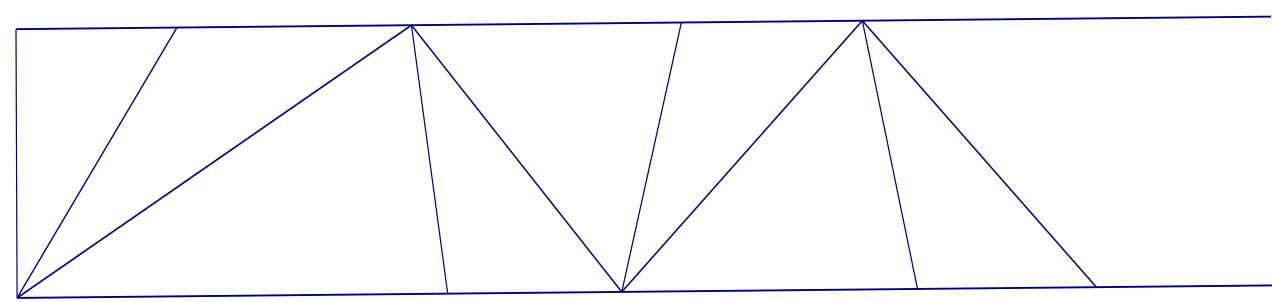


Folding Polygons

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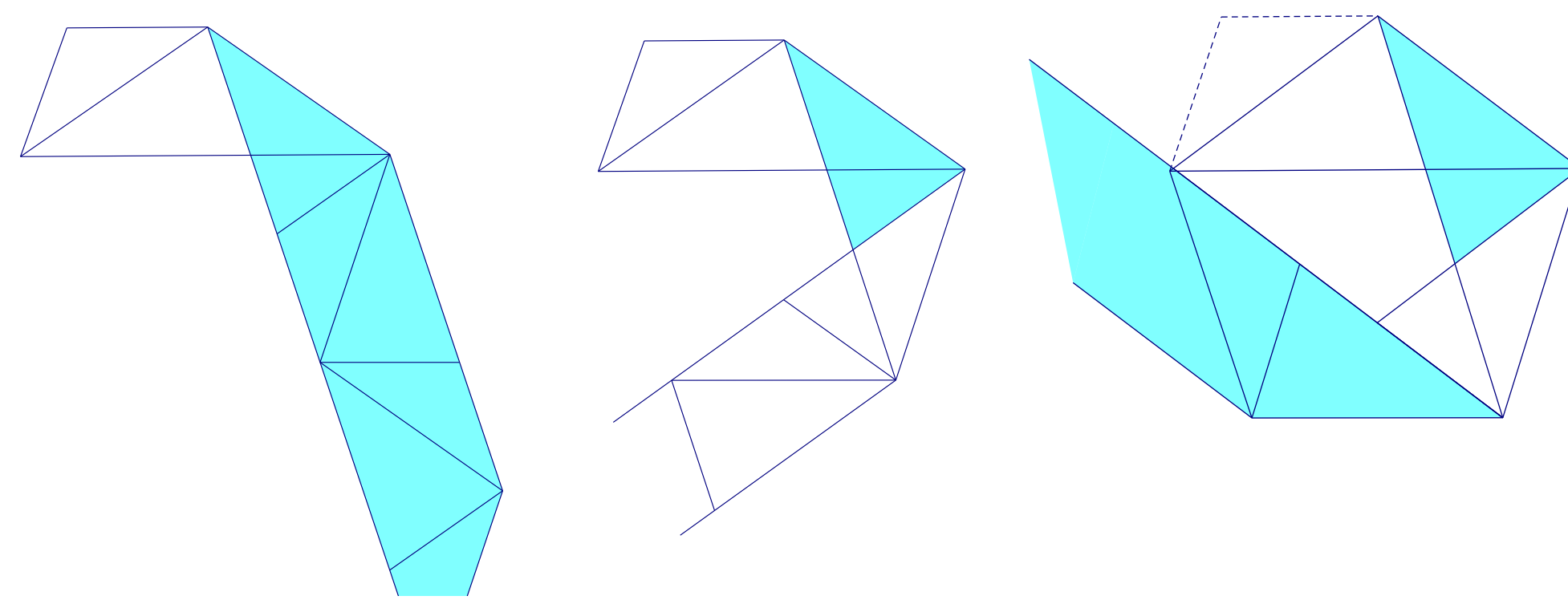
Generalizing

The strip below was created using an up-up-down-down (UUDD) folding pattern

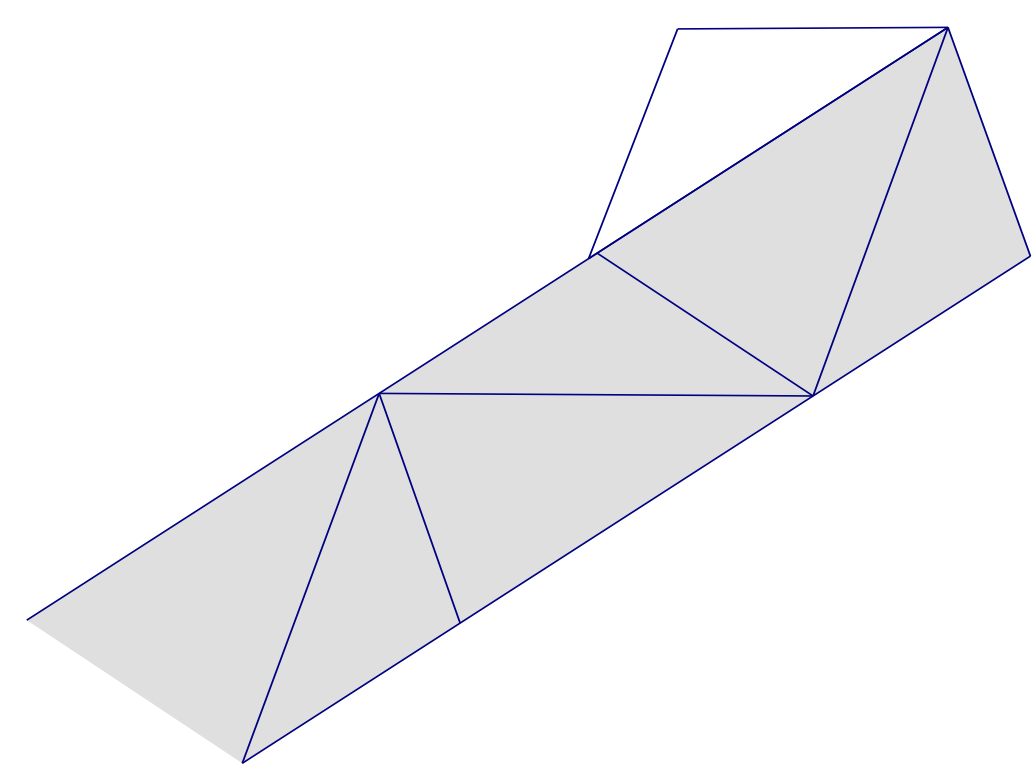


It is harder to identify the angles, but we can use the idea of folding polygons to make a conjecture about the measure of the angles formed. There are long fold lines and short ones. After throwing away the first few triangles, try folding along each of the two types of lines.

Folding along longer crease lines



Folding along shorter crease lines



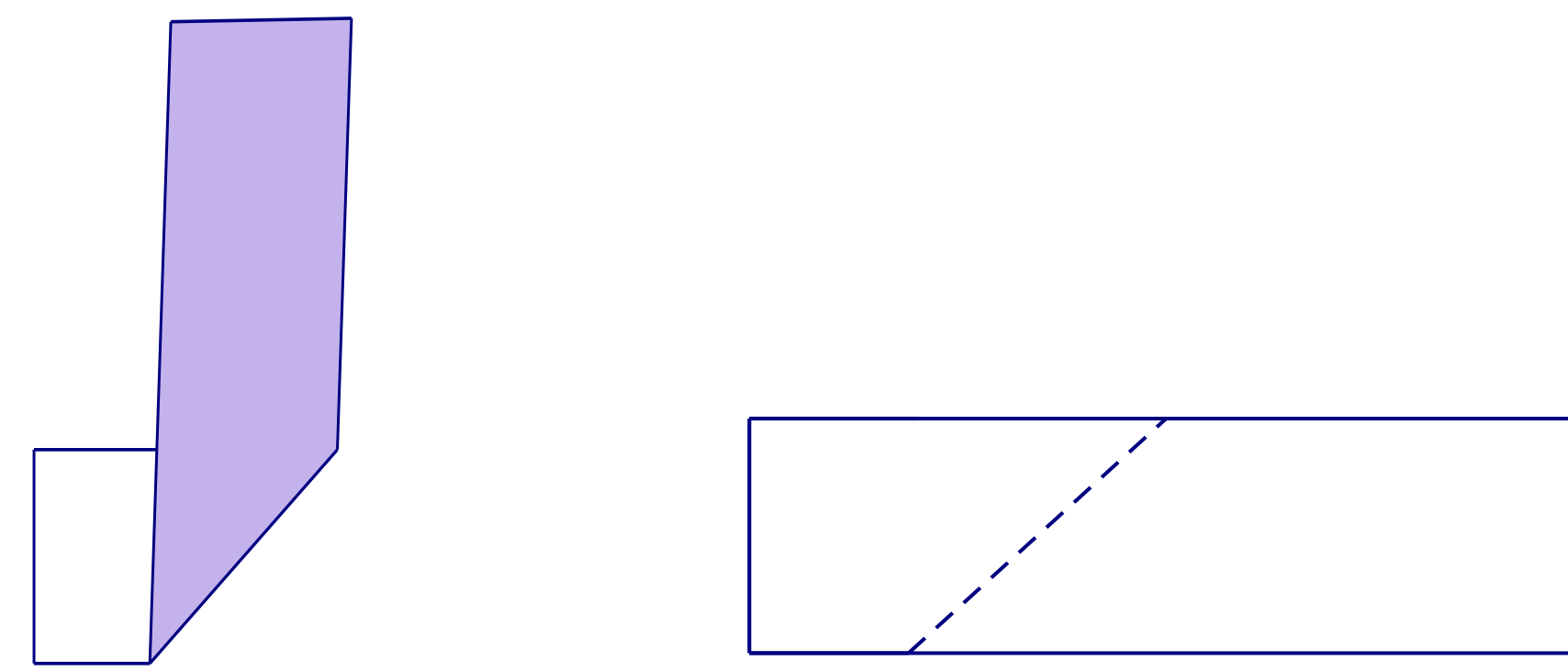
What do the resulting polygons tell you about the angles formed by the UUDD folding pattern. Can you prove your conjecture?

What polygon can you fold from an UUDDDD strip?

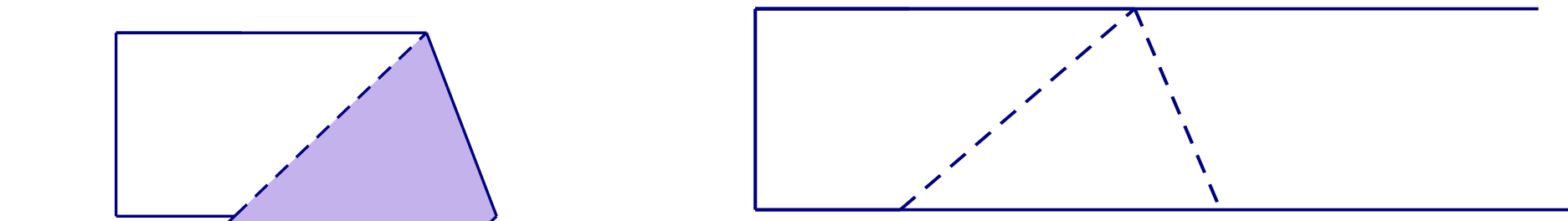
Can you find a folding pattern that will result in a regular 7-gon?

The up-down (UD) folding pattern

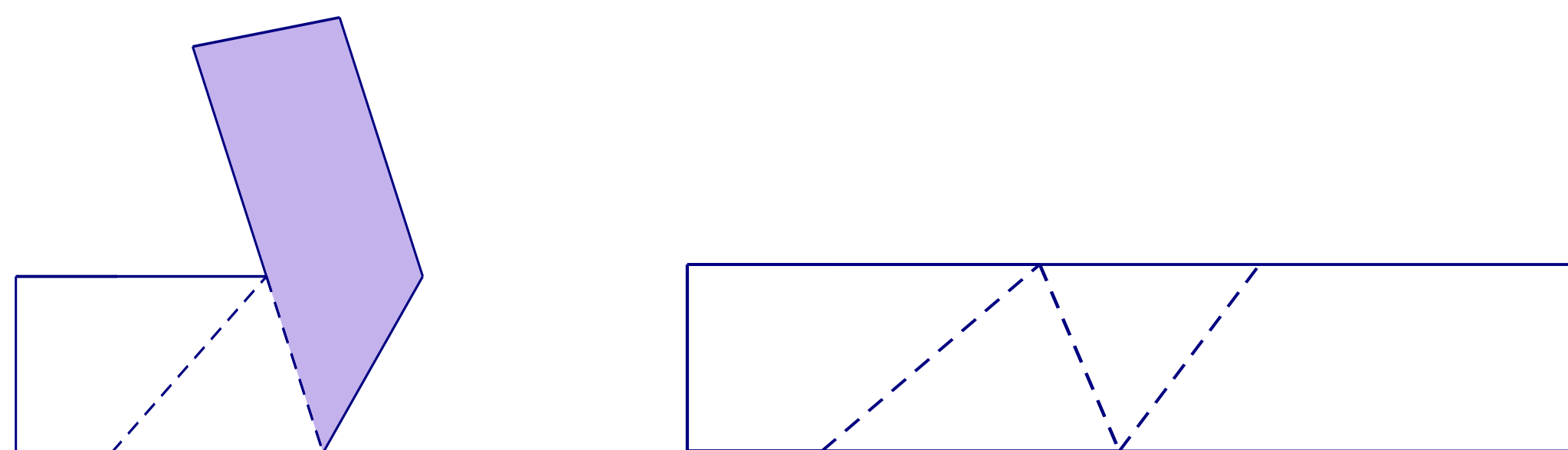
Take a strip of paper, fold it up to form a crease then unfold.



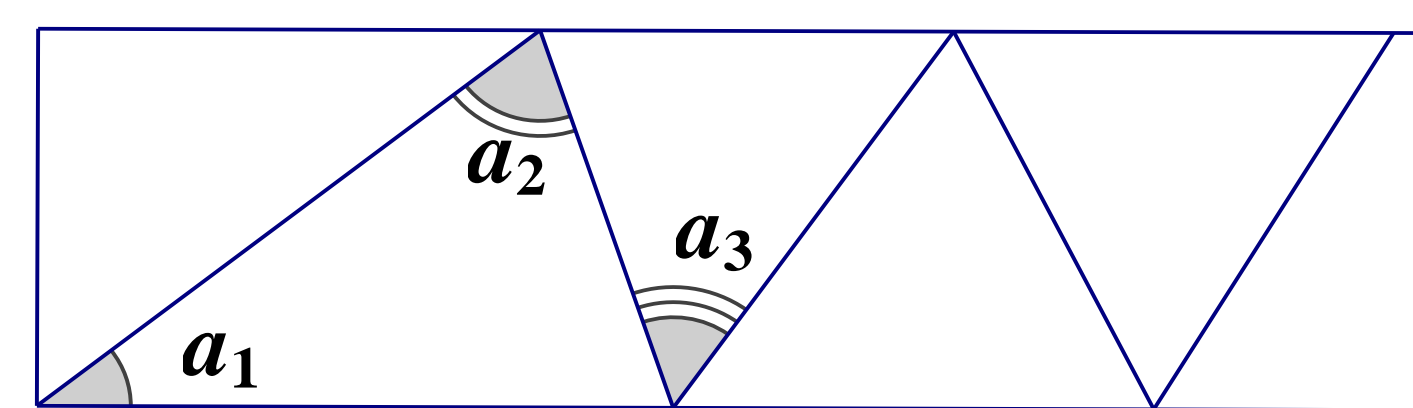
Now fold down so that the top edge of the paper falls along the crease line. Crease again, then unfold.



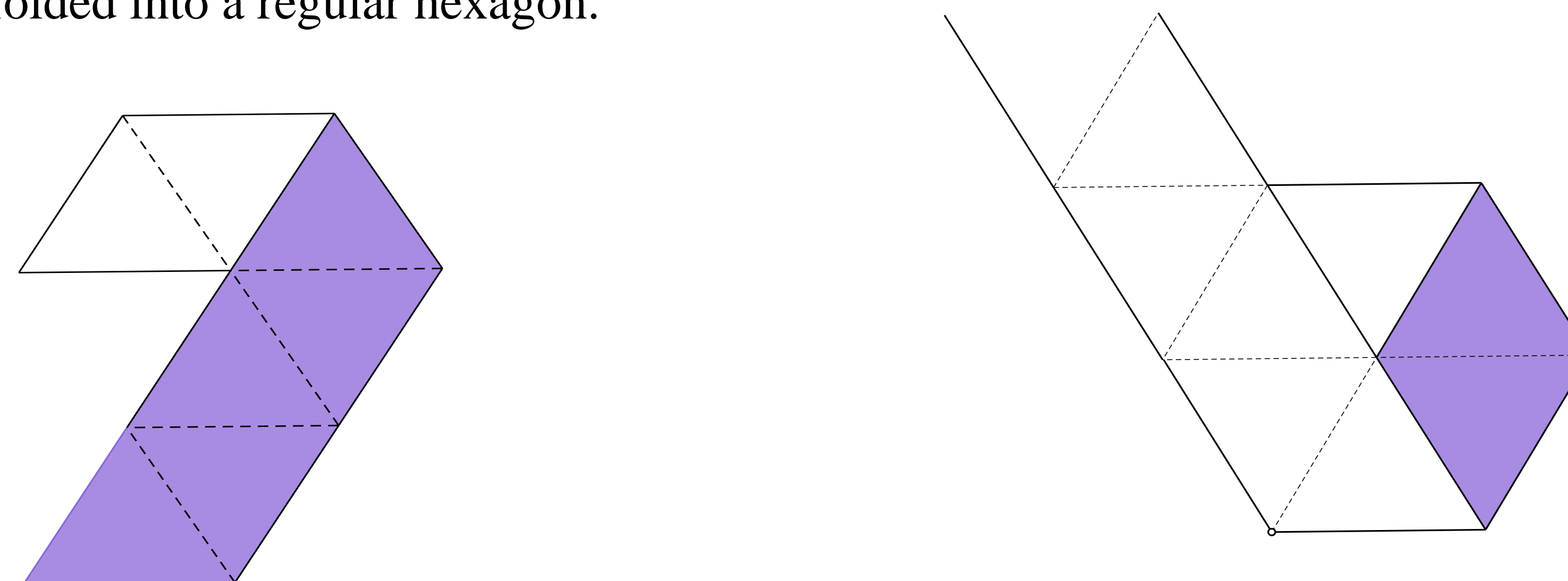
This is one iteration of the folding pattern. Now repeat several times so that you have a strip with several triangles formed by crease lines.



It appears that the triangles formed by this folding procedure become increasingly regular. We can prove that this is so in several ways. What relationships exist between the marked angles? How can these relationships be used to prove that the sequence of angles approaches an angle whose measure is 60 degrees?



After the first few non-regular triangles have been thrown away, the remaining strip of paper can be folded into a regular hexagon.

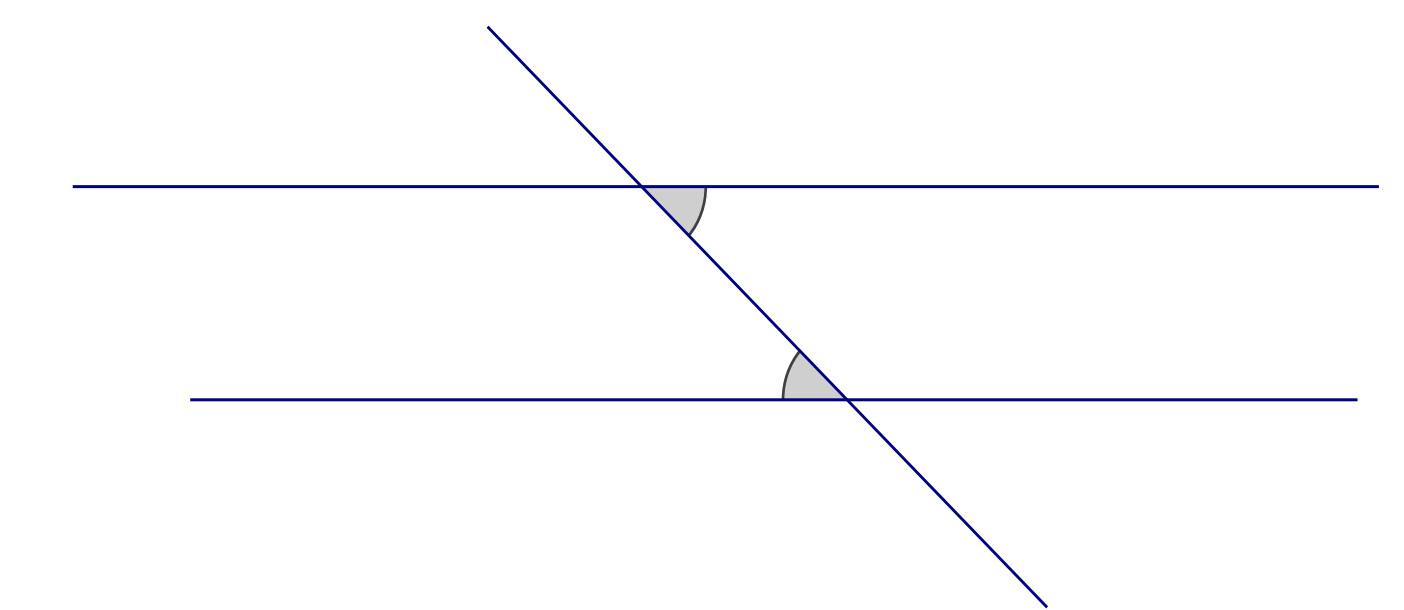


Mathematical Resources

This activity incorporates several geometric concepts accessible to middle school students. Some of these results are highlighted below. In addition, more advanced students can use limits of sequences, geometric series, and recursion to prove their conjectures and to investigate more general folding patterns.

Properties of parallel lines

When parallel lines are intersected by a transversal, the alternate interior angles created are congruent.



Properties of triangles

The sum of the measures of angles in a triangle is 180 degrees.

Properties of regular polygons

The interior angles in a regular polygon measure $180(n-2)/n$ degrees.

The exterior angles in a regular polygon measure $360/n$ degrees.

