Puzzles using Pattern

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Blocks



Pattern blocks





Free play



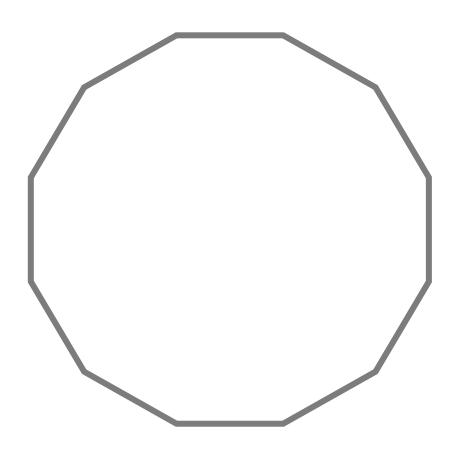








Make a regular dodecagon





Side length: 1 unit = side of green triangle



Dodecagon gallery













What do you notice? What do you wonder?

Wonderings regular dodecagon Does it always take 12 of each shape? Could we use other shapes to make the dodecagon? How many different petterns can you make of the regular didecagon? why are dodecasons circular looking, what ways can the triangles & rhombi touch? If the docle cagon, regular, must it be symmetrica () What type of symmetry exists)





Can we make a smaller dodocazon? Does the number of triangles dictate the number of sides? If the polygon is regular must there be symmetry? what if squares were used to build a dodecagon? why did we use only two shapes? Does an odd number of pes (mombi + triangles) determines an odd number of sides) If a dodecogon is symmetricely is it then regular? shat if the least humber of prs (Ret) to make & doderacion? a regular Jodecagon?

More observations

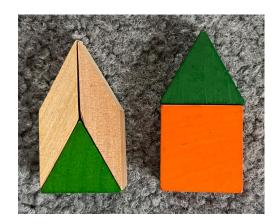
A few questions to consider

- Why do all the unit side regular dodecagons seem to consist of 12 triangles and 12 rhombuses?
- Is the center of the dodecagon always situated at the boundary of a piece?
- Do the perimeter pieces determine the arrangement?
- Can you make equiangular dodecagons using these two kinds of pieces?



Why 12 pieces each?

Idea: AREA





 $6 + 3\sqrt{3} = n\frac{1}{2} + k\frac{\sqrt{3}}{4}$



Generalization

Isogonal dodecagon – all angles are the same and there are two – alternating – side lengths.



Regular dodecagon again

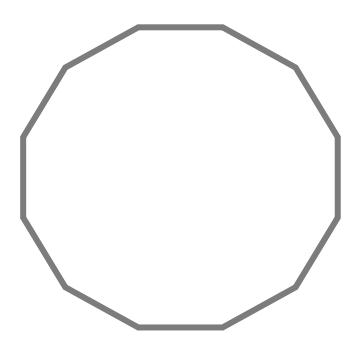
Can we make it with 6 blue (120°-60°) and 12 tan (150°-30°) rhombuses?



Rhombic polygons

• A convex 2n-gon is called rhombic if its sides are of unit length and its opposite sides are parallel.



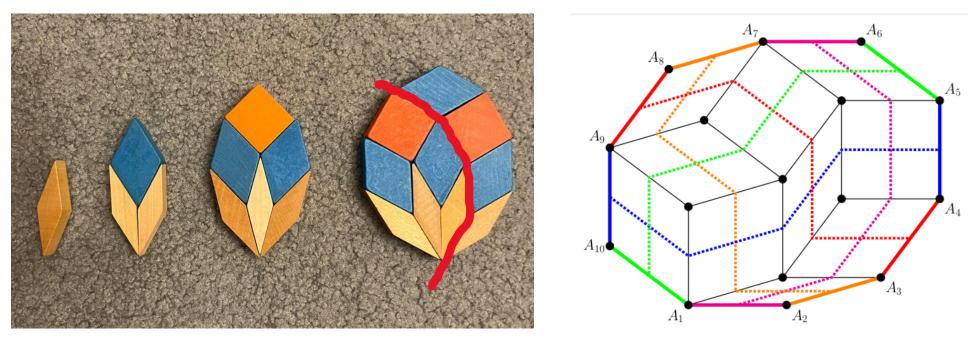




Rhombic polygons and triangular numbers

Every 2n-sided convex rhombic polygon is made up of

$$1 + 2 + ... + (n - 1) = \frac{n(n-1)}{2}$$
 rhombuses.

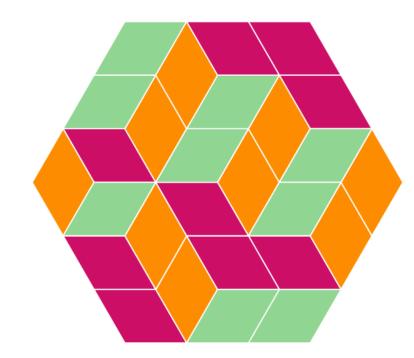




One kind of rhombus only - calissons

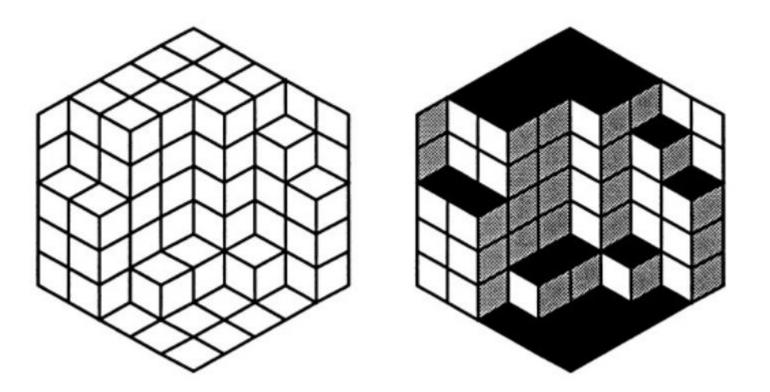
French rhombus-shaped sweets in a hexagonal box – different colors in different orientations – how many of each?







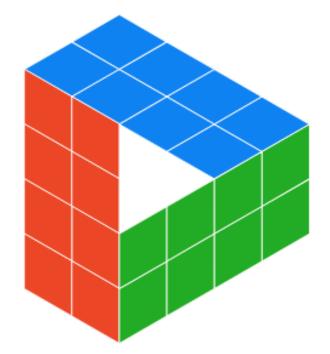
Proof without words





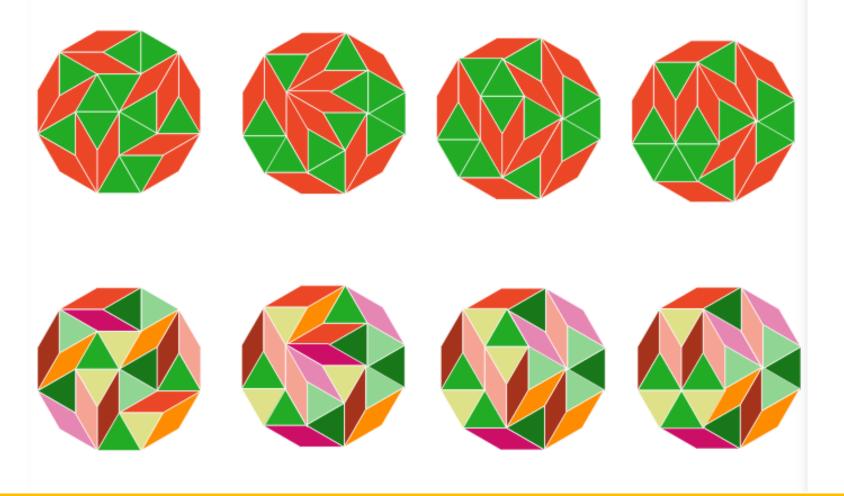
Generalization

• If a figure can be covered by congruent rhombuses, then the number of rhombuses of each orientation is independent of the covering.





Back to dodecagons



Triangles: 3-3-3-3

Rhombuses: 1-1-2-2-3-3 2-2-2-2-2-2

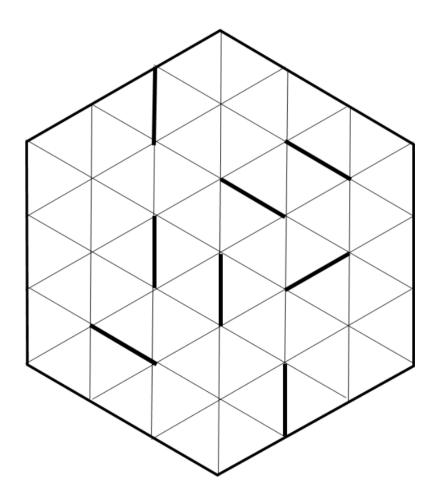


The calissons puzzle

Invented by Olivier Longuet, analyzed by Yan Gerard et.al.

Tile the grid with calissons in such a way that the edges given as input are not overlapped by a calisson and are adjacent to two calissons of different colors.

Play at: https://mathix.org/calisson/blog/





Thank you!

References and sources:

Ed Southall, Vincent Pantaloni: Geometry Snacks Kömal Problem B. 5156 February 2021; Guy David and Carlos Tomei, The Problem of the Calissons Roger Nelsen: Proofs without Words E. W. Dijkstra: On Covering a Figure with Diamonds, http://www.cs.utexas.edu/users/EWD/ewd10xx/EWD1055c.PDF https://mathix.org/calisson/blog/ https://szimmetria-airtemmizs.tumblr.com/post/151438386823/problemof-calissons



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