

A "Math Without Words" Puzzle Leading to Research Questions

Jane H. Long and Clint Richardson

The Puzzle

Doing Research? You Might Be Glad to Have These Tools!

Research Experiences and Possibilities

References

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Joint Mathematics Meetings

January 5, 2024



Hello! From the East Texas Math Teachers' Circle

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Outline

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1 The Puzzle

2 Doing Research? You Might Be Glad to Have These Tools!

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The Puzzle – Try It!

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The Puzzle – What's Going On? Can We Always Do It?

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Zooming in to Explain Vertex Labels

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The number on each vertex counts the number of edge endpoints connected to that vertex.

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Is this "It"?

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NO!

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NO!

For a completed puzzle with *n* vertices,

 $\frac{n(n+1)}{2}$ k=1

must be even.

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NO!

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must be even.

Can you see why?



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NO!

For a completed puzzle with *n* vertices,



must be even.



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NO!

For a completed puzzle with *n* vertices,

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

must be even, if and only if *n* is equivalent to 0 or 3 modulo 4.

The summation gives the number of places endpoints can land. Each edge contributes 2 endpoints, so a completed puzzle must have an according to Research Question lange H. Long and Clint Richardson A "Math Without Words" Puzzle Leading to Research Question



Mathematical Tools

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- Vocabulary (vertex, parity, invariant)
- Summation Notation
- Modular Arithmetic
- Logic (contraposition rules out n with odd summation)
- Mathematical Induction



Problem-Solving Tools

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- Solve a simpler problem: fewer vertices
- Look for patterns: arithmetic series formula, 'Yes/No' table
- Adding/removing assumptions and restrictions



Metacognitive Tools

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- What mathematical ideas does this puzzle suggest to you?
- What other questions could we investigate?
- 'I can't find a solution' versus 'A solution doesn't exist'
- What conclusions does our work allow us to make?
- What conclusions does our work not allow us to make?



Example

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Requirement that summation

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

must be even rules out some cases but *does not guarantee* existence of solutions for the cases it does not rule out

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A Constructive Algorithm*

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* Discovered by undergraduate math major Kaleb Fields and independently by R. (a + b + b) = (a + b) + (a

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Another Constructive Algorithm



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Graph Theory Questions, in Roughly Increasing Order of Difficulty

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- 1 Must vertex 1 always connect to vertex 2? (No)
- 2 Does the placement of vertices matter? (No)
- **3** For which n, if any, do solutions without loops exist?
- 4 For which *n*, if any, do solutions without cycles exist?
- 5 For which *n*, if any, do planar solutions exist?



Combinatorial Questions

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- How many edges does a completed drawing contain? $\left(\frac{1+2+\dots+n}{2}\right)$
- **2** In a given completed drawing, how many different paths exist between two given vertices?
- **3** How many solutions are possible for each n?
- 4 How many (connected, disconnected, planar, no-loop,...) solutions are possible for a given n?
- 5 What is the minimal number of repeated edges in a completed drawing having n vertices?



Other Big Questions

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- What makes solutions equivalent? (Distinct?)
- Can known invariants be used to distinguish solutions?
- Can new invariants be generated?



Are These Solutions Equivalent?

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What About These?

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What Is the Purpose of Research?

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Experience?

New results?

- New methods?
- JOY?
- What goals do you want to meet?

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American Institute of Mathematics. MTCircular, Summer/Autumn 2016, p. 19.

Long, Jane H. and Richardson, Clint (2021) A Math Without Words Puzzle, Journal of Math Circles: Vol. 2: Iss. 1, Article 5. Available at: https://digitalcommons.cwu.edu/ mathcirclesjournal/vol2/iss1/5

Tanton, J. Without Words: Mathematical Puzzles to Confound and Delight. St. Albans: Tarquin Books, 2015.



Thanks to...

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References

- Our session organizers
- Clint Richardson
- American Institute of Mathematics
- Journal of Math Circles
- James Tanton!

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