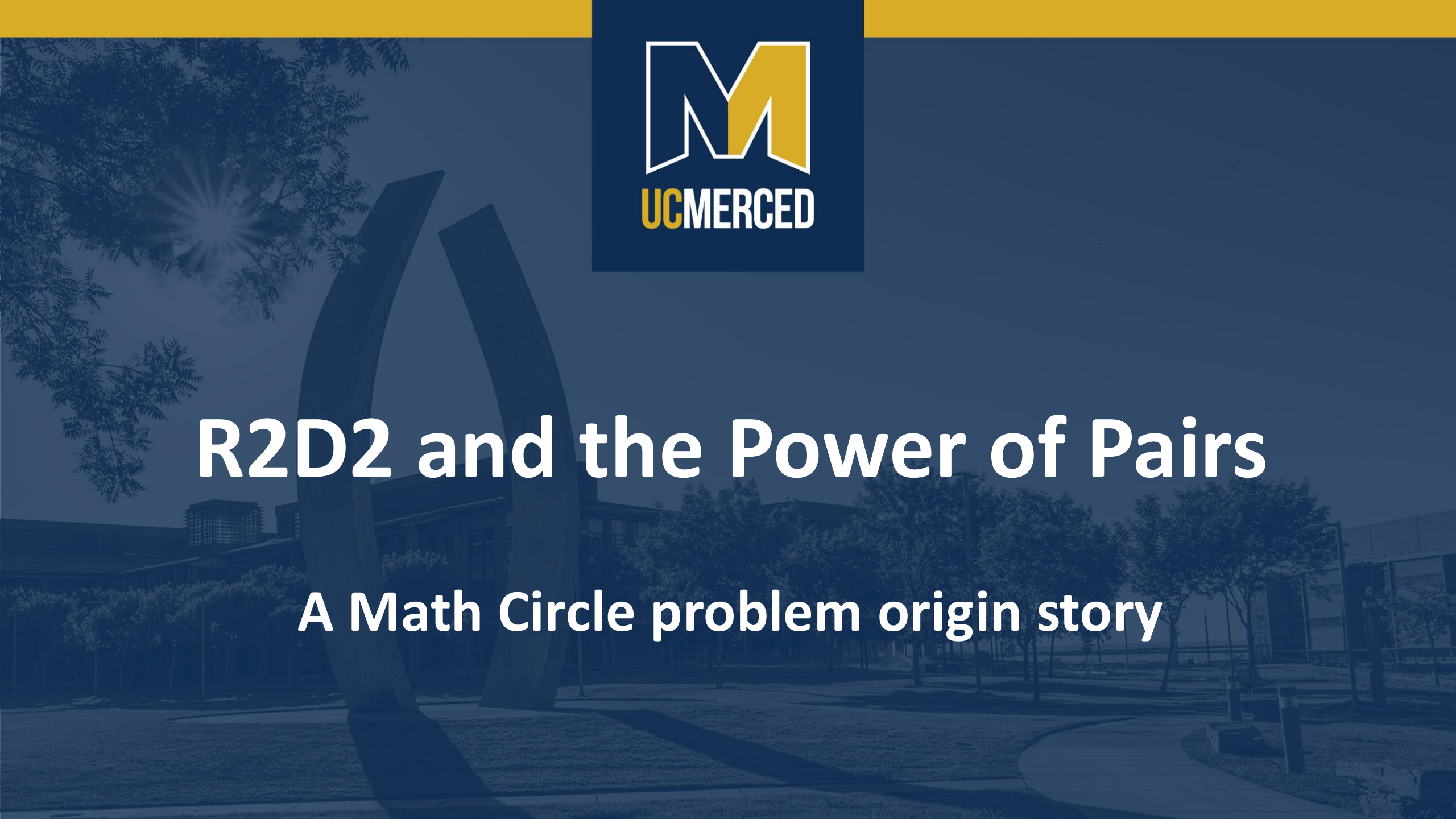

R2D2 and the Power of Pairs

A Math Circle problem origin story

R2D2 (Remove2Draw2)

- **R2:** click on the edge connecting two filled nodes to make them both empty
- **D2:** click on the edge connecting two empty nodes to make them both filled

Try Our Fun Online Puzzle Game

https://r2d2math.com/

Our Team

Department of Applied Mathematics

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EunSang (Patrick) Park

Mario Velez

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RANKED #1

THE WALL STREET JOURNAL

#18 BEST UNIVERSITY IN THE NATION // THE WALL STREET JOURNAL

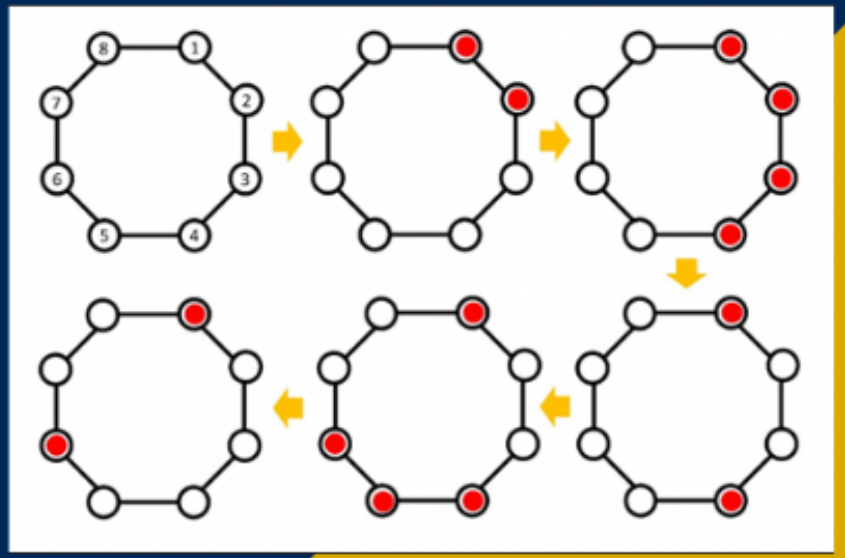
Merced Math Teachers' Circle

UCMERCED

SCHOOL OF NATURAL SCIENCES
Applied Mathematics

PROBLEM OF THE MONTH
March 2023

There are 8 holes located in vertices of an octagon (labeled 1 through 8) and you have 8 identical balls. Each hole either can hold one ball or it can be empty. If there are two empty neighboring holes, then you are allowed to put two balls into these holes. If there are two neighboring holes with balls, then you are allowed to remove those two balls. When all 8 holes are initially empty and you keep performing these allowed actions, what is the total number of configurations that can be found? Some possible configurations are shown in the figure. Can you generalize this result to the case with 2k holes (k=2,3,4, ...)?



To submit your solutions for a chance to win an Amazon gift card, and to find out detailed contest rules,

- *scan the QR code to the right, or*
- *go to https://appliedmath.ucmerced.edu/news-events/problem-month*

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MATH CLUB

Our first meeting is on September 9!

Where: Granite Pass 170

When: Every other Monday from 5-6:30pm

Join us for pizza and fun math activities!

Join us on discord!

$b^2 - 4ac$

$\sin(\theta) = \frac{opp}{hyp}$

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Original Math-Circle-Worthy problems?

LOW FLOOR HIGH CEILING

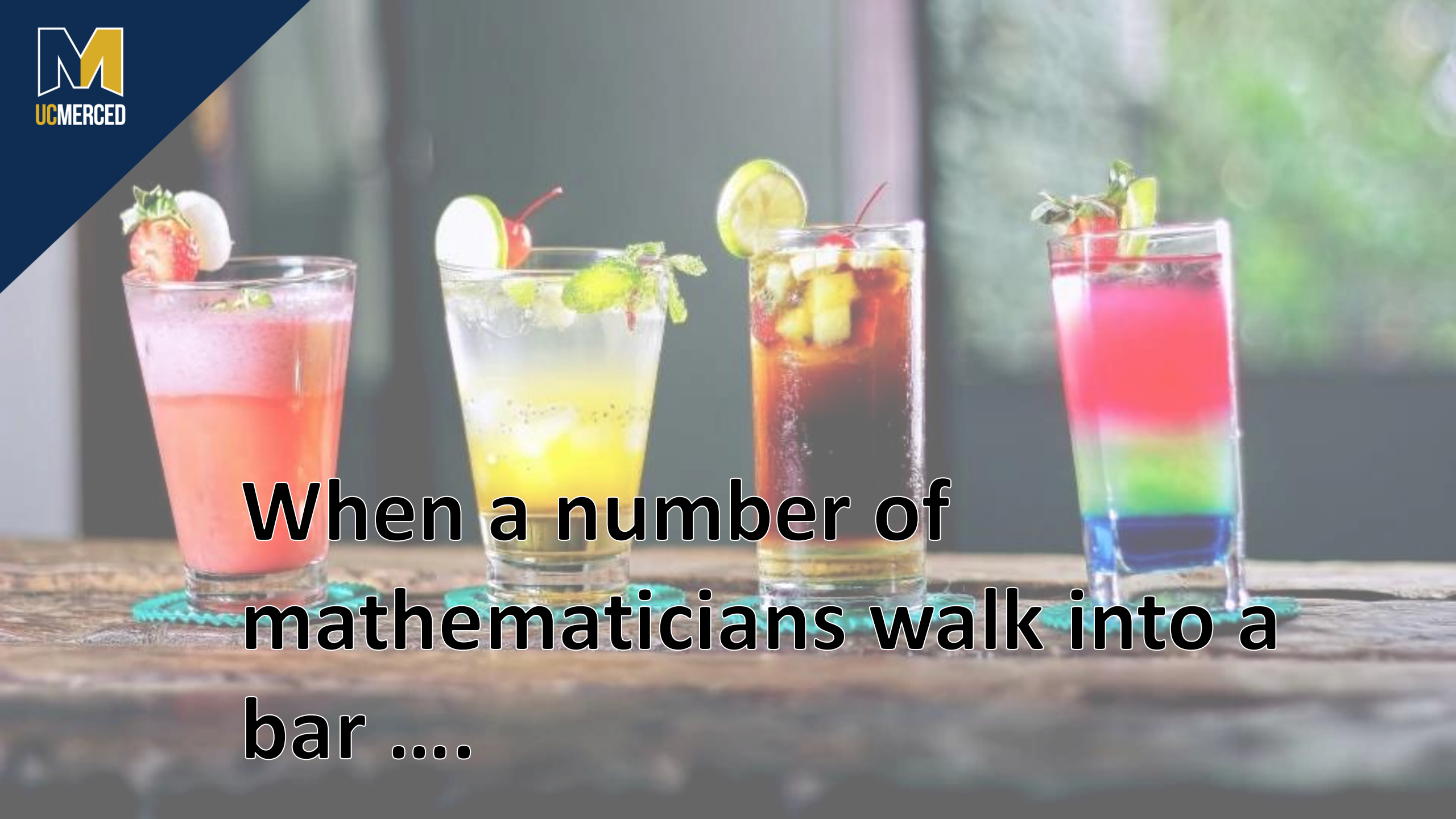
EXPLORATION & CONJECTURE

MULTIPLE SOLUTION STRATEGIES

DISCUSSION & COLLABORATION

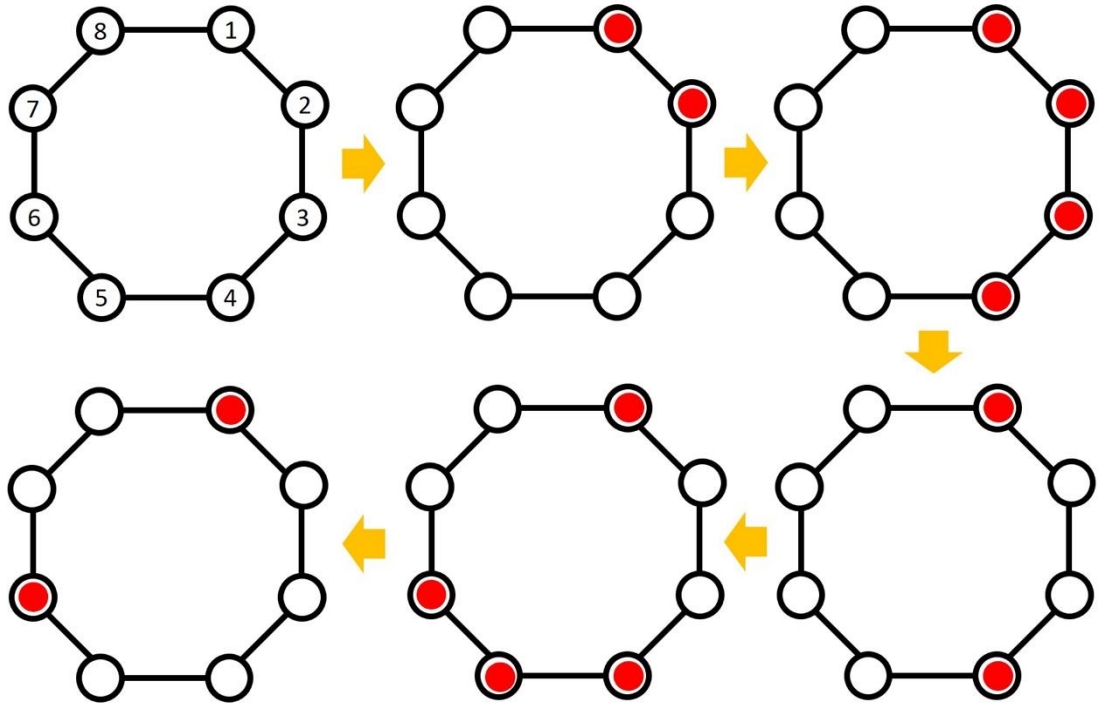
Online: Math Circle Network, Problem of the Math archives, YouTube, TikTok, …

Magazines: Math Circular, Math Horizons, …


When a number of mathematicians walk into a bar ….

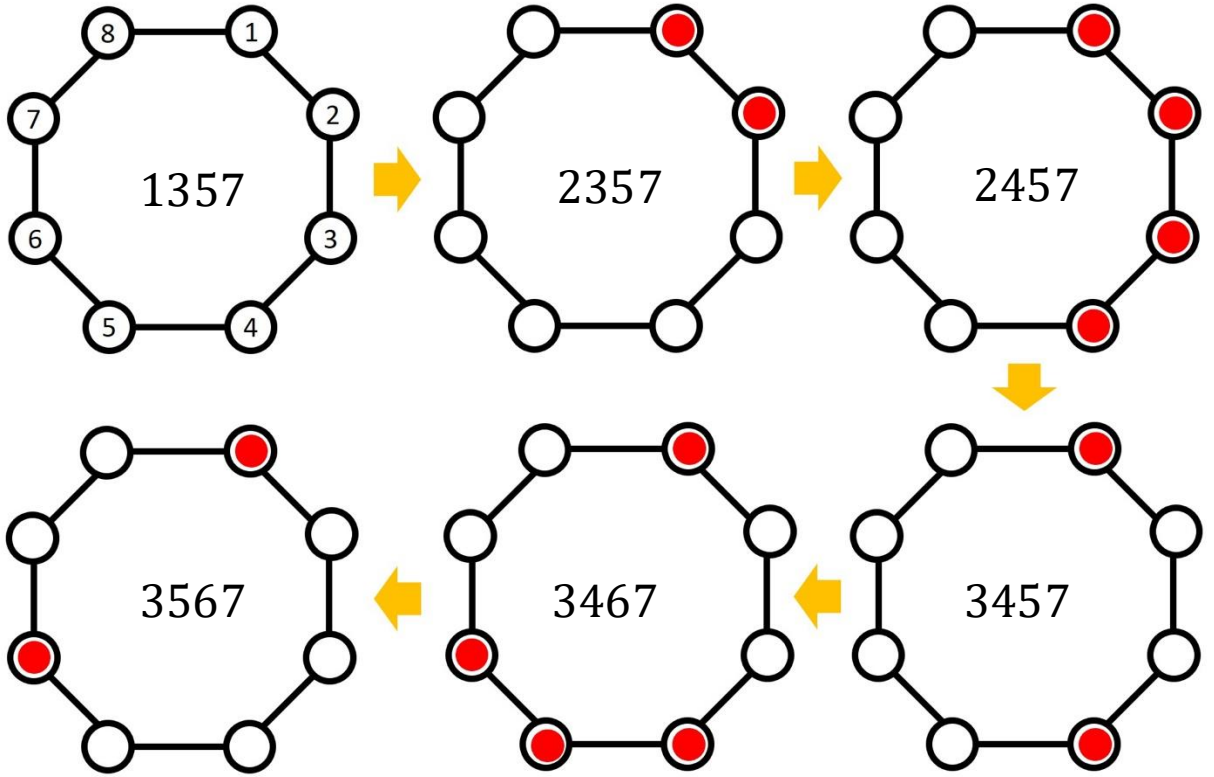
On a cycle graph with an even number, $2n$, of vertices, how many possible configurations can be reached starting from empty?

Computer simulations says $\binom{2n}{n}$. Can we prove it?



A 1-1 Correspondence

Select empty odd-indexed vertices and filled even-indexed vertices

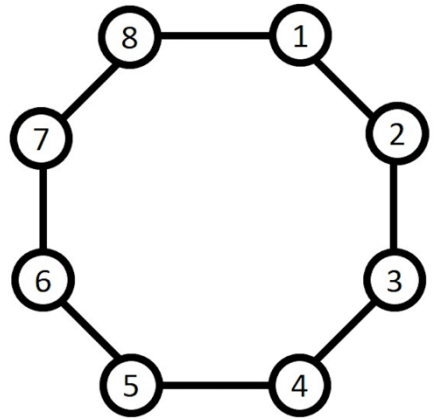


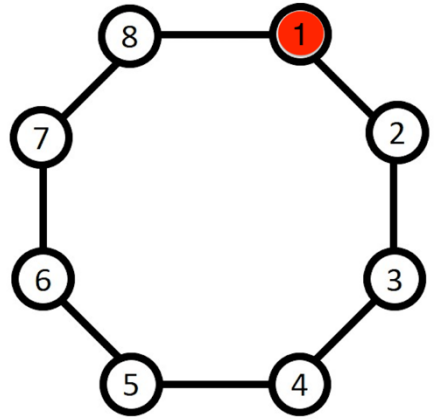
Changho kept asking questions

- What if the cycle graph has an odd number, $2n + 1$, of vertices?
- What if we have a path graph instead of a cycle graph?
- What if we don't start with an empty configuration?
- What about higher dimensional lattices?

→ complete characterization of mutually accessible classes of configurations, depending on the graph

One Easy-to-Spot Invariant of R2D2

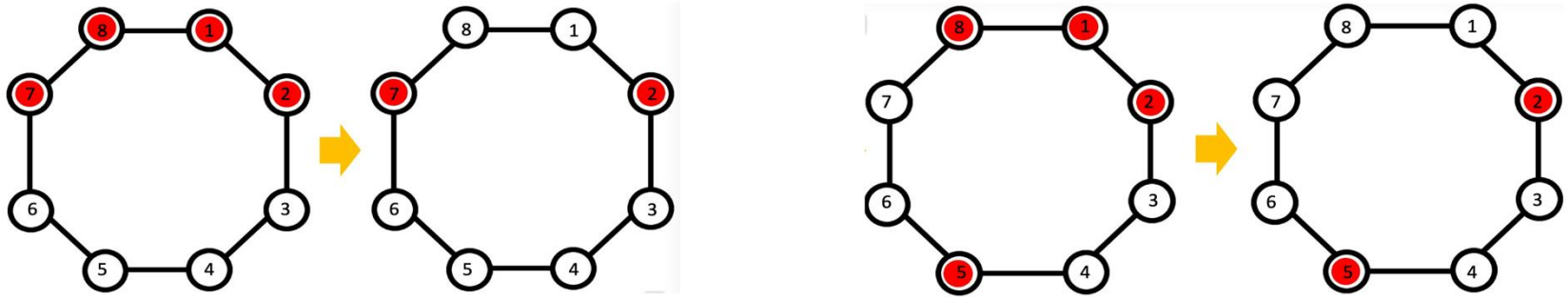




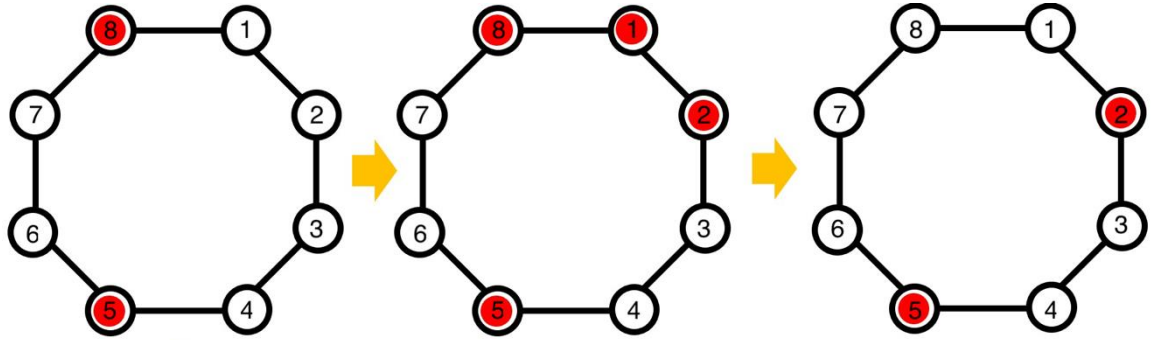
If only one filled -- not possible

Less Obvious Invariant of R2D2

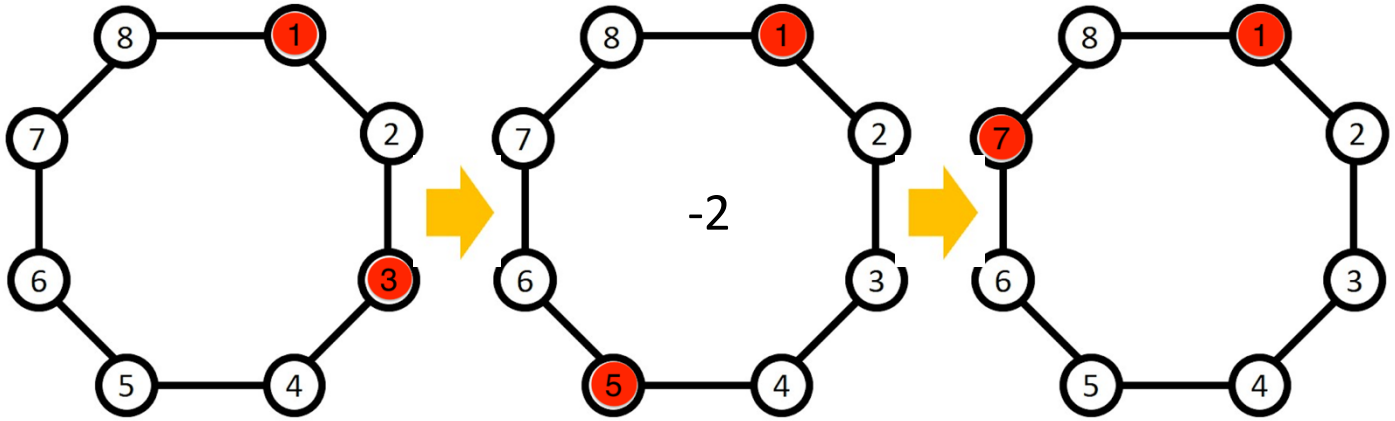
OBSERVATION 1. Any given configuration can be changed to another one with only isolated single filled vertices.

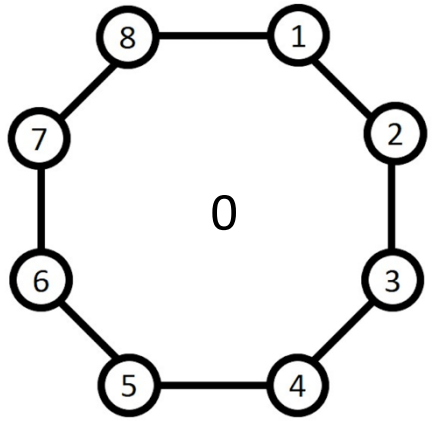


OBSERVATION 2. An isolated single filled vertex can be "single-skipped" two vertices over if those vertices are empty.



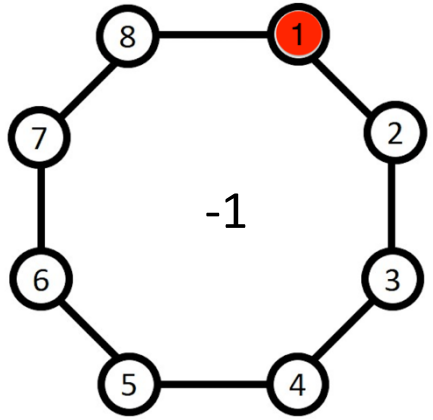
Case 1: Single-Skips Preserve Parity





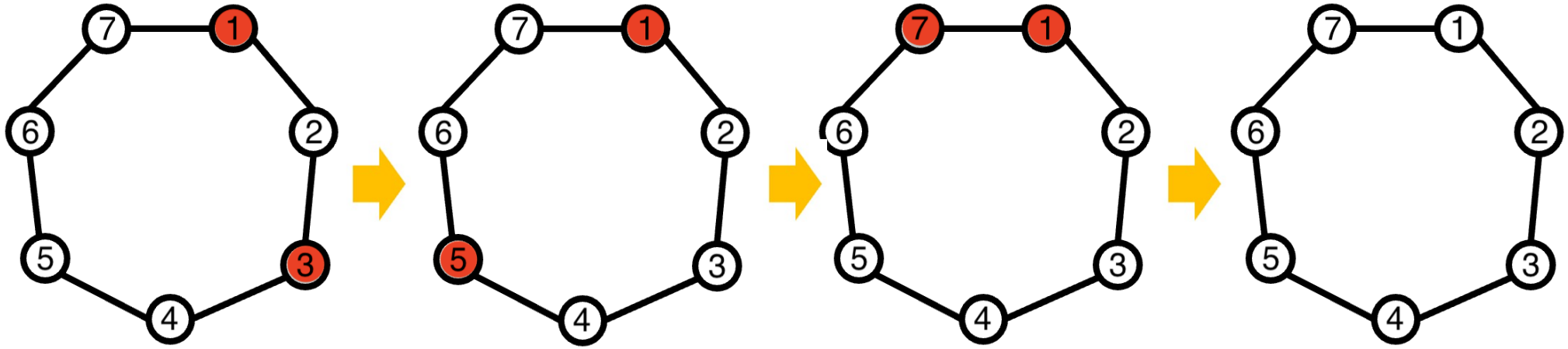
Two configurations are mutually accessible iff they have the same value of

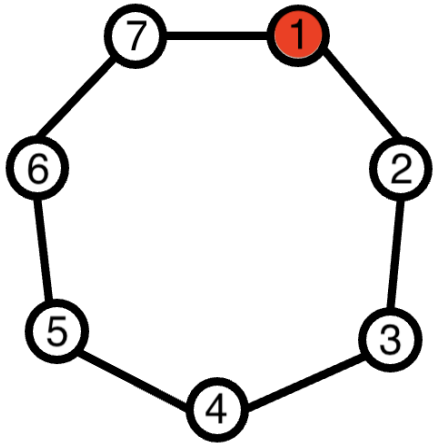
B = #(even filled) - #(odd filled)



Bipartite Graphs

Case 2: Single-Skips Does NOT Preserve Parity





Only TWO mutually accessible classes:
odd or even total # of filled vertices

Non-Bipartite Graphs

Math-Circle Worthy?

- ✓ LOW FLOOR HIGH CEILING
- ✓ EXPLORATION & CONJECTURE
- ✓ MULTIPLE SOLUTION STRATEGIES
- ✓ DISCUSSION & COLLABORATION

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Further Questions to Explore

- Shortest path between two accessible configurations
- Dynamical progression toward equilibrium coverage of a lattice
- Use heterogeneous pair (RabDab)
- What about R3D3 (or even, RkDk) problems (no chemistry, math curiosity)

Thank You!

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