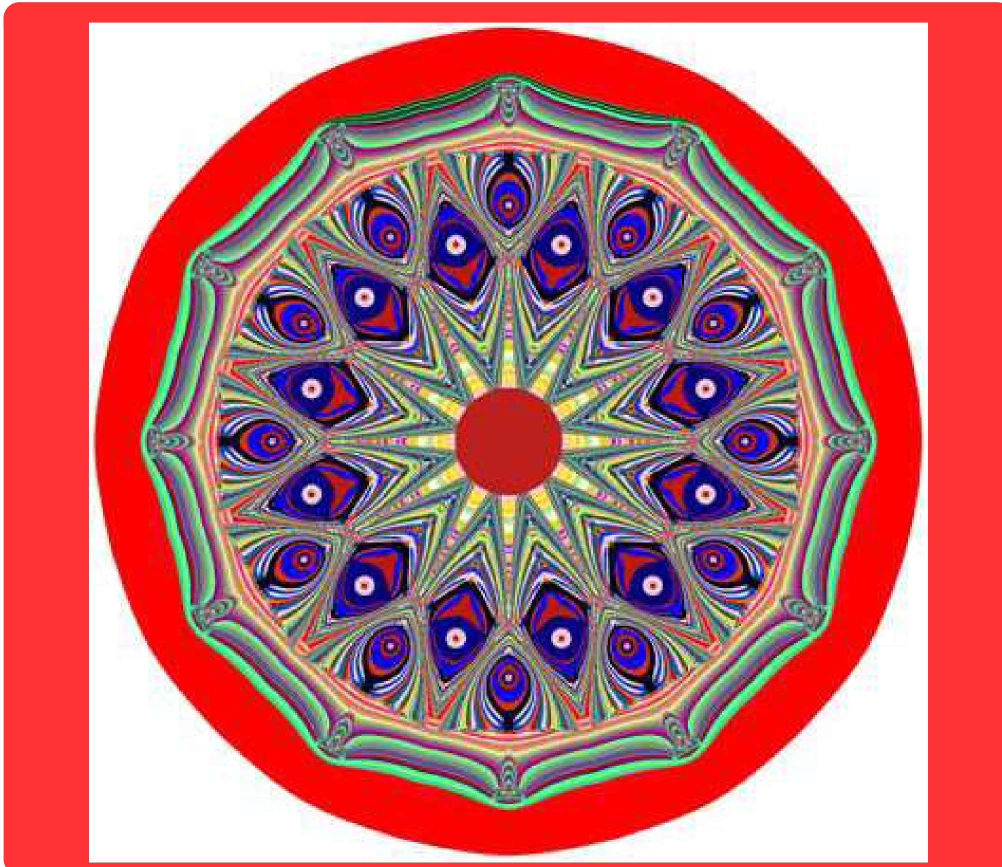
Polynomiography In Design

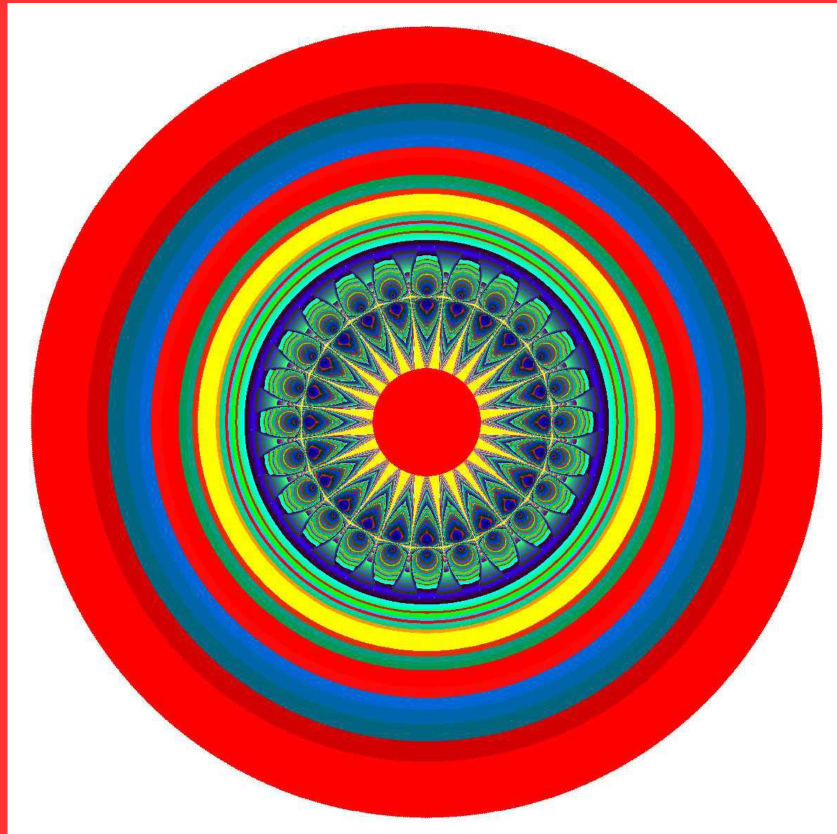
Designing A Carpet

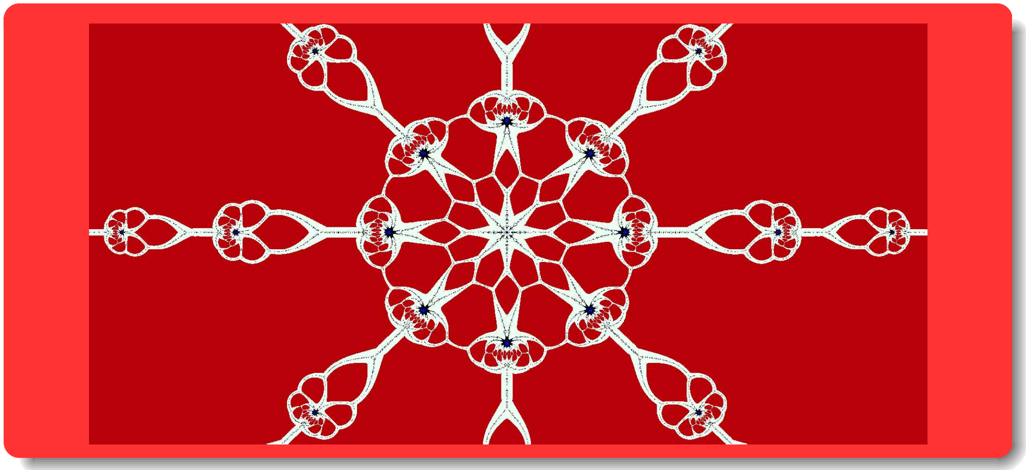


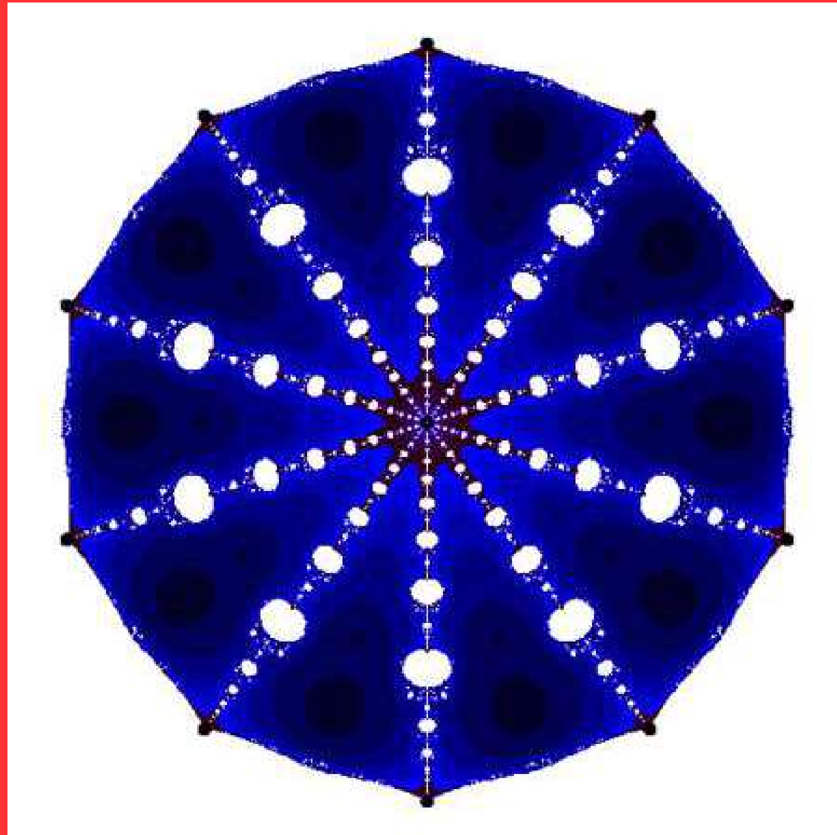


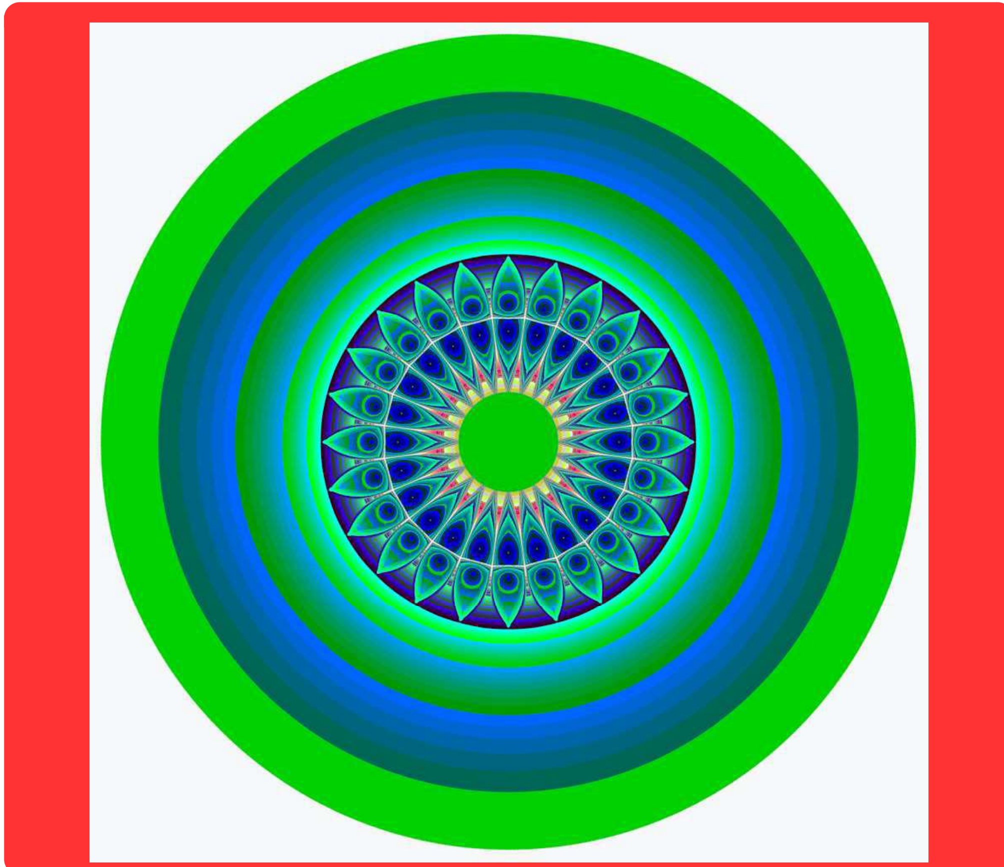

POLYNOMIOGRAPHY

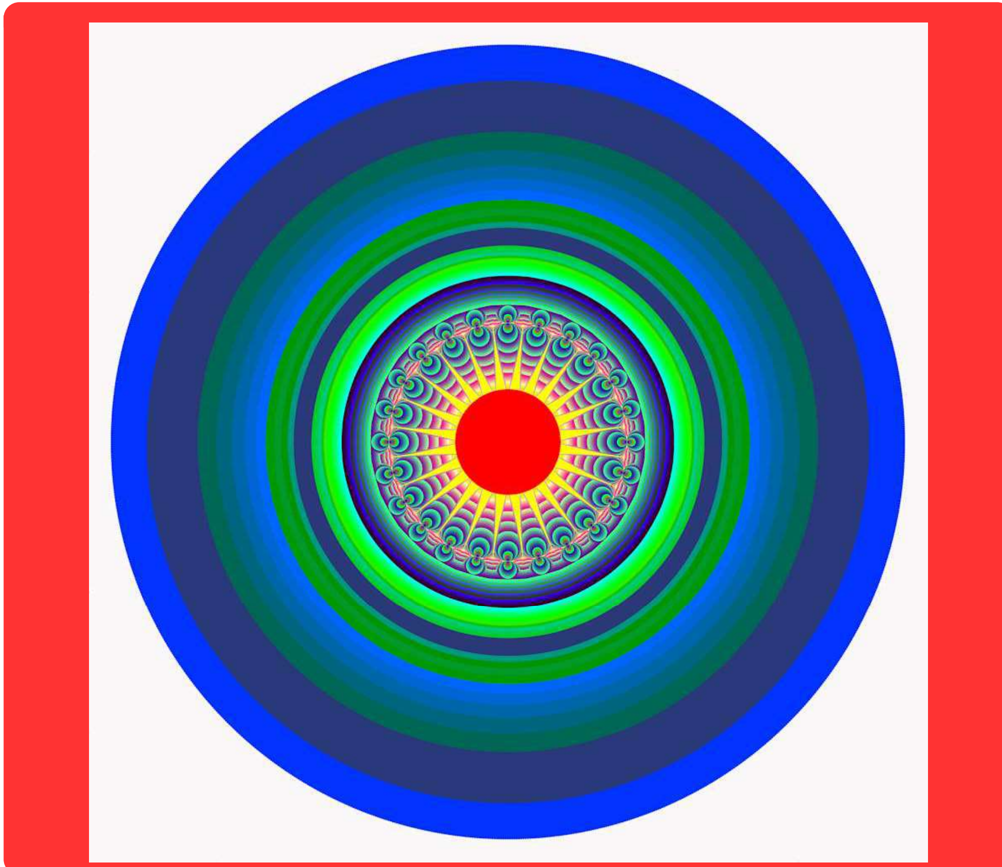




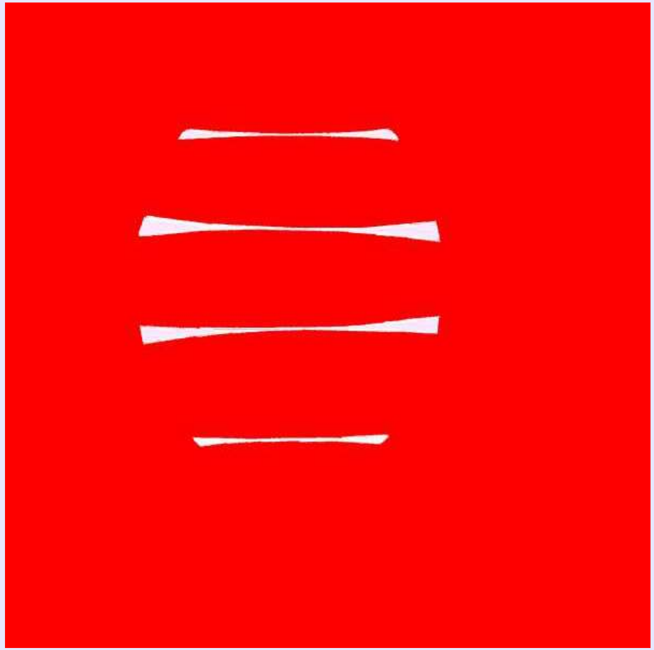


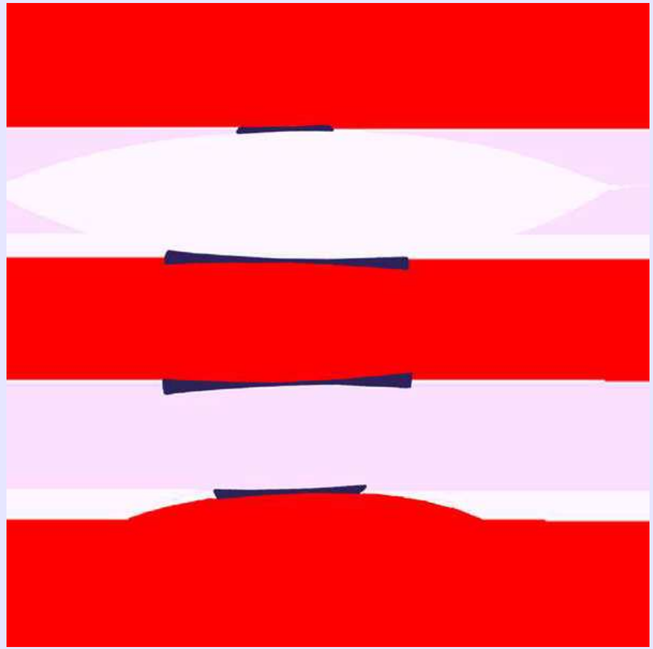


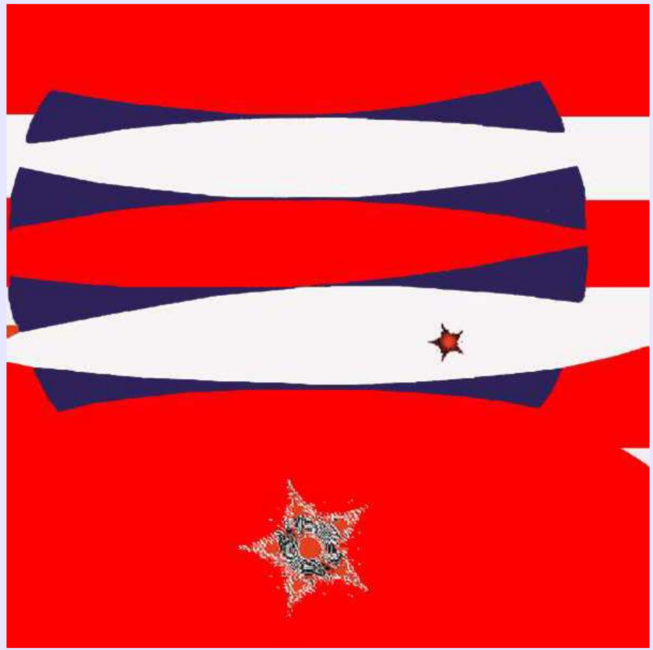


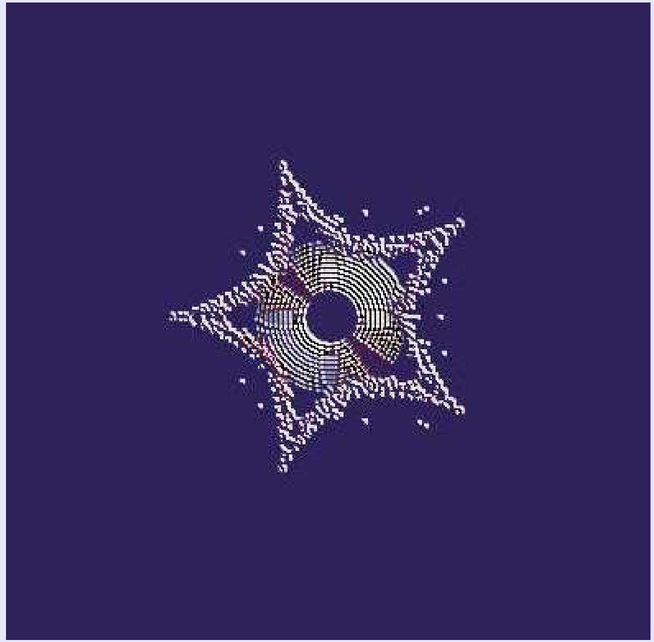


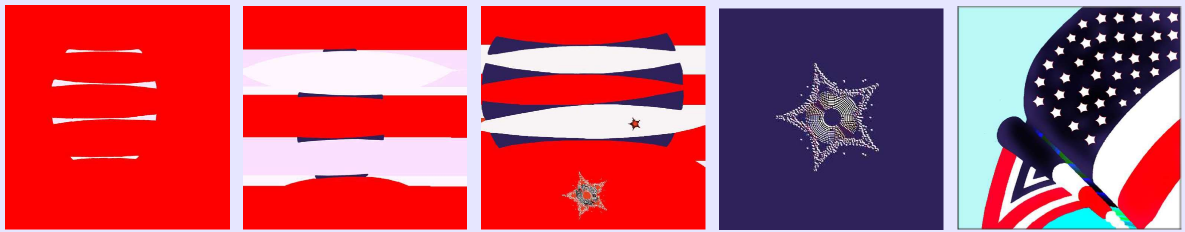
Making US Flag through Polynomiography- Inspired by Jasper Johns









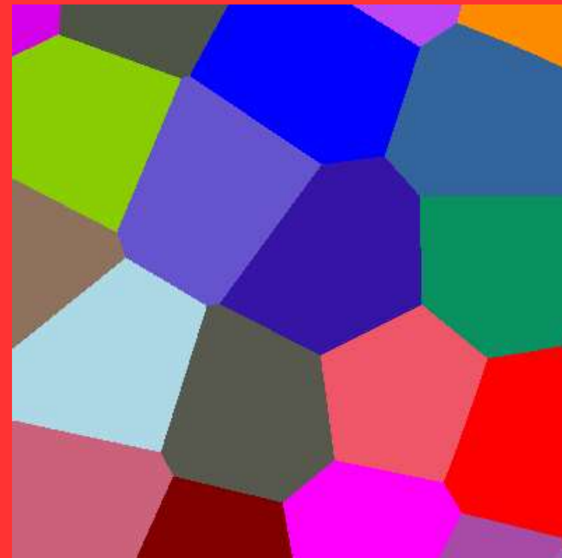


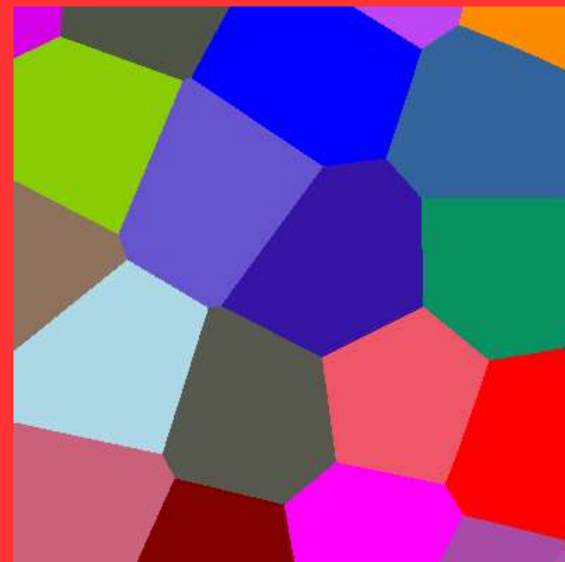
Evolution of Stars and Stripes

Artists and Polynomiography

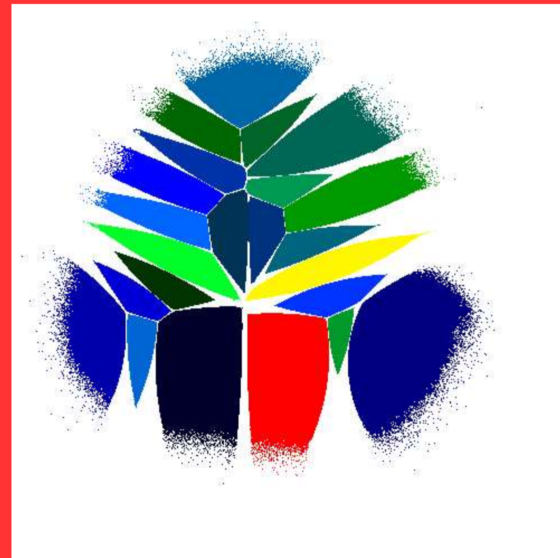
Artists and Polynomiography

Can We Connect Artists with Polynomiography?

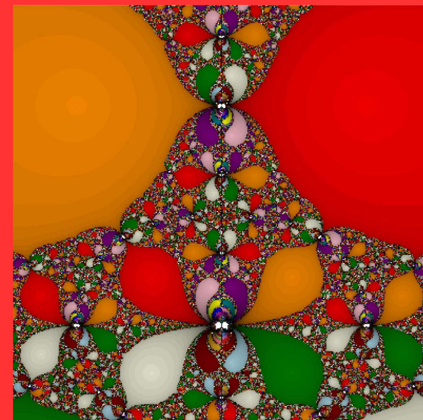



Klee and Polynomiography

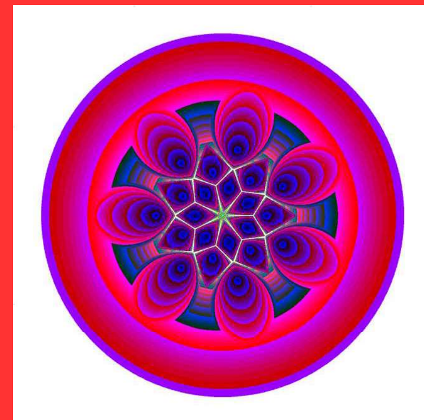


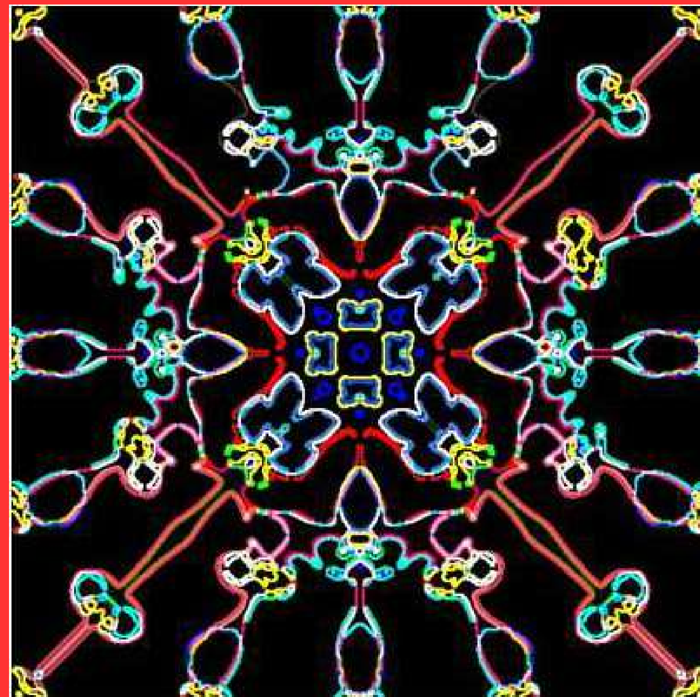
Picasso and Polynomiography

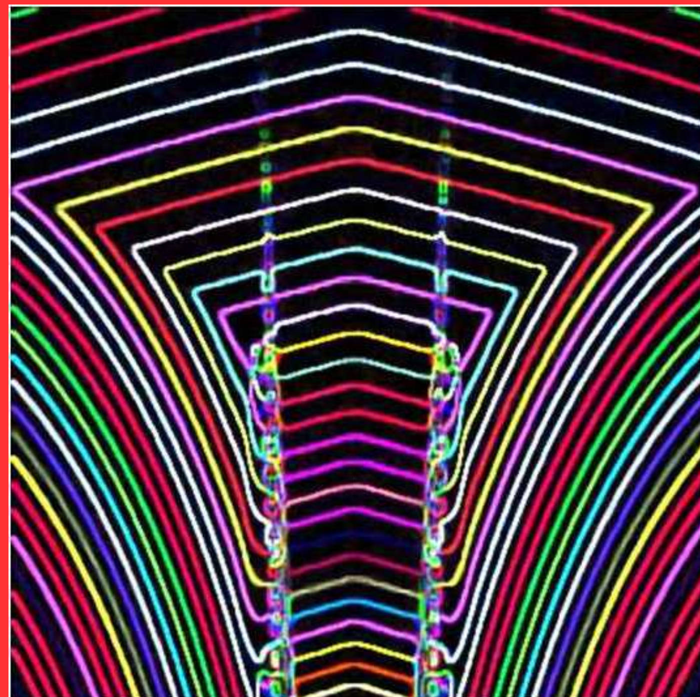


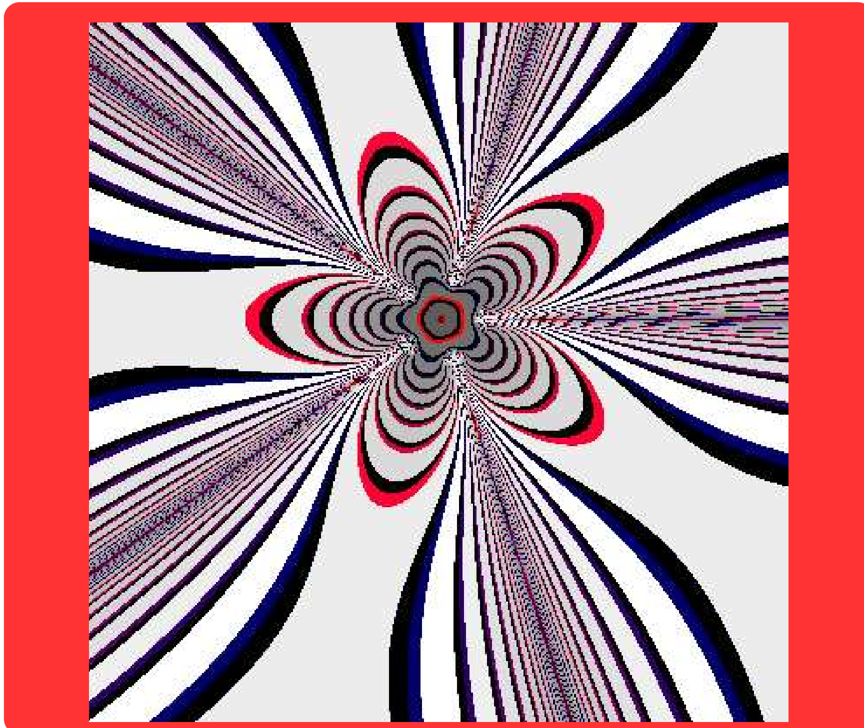


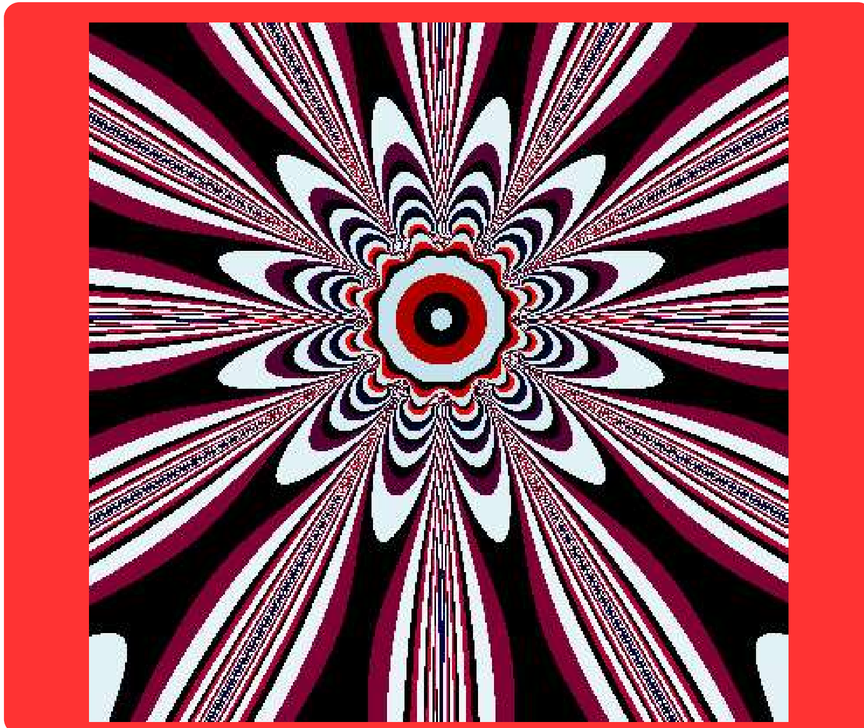
"Brain" Lillian F. Schwartz. Copyright 2003 Lillian F. Schawrtz

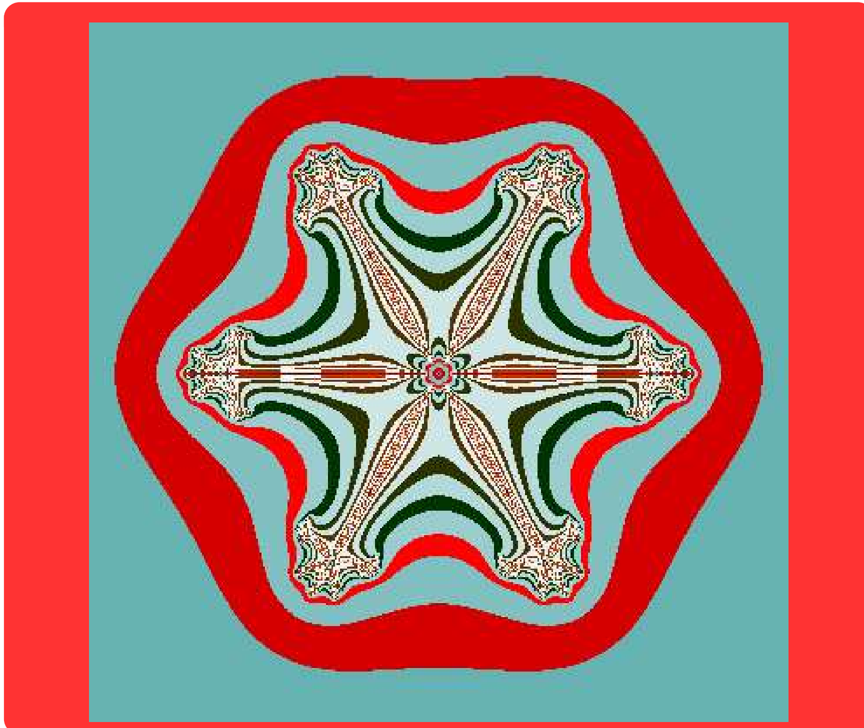


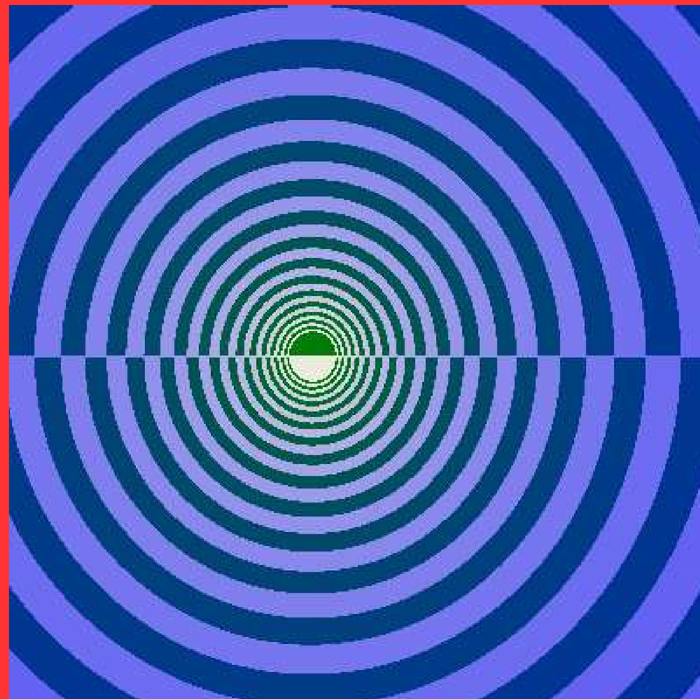


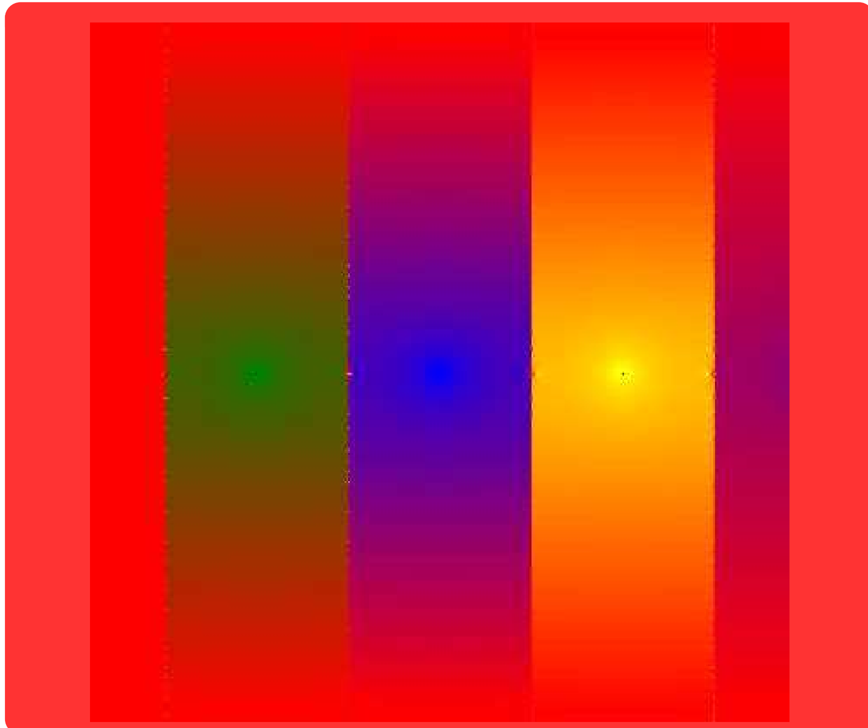




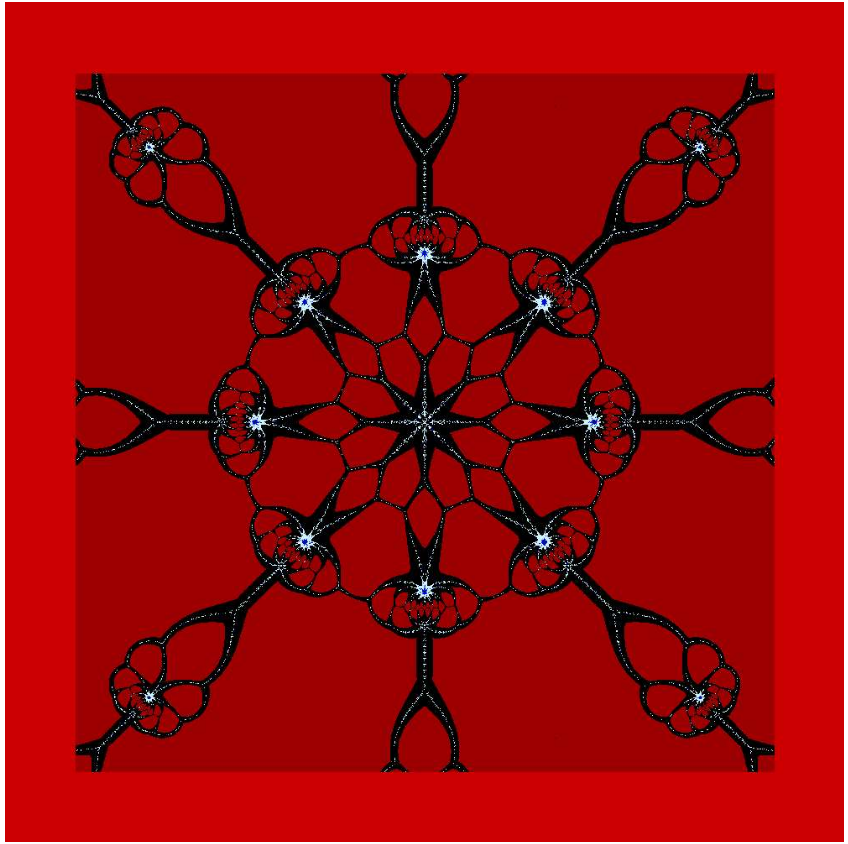


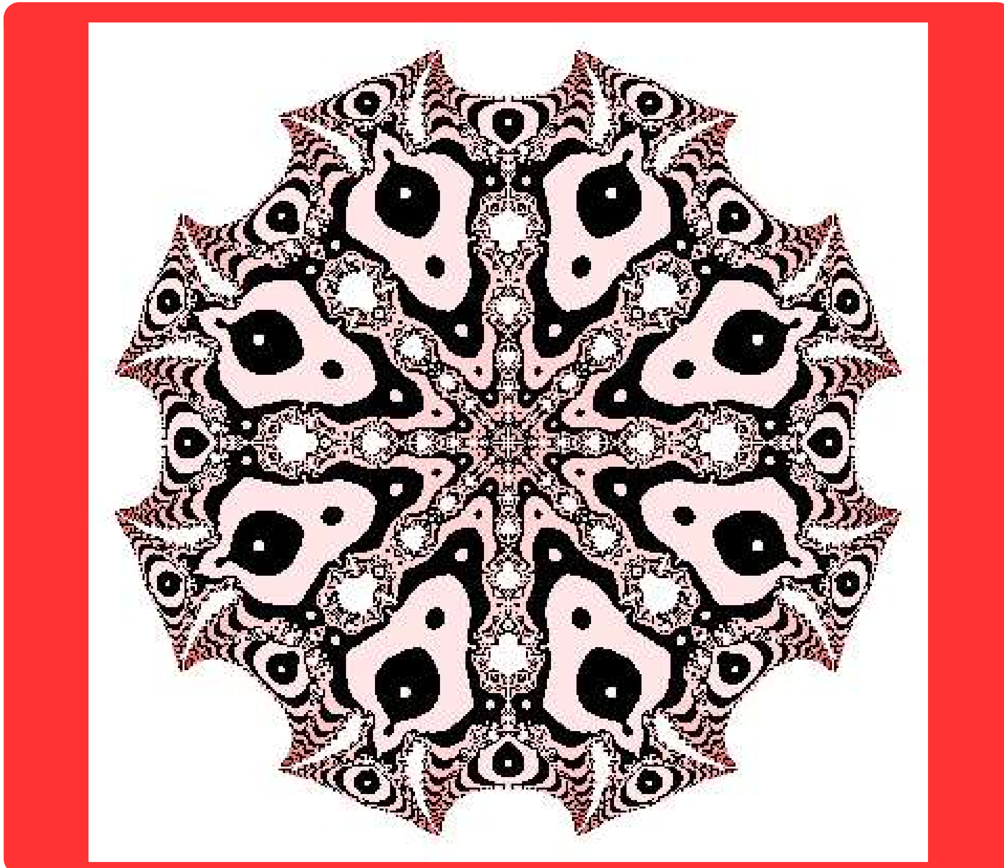


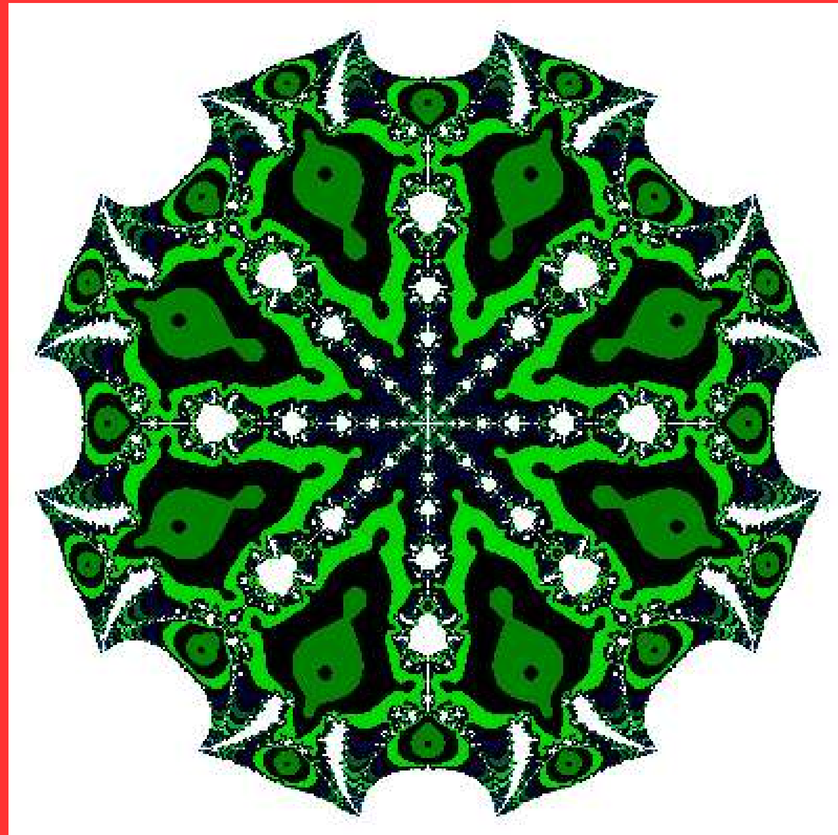


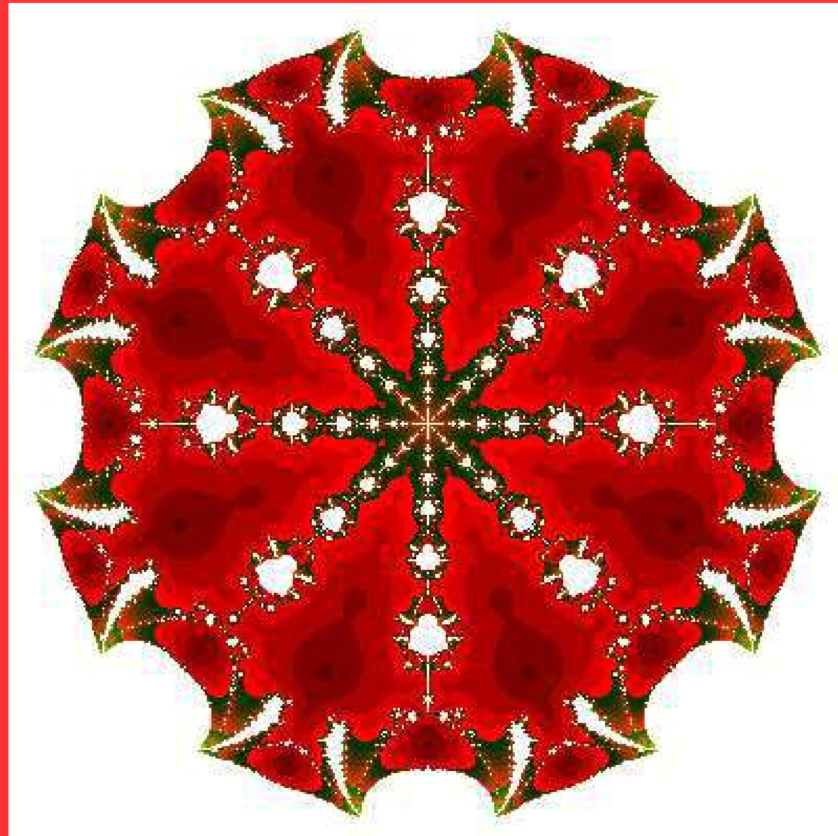


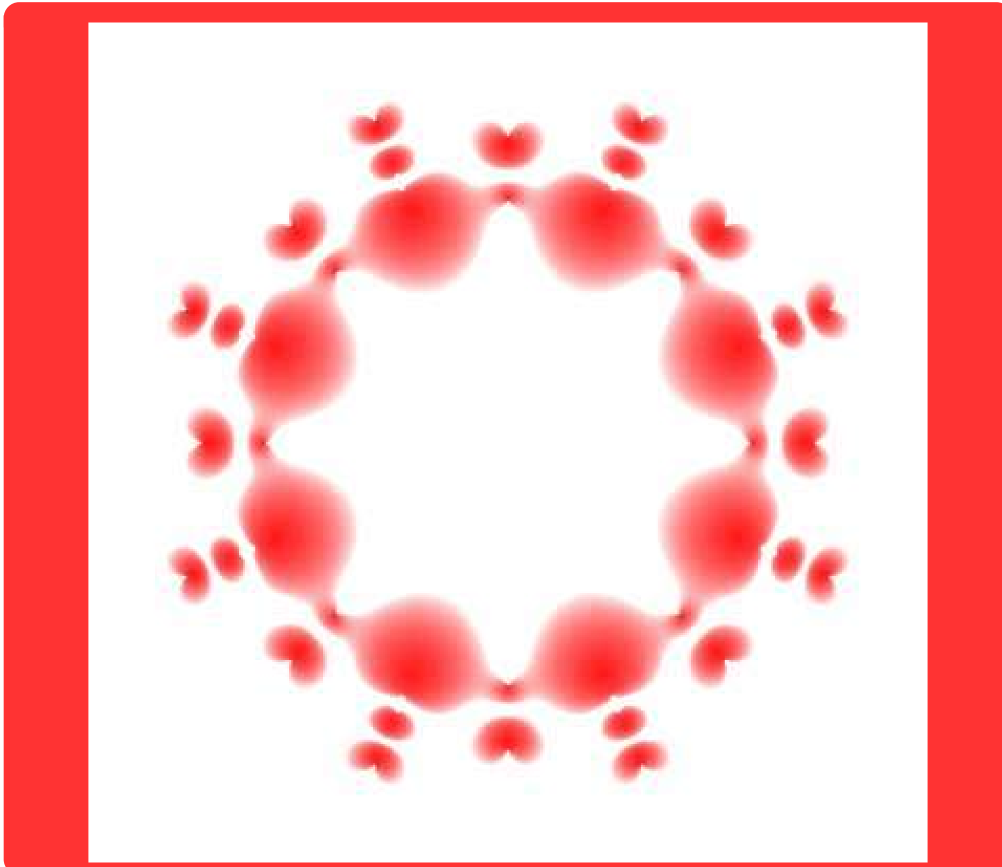
Endless Designs with a Single Polynomial

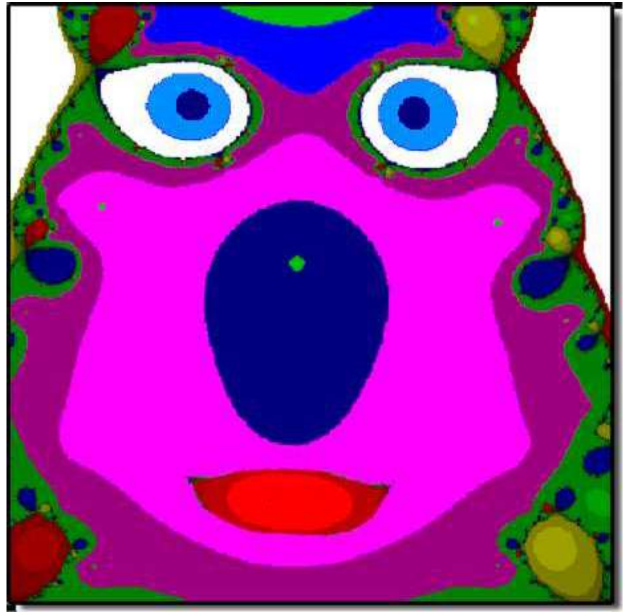




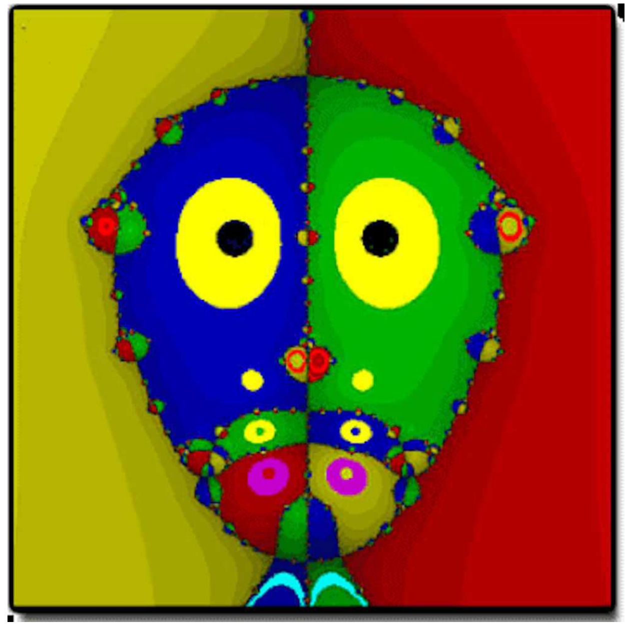




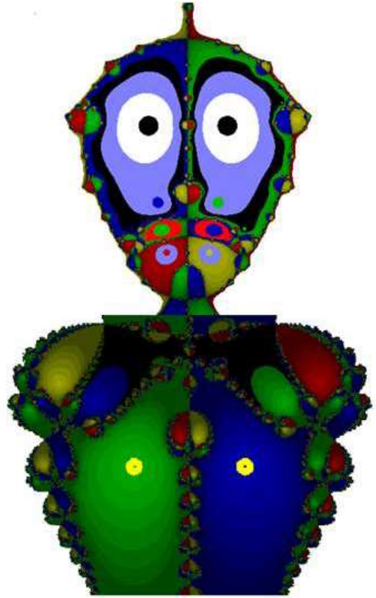




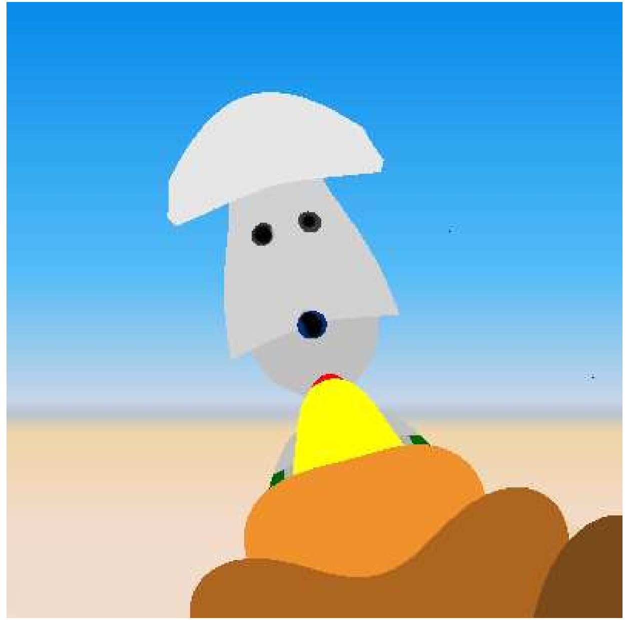
Ms. Poly



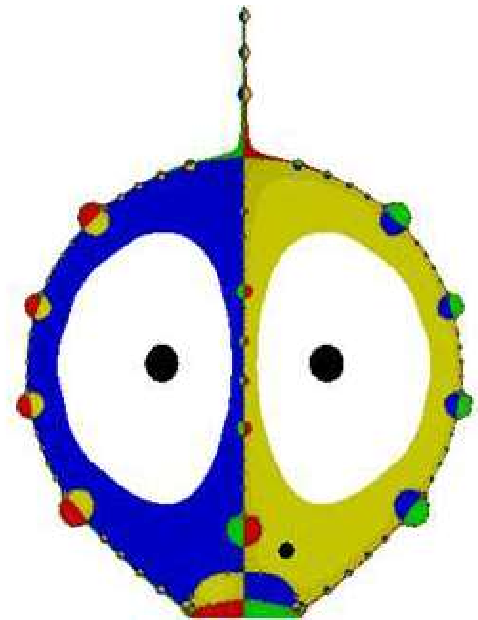
L3



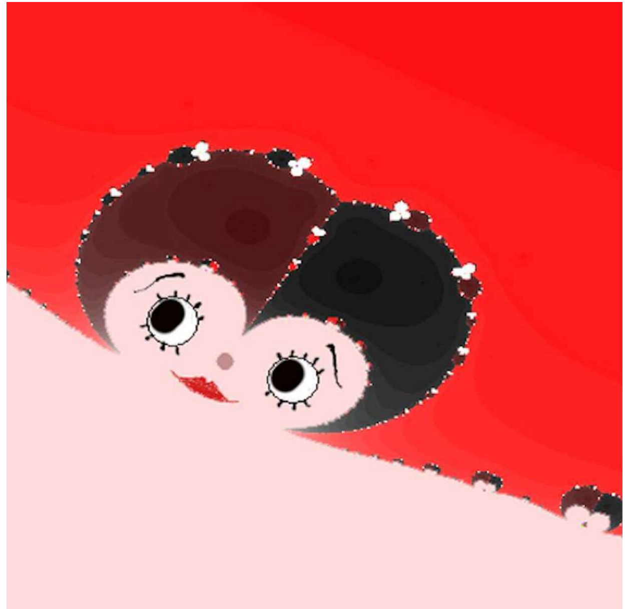
Don Quixote

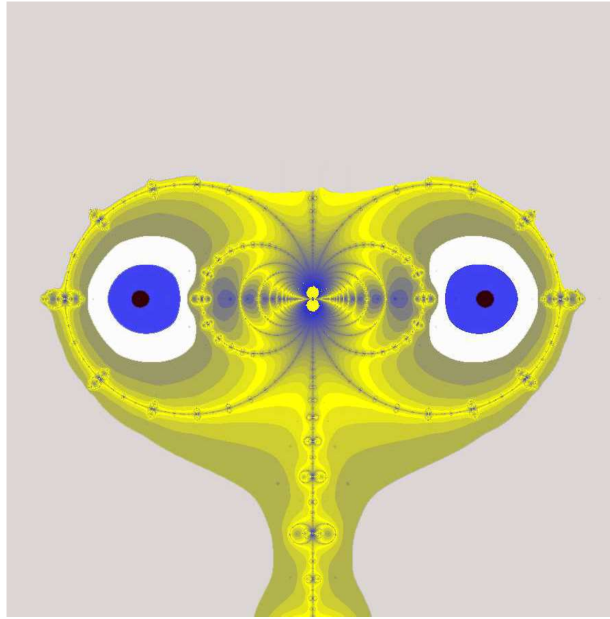


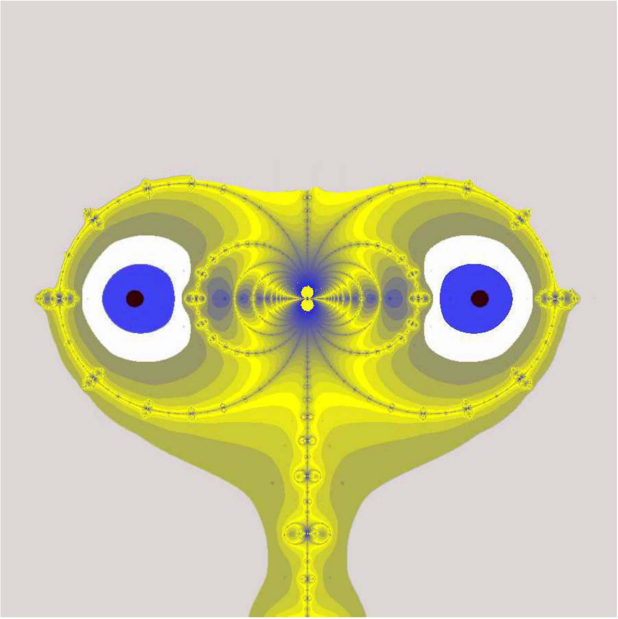
Snoopy on a ride



Hi!


Pretty Betty





They call me Z.T.