CONTINUED FRACTIONS

MATH FEST
AUGUST 5, 2023

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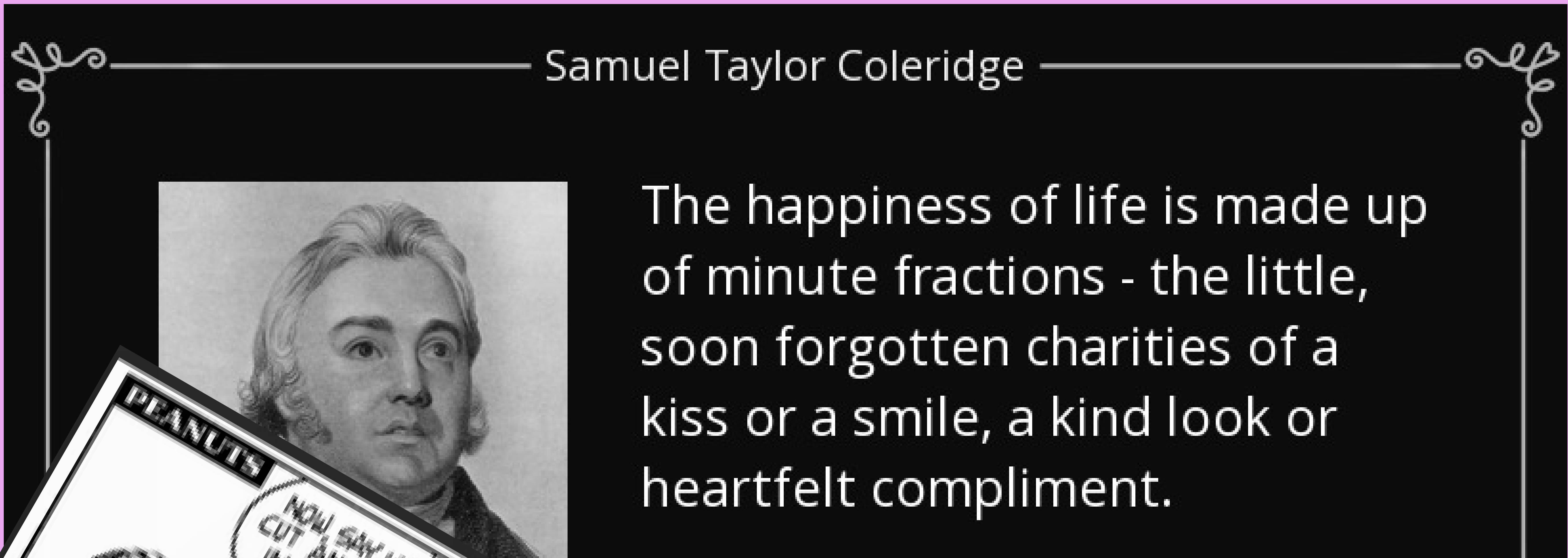
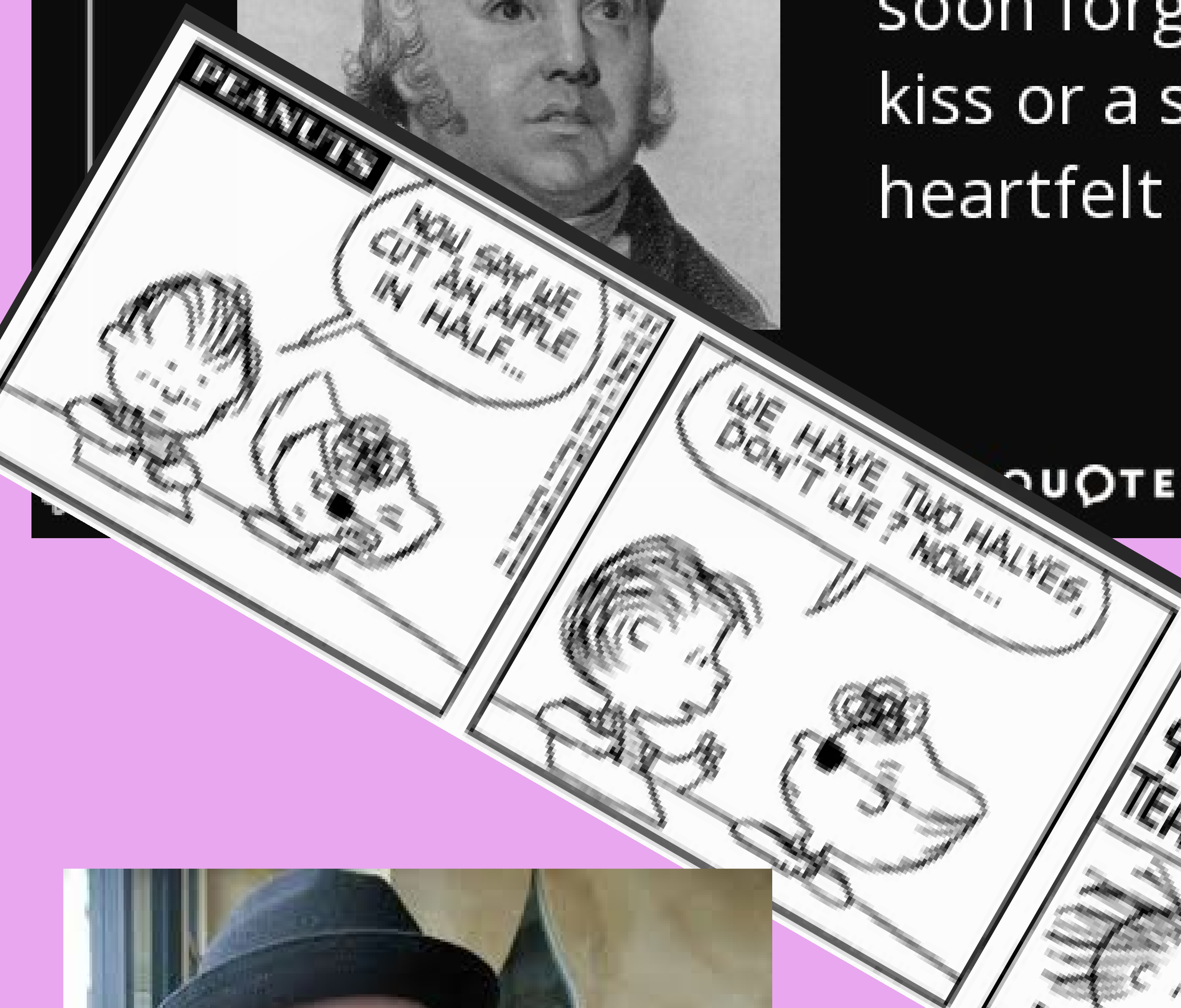




Matt Fraction

Matt Fraction

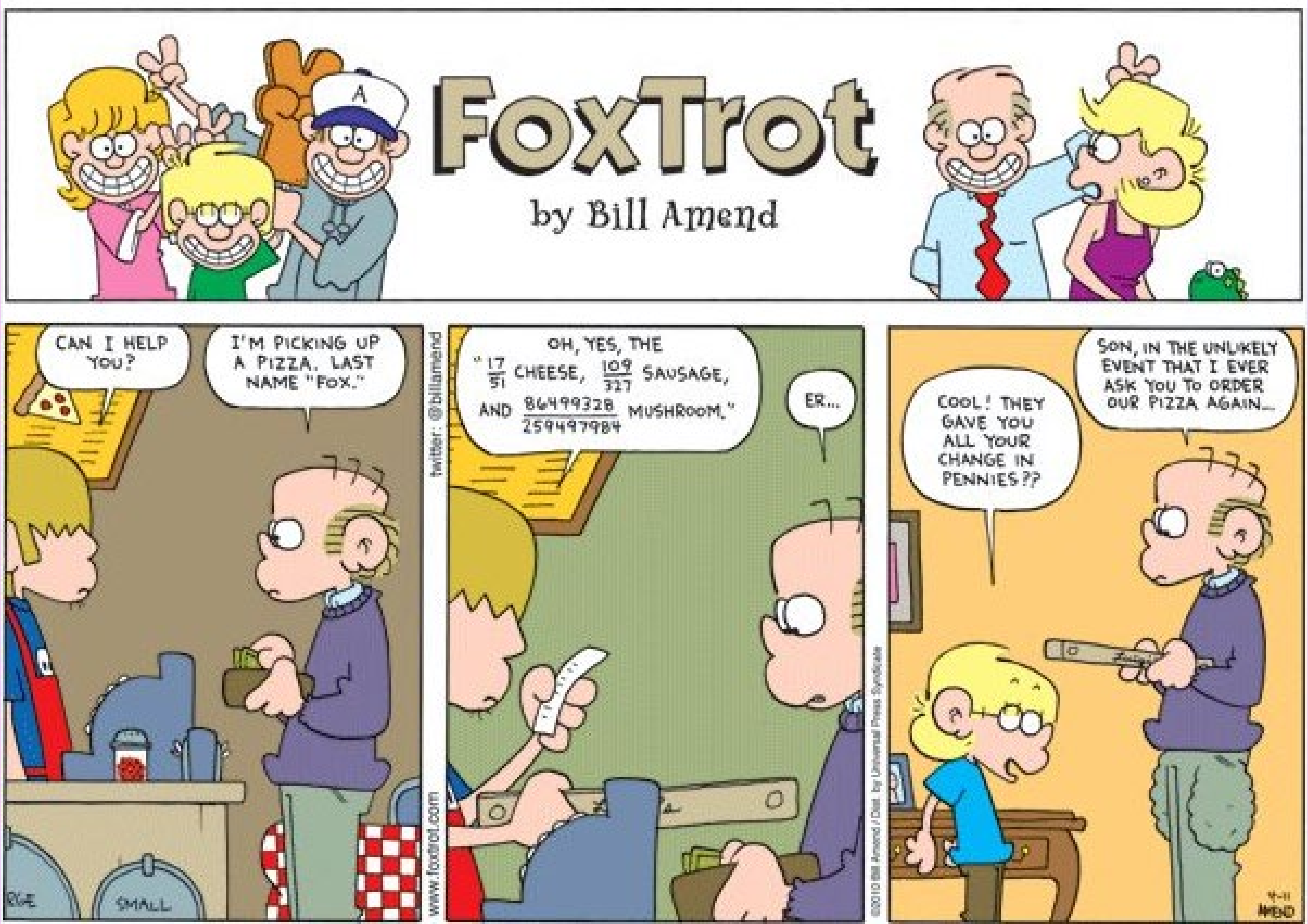
Fraction at the Midtown Comics booth at the 2011 New York Comic Con

Born	Matt Fritchman December 1, 1975 (age 47) Chicago Heights, Illinois, U.S.
Area(s)	Writer
Notable works	*Hawkeye* *Sex Criminals* *The Invincible Iron Man* *The Immortal Iron Fist* *Casanova* *Uncanny X-Men* *FF*
Awards	"Favourite Newcomer Writer" Eagle Award (2007) "Best New Series" Eisner Award (2009) Inkpot Award (2016)[1]
Spouse(s)	Kelly Sue DeConnick

http://www.mattfraction.com

We cannot fully explore
ala math circle
Continued Fractions
Here!
But
we can
show you
some cool stuff.

$$\frac{86499328}{259497984} = \frac{109}{327} = \frac{17}{51} = \frac{1}{3}$$



$$\frac{1011794048256517}{123389815563513} = 8+\frac{24675523748413}{123389815563513}$$

=L[8;5,2023,9,121,333,33602]

$$= 8+\cfrac{1}{\cfrac{123389815563513}{24675523748413}}$$

$$= 8+\cfrac{1}{5+\cfrac{12196821448}{24675523748413}}$$

$$= 8+\cfrac{1}{5+\cfrac{1}{\cfrac{24675523748413}{12196821448}}}$$

$$= 8+\cfrac{1}{5+\cfrac{1}{2023+\cfrac{1353959109}{12196821448}}}$$

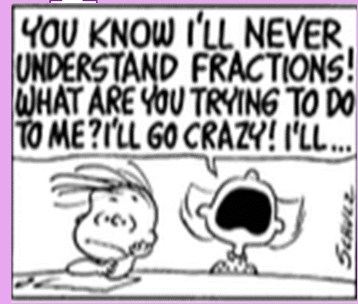
$$= 8+\cfrac{1}{5+\cfrac{1}{2023+\cfrac{1}{9+\cfrac{111189467}{1353959109}}}}$$

$$= 8+\cfrac{1}{5+\cfrac{1}{2023+\cfrac{1}{9+\cfrac{1}{121+\cfrac{33602}{111189467}}}}}$$

$$= 8+\cfrac{1}{5+\cfrac{1}{2023+\cfrac{1}{9+\cfrac{1}{121+\cfrac{1}{333+\cfrac{1}{33602}}}}}}$$

8/5/2023 – 9am Room 121 – 333 S Franklin Street Tampa Bay FL 33602

That was a finite continued fraction You know, you don't have to stop if you don't care to....



$$405+\cfrac{1}{6+\cfrac{1}{28+\cfrac{1}{7+\cfrac{1}{28+\cfrac{1}{7+\cfrac{1}{28+\cfrac{1}{7+\cfrac{1}{28+\cfrac{1}{7+\cfrac{1}{28+\cdots}}}}}}}}}}$$

$$x=\cfrac{1}{28+\cfrac{1}{7+x}}$$

Error $\approx 2.5\times10^{-8}$

$$\Rightarrow x=\frac{-7+5\sqrt{2}}{2}$$

So the above is equal to $405+\frac{1}{6+x}$

$$=\mathbf{L}[405;6,\overline{28,7}]=\frac{2023+\sqrt{8}}{5}\approx 405+\cfrac{1}{6+\cfrac{1}{28+\cfrac{1}{7}}}$$

$$\frac{2023+\sqrt{8}}{5} = \mathbf{L}[405; 6, \overline{28,7}]$$

$$\frac{2023+\sqrt{5}}{8} = \mathbf{L}[253; 6, \overline{2,8}]$$

$$\frac{8+\sqrt{2023}}{5} = \mathbf{L}[10; \overline{1,1,2,8,1,1,2,17}]$$

$$\frac{5+\sqrt{2023}}{8} = \mathbf{L}[6; \overline{4,22,4,10,1,358,1,10}]$$

$$\frac{5+\sqrt{8}}{2023} = \mathbf{L}[0; 258,2, \overline{2,1,1,13,6,2,6,13,1,1,2,1,1,672,1,1}]$$

$$\frac{8+\sqrt{5}}{2023} = \mathbf{L}[0;197,1,1,\overline{1,2,1,3,1,1,2261,7,13,1,2,37,1,163,1,1,12,2,2,8,}$$
$$\overline{2,5,2,4,18,2,7,2,1,17,10,1,2,2,1,9,59,1,4,3,4,1,1,10,2,1,3,}$$
$$\overline{3,2,1,4,1,2,13,7,1,3,45,1,9,11,1,2,1,6,1,2,25,1,1,2,1,5,1,1,}$$
$$\overline{4,2,9,2,2,1,8,1,1,1,1,7,4,1,1,43,1,1,2,1,2,4,2,2,2,9,3,3,3,}$$
$$\overline{1,4,4,25,5,1,2,4,2,2,1,2,7,2,5,2,1,8,46,22,1,7,1,1,37,3,12,}$$
$$\overline{3,3,1,1,1,1,1,26,1,2,2,7,2,1,1,24,1,1,1,1,6,32,22,2,1,3,9,2,}$$
$$\overline{4,14,2,1,2,2,11,2,7,7,1,2,3,1,25,1,3,5,2,1,45,2,8,1,1,14,1,}$$
$$\overline{1,1,6,1,7,1,3,1,2,1,1,3,1,1,3,1,7,21,1,118,11,1,1,2,1,4,6,1,}$$
$$\overline{7,4,1,3,1,1,9,6,3,1,3,62,7,1,4,3,1,2,1,2,2,1,1,7,1,3,6,1,4,}$$
$$\overline{1,45,3,28,33,2,1,6,2,4,3,7,1,1,15,15,25,2,2,1,1,3,4,1,1,1,2,}$$
$$\overline{6,1,2,2,8,1,2,475,1,4,1,1,7,3,1,1,3,1,1,4,4,2,1,1,2,1,1,6,15,}$$
$$\overline{35,1,45,5,2,1,3,2,6,5,5,7,1,1,1,2,1,2,2,1,1,1,1,6,1,13,1,2,}$$
$$\overline{1,1,1,1,1,1,4,2,89,8,26,2,2,5,1,8,1,6,1,12,1,5,1,9,1,1,4,50,}$$
$$\overline{3,9,7,1,1,4,5,1,1,45,1,1,1,1,2,22,1,6,1,6,1,1,1,2,1,2,2,1,100,}$$
$$\overline{1,19,14,3,2,9,1,1,1,2,1,2,4,1,1,1,10,2,1,1,2,1,7,9,1,1,2,}$$
$$\overline{2,1,38,1,4,24,1,6,6,2,1,28,1,46,2,2,45,1,3,7,1,1,2,1,5,5,5,}$$
$$\overline{15,1,2,2,2,1,1,2,1,2,1,2,2,1,14,3,1,1,1,2,12,1,1,1,3,4,5,1,}$$
$$\overline{6,1,73,1,8,1,2,1,2,1,1,5,1,2,2,1,7,40,1,152,2}]$$

$$n+\cfrac{1}{n+\cfrac{1}{n+\cfrac{1}{\dots}}}=[n;\overline{n}]$$

$$\Longrightarrow x=n+\frac{1}{x}$$

$$\Longrightarrow x=\frac{n\pm\sqrt{n^2+4}}{2}$$

$$\Longrightarrow \frac{n+\sqrt{n^2+4}}{2}=[n;\bar{n}]$$

$$\Longrightarrow \frac{\sqrt{n^2+4}-n}{2}=[0;\bar{n}]$$

GOLDEN RATIO

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots}}}=[1;\overline{1}]$$

$$\Rightarrow x = 1+\frac{1}{x}$$

$$\Rightarrow x = \frac{1+\sqrt{5}}{2}$$

$$[1;\underbrace{1\cdots 1}_{K}\,] = \frac{F_{k+1}}{F_k}$$ *Fibonacci Numbers!*

GOLDEN STRING USES CONCATENATION INSTEAD OF ADDITION & IS IN BASE 2

1 10 101 10110 10110101

In Base 2 we have a non- repeating decimal

$$0.1101011011010110101\ldots$$

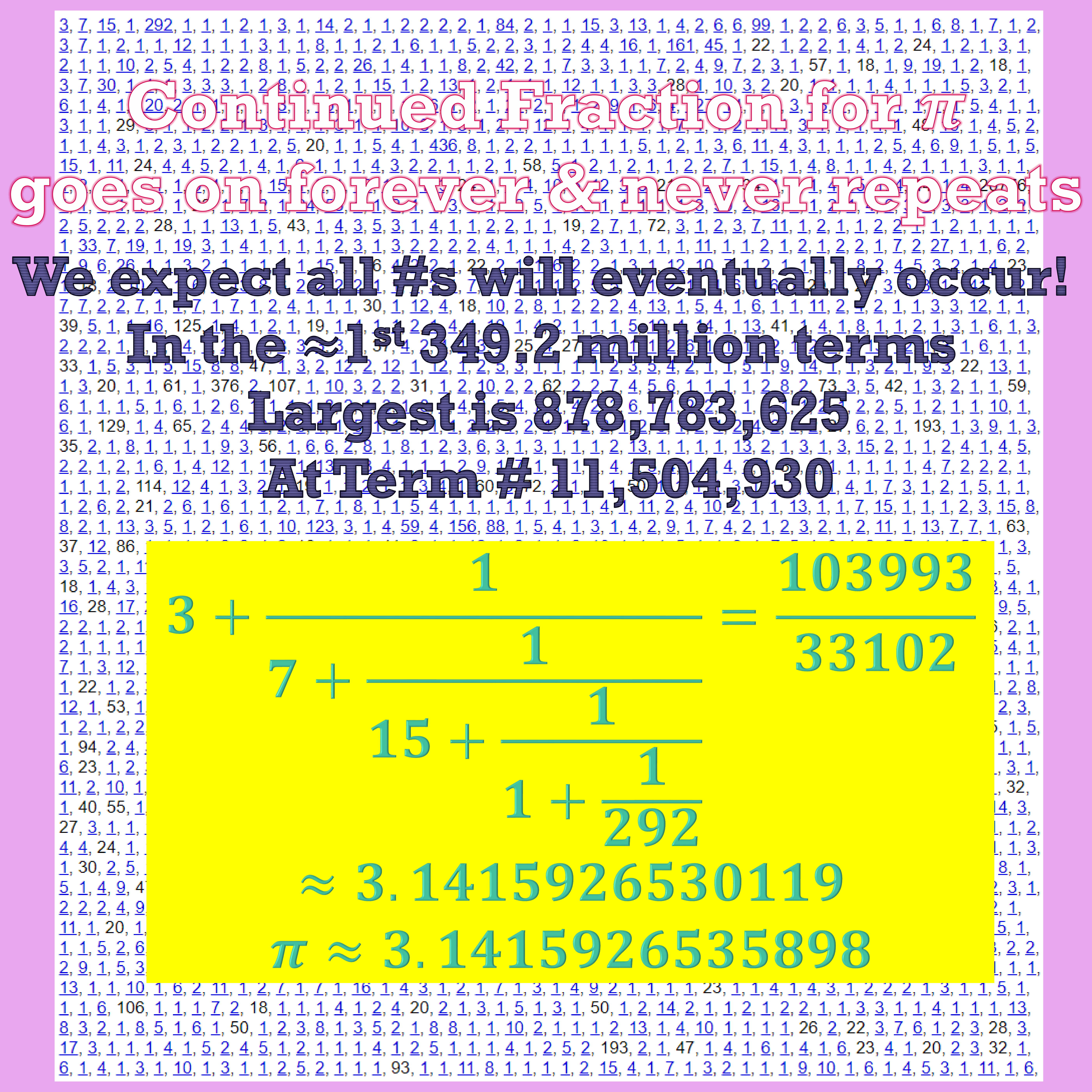
Called the Rabbit Constant!

$$\approx 0.709803442861291 3\ldots$$

$$= [0; 1, 2, 2, 4, 8, 32, 256, \ldots]$$

$$= [0; 2^0, 2^1, 2^1, 2^2, 2^3, 2^5, 2^8 \ldots]$$

$$= [0; 2^{F_0}, 2^{F_1}, 2^{F_2}, 2^{F_3}, 2^{F_4}, 2^{F_5}, 2^{F_6}, \ldots]$$



Continued Fraction for π
goes on forever & never repeats
We expect all #s will eventually occur!
In the ≈1st 349.2 million terms
Largest is 878,783,625
At Term # 11,504,930
$$3+\cfrac{1}{7+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{292}}}}=\frac{103993}{33102}$$
$$\approx 3.1415926530119$$
$$\pi \approx 3.1415926535898$$

Name That Tune

$$L[2; 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, \vdots 1, 2n, 1, \cdots] = e$$

Other questions! (no answers)

There are non-standard
continued fractions
With different numerators besides 1
With negatives.
What numbers (e.g. cube roots)
Have identifiable patterns?

What is
$[1; 2, 3, 4, 5, \ldots]$
$= K_{n=1}^{\infty} n \approx 1.4331274267223$
We can find recursive sequences
For numerator and denominator
But does the limit have a closed form?

References

Exploring Continued Fractions
By Andrew J Simoson
MAA/AMS Press

University of Surrey
https://r-knott.surrey.ac.uk/Fibonacci/cfCALC.html