The Melrose Math Circle

Meets weekly at the Roosevelt Elementary School for two groups of students:
- First and second graders
- Third and fourth graders

Our intention is to have an experience that is:
- Interactive and informal
- Relaxed and fun
- Supportive and encouraging

What works:
- buy-in from building principal;
- working directly with teachers;
- additional “math time” with teachers to prepare for the meetings and to discuss larger mathematical topics;
- Parents pay small “tuition” for the program.
Website [http://sites.google.com/site/melrosemathc/](http://sites.google.com/site/melrosemathc/) where you can read about our weekly sessions.

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Topics for spring 2011:
- **Session I**
  The Handshake problem and graph theory
- **Session II**
  Four Color Theorem; Konigsburg Bridge Problem

Topic for fall 2011:
The game of SET
The Handshake Problem

The problem:
If everyone in a group shakes hands, how many total handshakes are there?
First questions:
• How many people are in the room?
  We start with some small examples to see the pattern

• What are the rules?
  ➢ You don’t shake hands with yourself;
  ➢ You only shake hands with a person once: If Bob and Jane shake hands, then they don’t shake hands again.
One approach: work out small examples to see the pattern.

If there are 2 people, the number of handshakes is 1.
If there are 3 people, how many handshakes are there?

3
If there are 4 people, how many handshakes are there?

6
If we add another person so that there are 5 people, how many handshakes are there?

10
So far, this is our data:

<table>
<thead>
<tr>
<th>People</th>
<th>Number of handshakes</th>
<th>Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3 = 1 + 2</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>6 = 3 + 3</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>10 = 6 + 4</td>
</tr>
</tbody>
</table>
When a new person joins the group, that person shakes hands with everyone in the group.

- If there are 5 people in the group and a 6th person joins, that adds 5 handshakes to the total.

- If there are 6 people in the group and a 7th person joins, that adds 6 handshakes to the total.
Can you predict how many handshakes there would be if there are **10 people in the room**?

What if there were **100 people in the room**?
If there are 10 people in the room, there will be

\[ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45 \]  
handshakes
If there are 100 people in the room, there will be

\[ 1 + 2 + 3 + 4 + \ldots + 96 + 97 + 98 + 99 \]

= ??????? handshakes
100 people:
$1 + 2 + 3 + 4 + \ldots + 96 + 97 + 98 + 99$

$\quad = (1 + 99) + (2 + 98) + (3 + 97) + (4 + 96) + \ldots + (49 + 51) + 50$

$\quad = 49(100) + 50$

$\quad = 4950$ handshakes.
Handshake Theorem: If there are $N$ people in a room, the total number of handshakes will be $\frac{1}{2} N(N - 1)$.

Why? We use the pattern to see there are this many handshakes:

$1 + 2 + 3 + ... + (N - 3) + (N - 2) + (N - 1)$

$= [1 + (N - 1)] + [2 + (N - 2)] + [3 + (N - 3)] + ...$

$+ \frac{1}{2}(N - 1) + \frac{1}{2}(N + 1) + \frac{1}{2}N$

$= \frac{1}{2} N(N - 1)$. 