

Pennies on a Chessboard

Jeffrey Musyt

Slippery Rock University

January 4, 2023

Outline

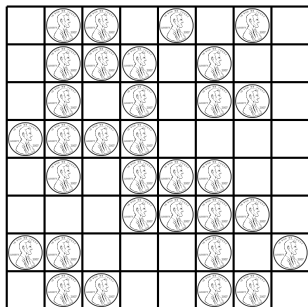
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 - All Heads on Board
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Why Pennies on a Chessboard? ... It just makes sense!

- Low barrier of entry to the problems
- Having manipulatives encourages solvers to engage with the problem
- Variable range of difficulty
- Variable range of mathematical rigor
- Solutions involve a wide range of mathematical ideas:
 - Discrete Math - parity, counting, pigeonhole principle, graphs
 - Game Theory - algorithms, strategy games, zugzwang
 - Geometry - symmetry, rotations, distances
 - Etc. - invariants, algebra, coding, probability
- Springboard for students to pose their own problems

Four in Each Row

- Can you arrange pennies on a chessboard so that each row has exactly four pennies and each column has a different number of pennies?



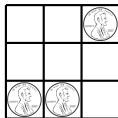
Six Pennies Apart

- Can you place 6 pennies on an 6×6 chessboard so that the distances between the center of any two pennies are all different?

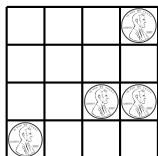
Correct
Solution



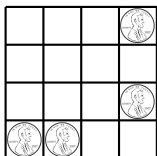
Correct
Solution



Incorrect
Solution



Correct
Solution

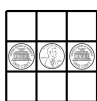
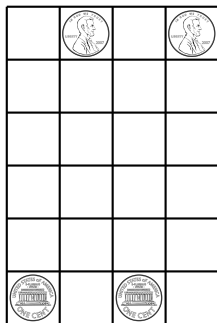


Six Pennies Apart

- Can the problem be solved for every size of chessboard?
- Some interesting history involved in the 7×7 case and the complete solution to the problem.
- Springboard for the use of computer programming in solving mathematical problems!

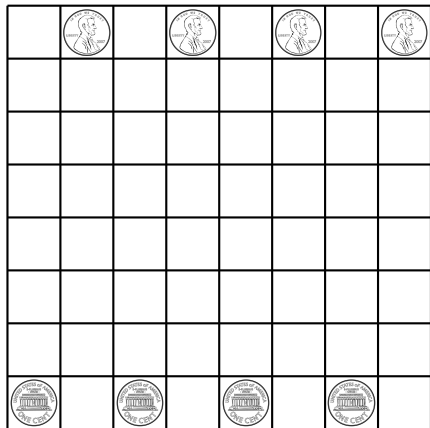
Heads vs Tails

- At the start of the game two heads are placed at the top of a 6×4 chessboard and two tails are placed at the bottom.
- The players alternate turns with the heads player sliding one of their coins one space down and the tails player sliding one of their coins one space up.
- Heads wins if all their pennies reach the other side, while tails wins if they capture any of the heads, which if a heads ever lies directly between two tails.



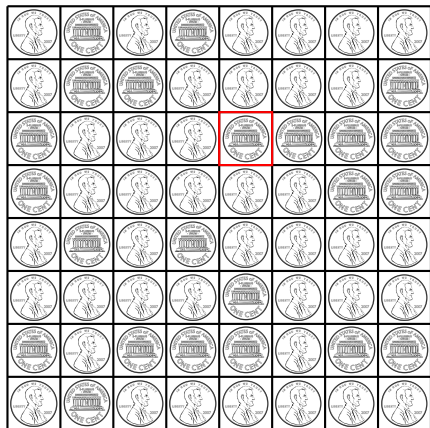
Heads vs Tails

- Can you find a strategy that works for either player?
- Can you rigorously prove that it works, perhaps by using an invariant?
- What if we change the size of the chessboard? What if we add more pennies?



All Heads on Board

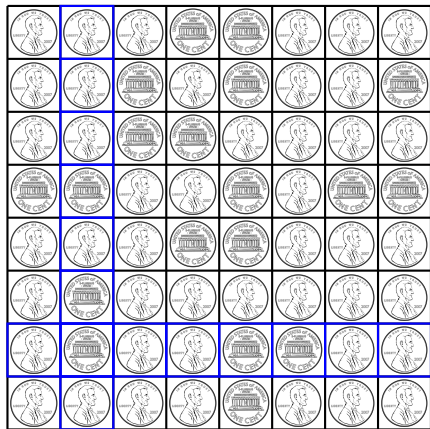
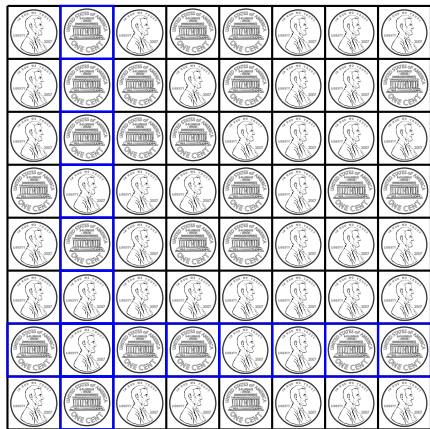
- Start a chessboard filled with pennies with a random amount of heads and tails.
- Using a series of Cross-Flips can you make the chessboard contain only heads.
- A Cross-Flip is performed by selecting a single penny and then flipping over the selected penny and every other penny that shares either the same row or column as the selected penny.



All Heads on Board

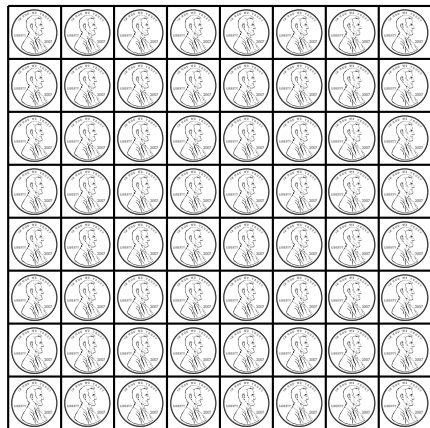


All Heads on Board



All Heads on Board

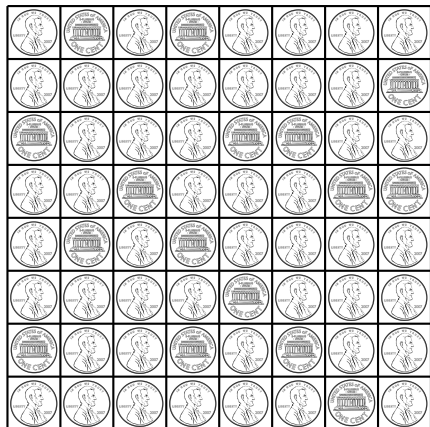
... Several Cross-Flips Later ...



- Does the starting position of the heads and tails matter? Is there always a series of Cross-Flips to reach a chessboard with all heads?
- What if we change the size of the chessboard or make it non-square?
- What if we use Diagonal-Flips?

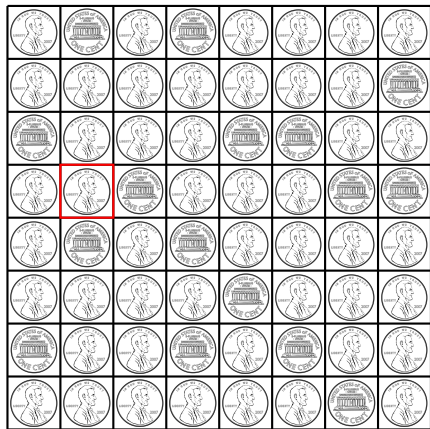
The Devil's Chessboard

- The Devil tells you and your friend that he has a chessboard full of pennies in the other room that are random arranged as heads or tails.
- The Devil takes you into the room and shows you his favorite penny on the board.
- You are then allowed to flip over exactly one penny on the board so that when your friend enters the room, they'll be able to tell which penny on the board was the Devil's favorite.



The Devil's Chessboard

- What strategy can you and your friend use so that you'll be able to communicate to your friend which coin is the Devil's favorite?
- For instance if the Devil told his favorite penny was in red square, what penny might you flip over?



My Two Cents

- I've had great success with these types of problems in various settings including:
 - Using them as first day warm-ups for my courses
 - Writing them on the board before class
 - Using them to introduce students to math research
 - Having them be a theme for homework assignments
 - Posting them as weekly problems for math club
 - Making them a table for a math festival or outreach event
- Great ways to introduce people to mathematical thinking and problem-solving.
- Plain old fun!

References

- Four in Each Row
 - Math Circles by the Bay: Topics for Grades 1-5
by Laura Givental, Marie Nemirovskaya, and Ilya Zakharevich
- 6 Pennies apart
 - The Colossal Book of Short Puzzles and Problems (Problem 5.26)
by Martin Gardner
- Heads vs Tails
 - Invitation to a Mathematical Festival (Problem 5, 7th Grade 1994)
by Ivan Yashchenko
- All Heads on Board
 - The Art and Craft of Problem Solving (Problem 3.4.21)
by Paul Zeitz
- The Devil's Chessboard
 - The Devil's Chessboard Problem - (Blog, June 29th, 2016)
by Brian Hamrick

Thank You!!!

jeffrey.musyt@sru.edu

If you do actually want to know some of the solutions,
come find me during the break!