Puzzles using Pattern Blocks

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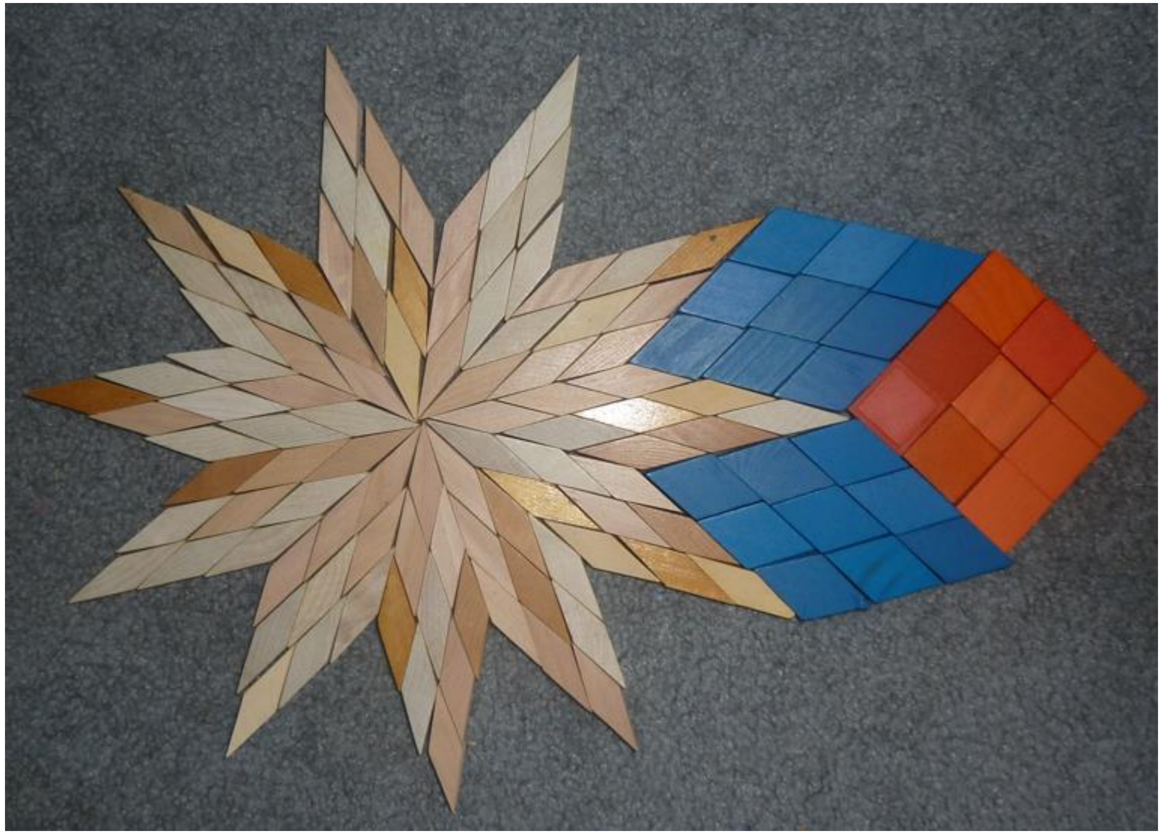
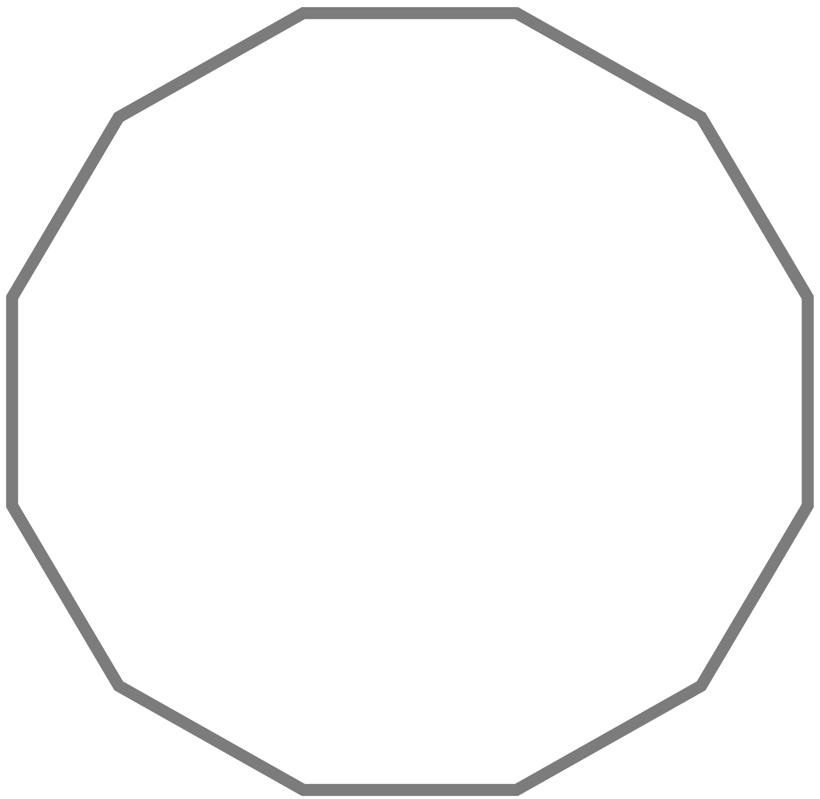
Pattern blocks



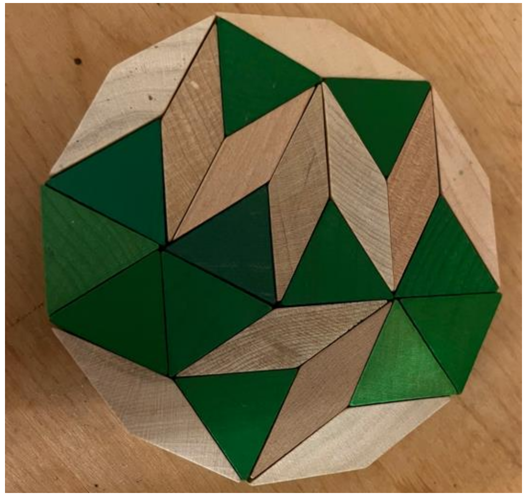
Free play

Make a regular dodecagon

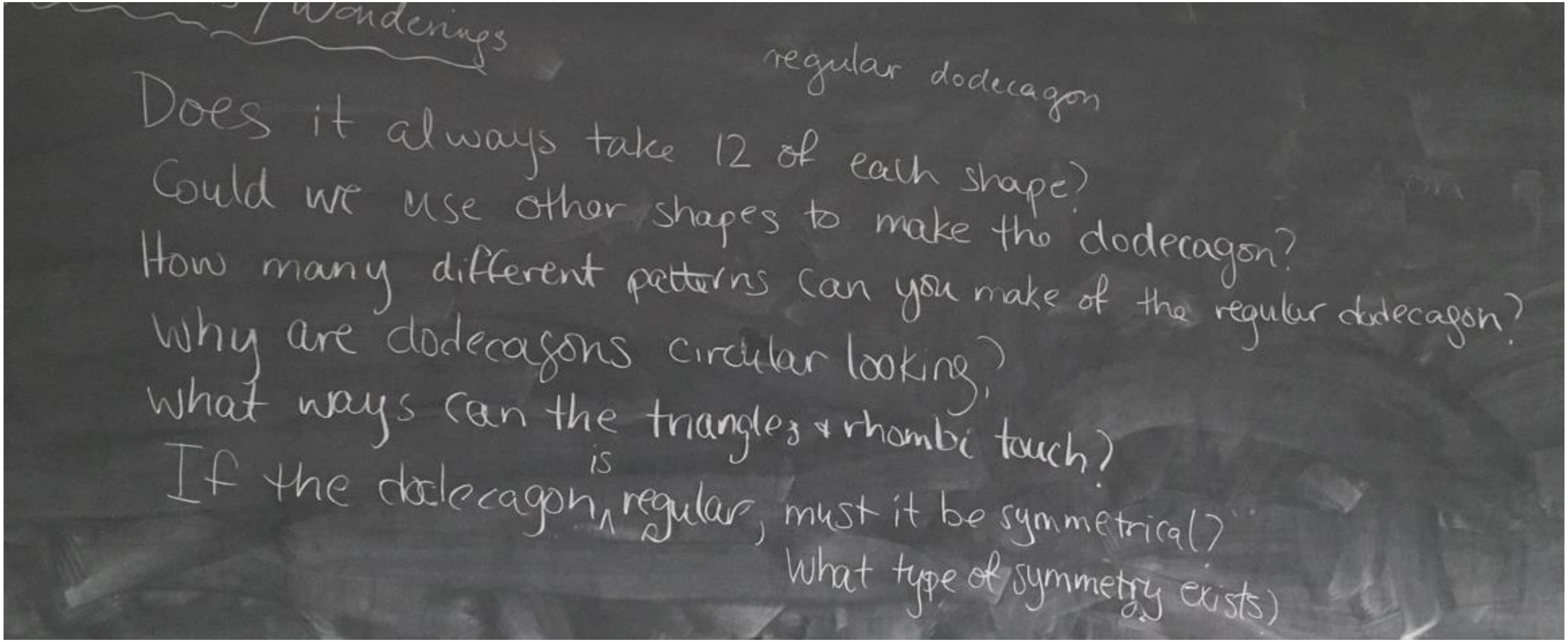


Side length: 1 unit = side of green triangle

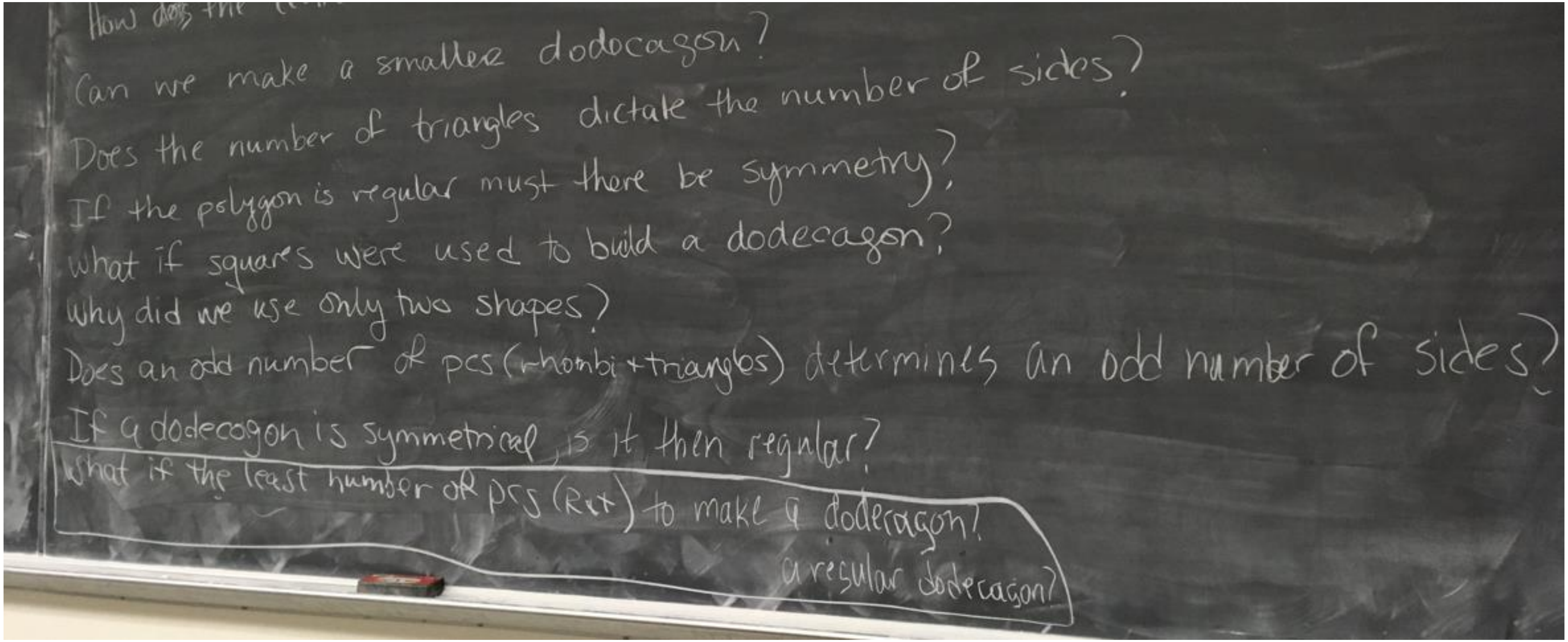
Dodecagon gallery



What do you notice? What do you wonder?



More observations



A few questions to consider

- Why do all the unit side regular dodecagons seem to consist of 12 triangles and 12 rhombuses?

- Is the center of the dodecagon always situated at the boundary of a piece?

- Do the perimeter pieces determine the arrangement?

- Can you make equiangular dodecagons using these two kinds of pieces?

Why 12 pieces each?

Idea: AREA

$$6 + 3\sqrt{3} = n\,\frac{1}{2} + k\,\frac{\sqrt{3}}{4}$$

Generalization

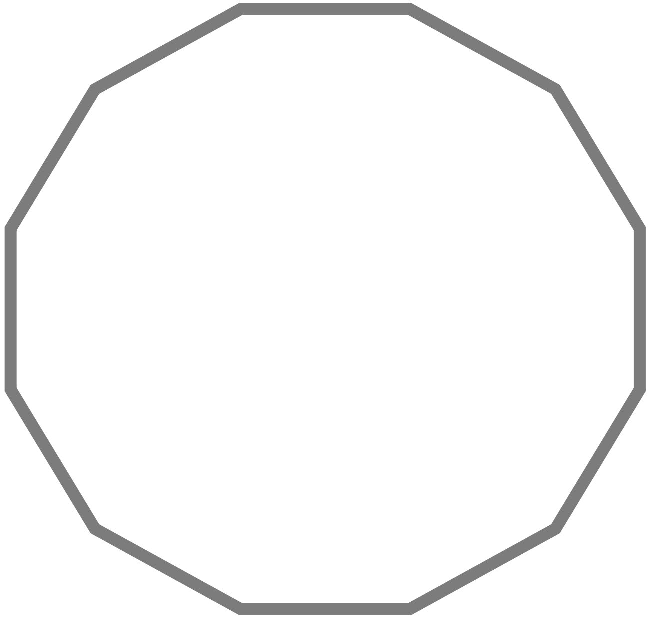
Isogonal dodecagon – all angles are the same and there are two – alternating – side lengths.

Regular dodecagon again

Can we make it with 6 blue (120°-60°) and 12 tan (150°-30°) rhombuses?

Rhombic polygons

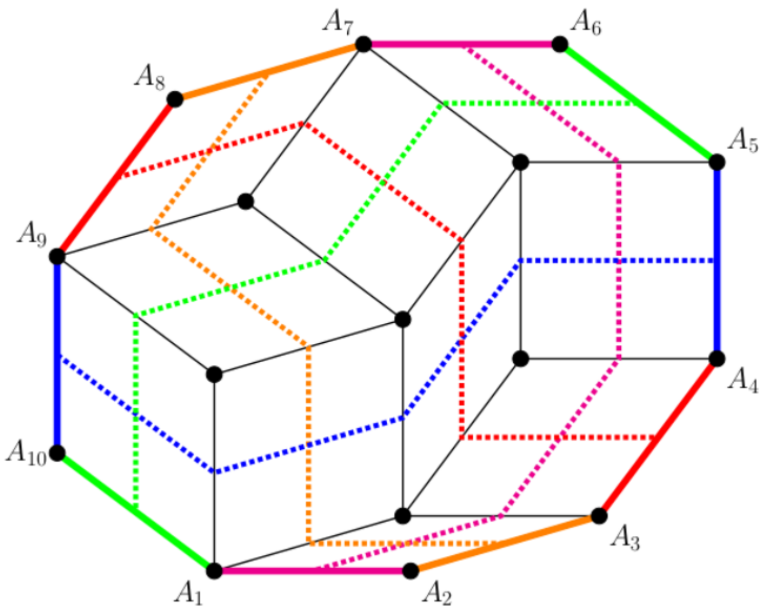
- A convex 2n-gon is called rhombic if its sides are of unit length and its opposite sides are parallel.



Rhombic polygons and triangular numbers

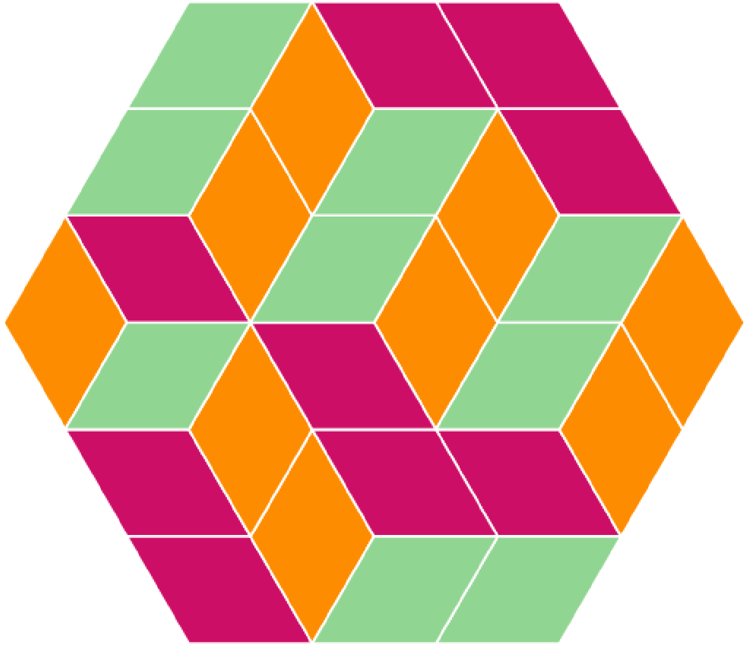
Every 2n-sided convex rhombic polygon is made up of

$1 + 2 + \ldots + (n-1) = \frac{n(n-1)}{2}$ rhombuses.

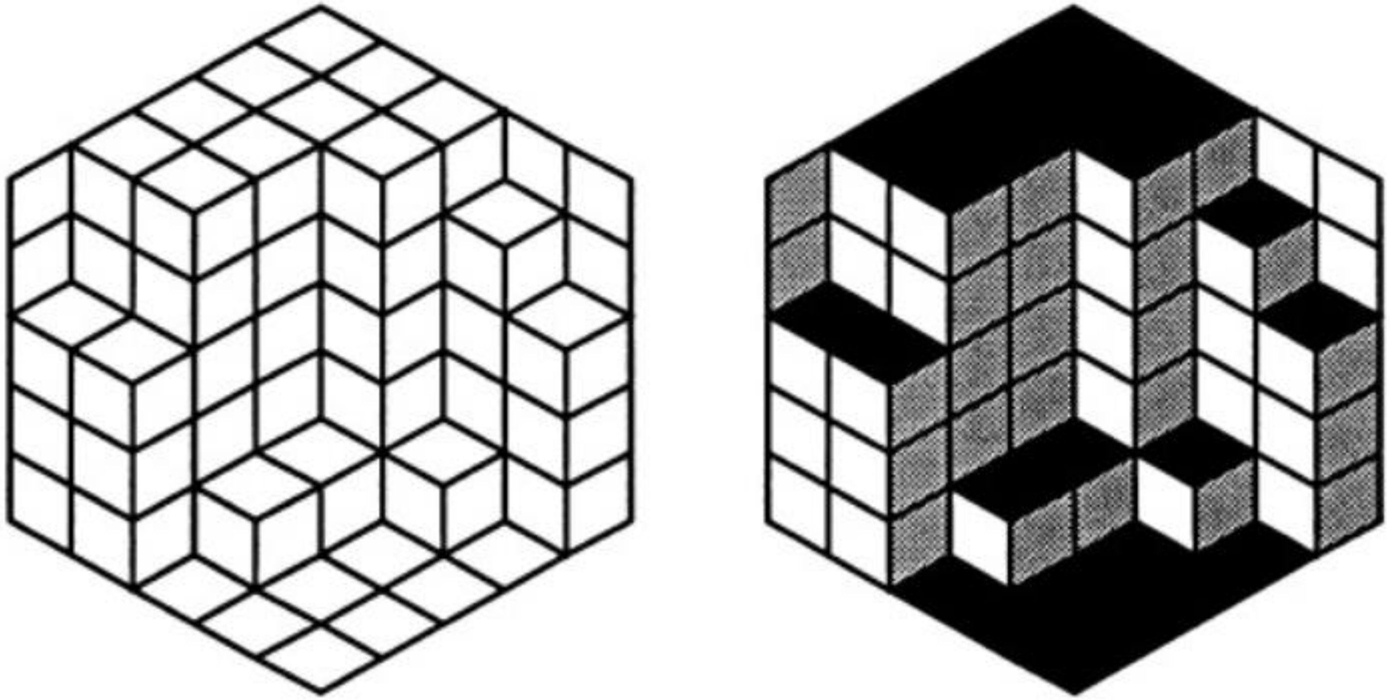


One kind of rhombus only - calissons

French rhombus-shaped sweets in a hexagonal box – different colors in different orientations – how many of each?

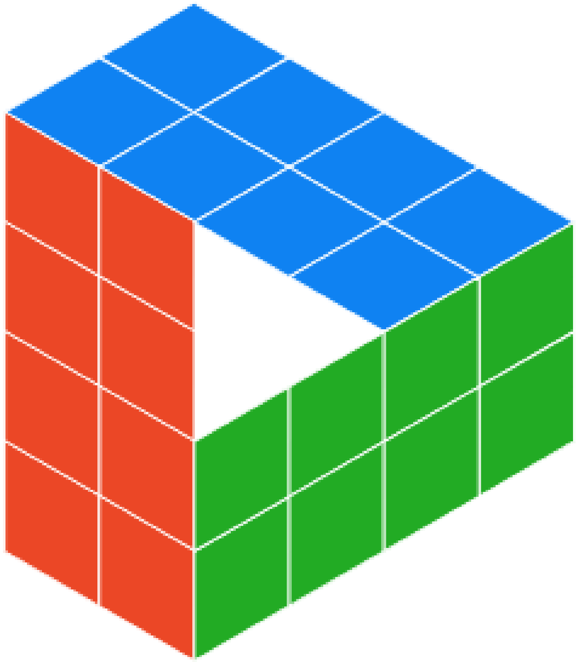


Proof without words

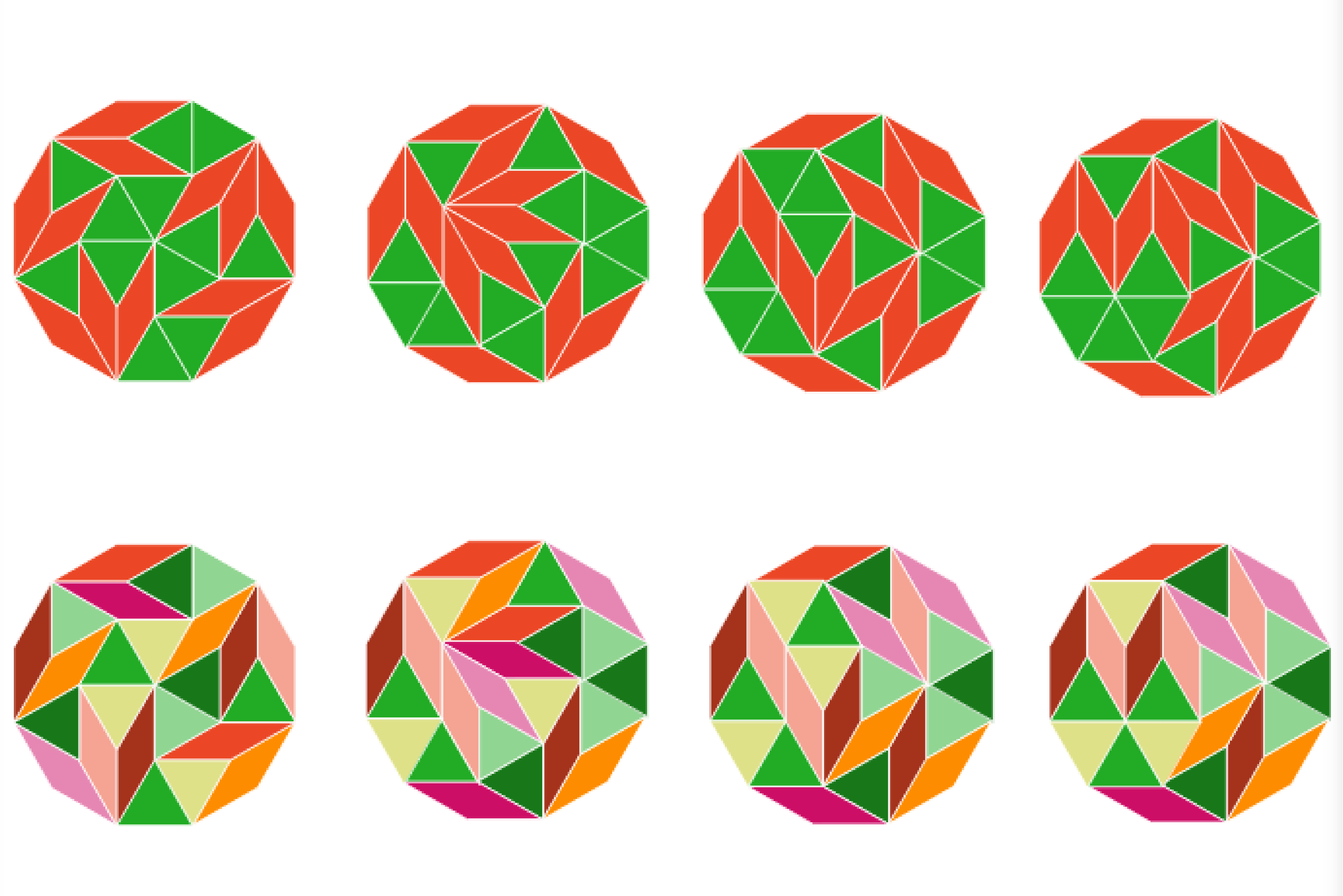


Generalization

- If a figure can be covered by congruent rhombuses, then the number of rhombuses of each orientation is independent of the covering.



Back to dodecagons



Triangles: 3-3-3-3

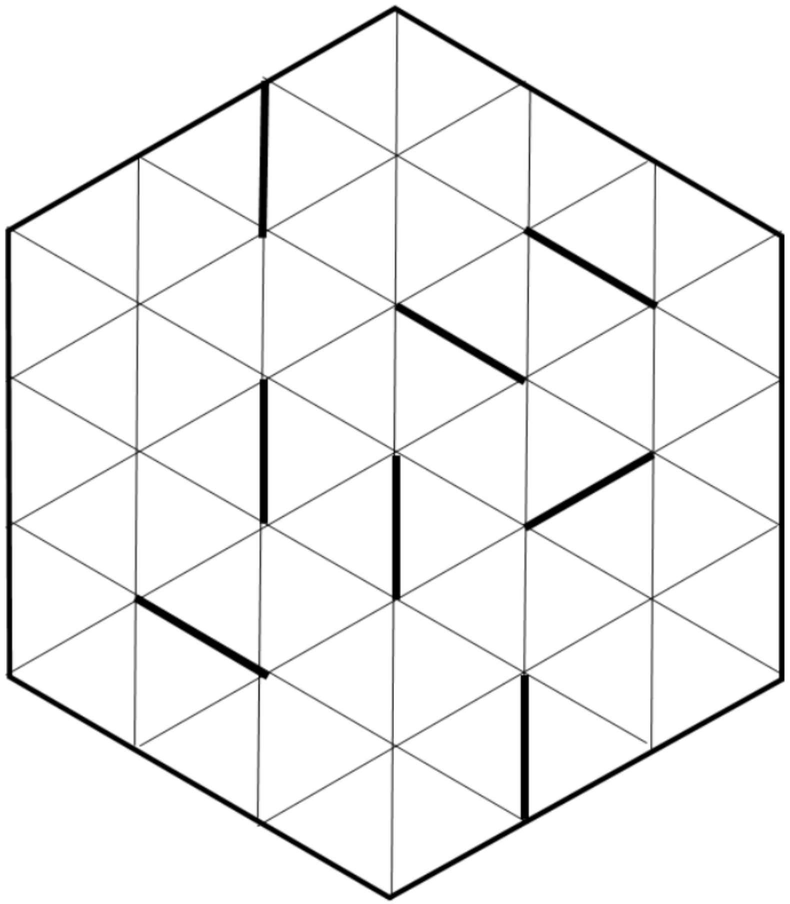
Rhombuses: 1-1-2-2-3-3
2-2-2-2-2-2

The calissons puzzle

Invented by Olivier Longuet,
analyzed by Yan Gerard et.al.

Tile the grid with calissons in such a way that the edges given as input are not overlapped by a calisson and are adjacent to two calissons of different colors.

Play at:
https://mathix.org/calisson/blog/



Thank you!

References and sources:

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