Melrose Math Circle

The Melrose Math Circle

Meets weekly at the Roosevelt Elementary School for two groups of students:

- First and second graders
- Third and fourth graders

Our intention is to have an experience that is:

- Interactive and informal
- Relaxed and fun
- Supportive and encouraging

What works:

- buy-in from building principal;
- working directly with teachers;
- additional "math time" with teachers to prepare for the meetings and to discuss larger mathematical topics;
- MSRI mini-grants (2010 – 2011 and 2011 – 2012) for seed money;
- Parents pay small "tuition" for the program.

Website http://sites.google.com/site/melrosemathc/
where you can read about our weekly sessions.

Contact us: Jack Reynolds (jack.jreynolds@verizon.net) and Maura Mast (maura.mast@gmail.com)

Topics for spring 2011:

Session I

The Handshake problem and graph theory

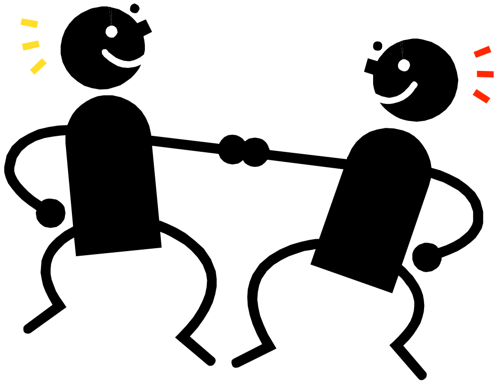
Session II

Four Color Theorem; Konigsburg Bridge Problem

Topic for fall 2011:

The game of SET

The Handshake Problem



The problem:
If everyone in a group shakes hands, how many total handshakes are there?

First questions:

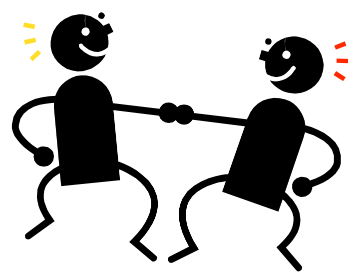
- How many people are in the room?

  We start with some small examples to see the pattern

- What are the rules?
  - You don't shake hands with yourself;
  - You only shake hands with a person once: If Bob and Jane shake hands, then they don't shake hands again.

One approach: work out small examples to see the pattern.

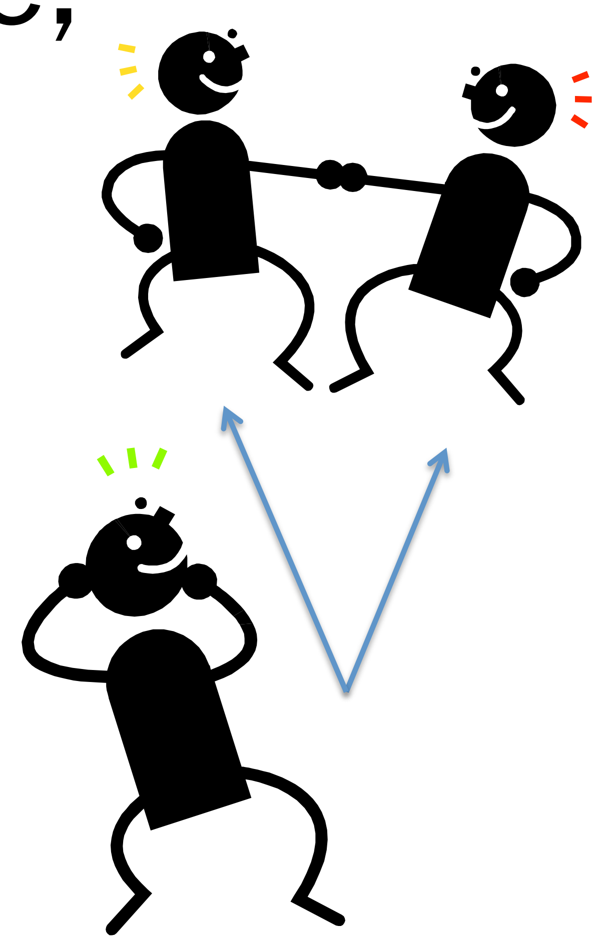
If there are 2 people, the number of handshakes is



1

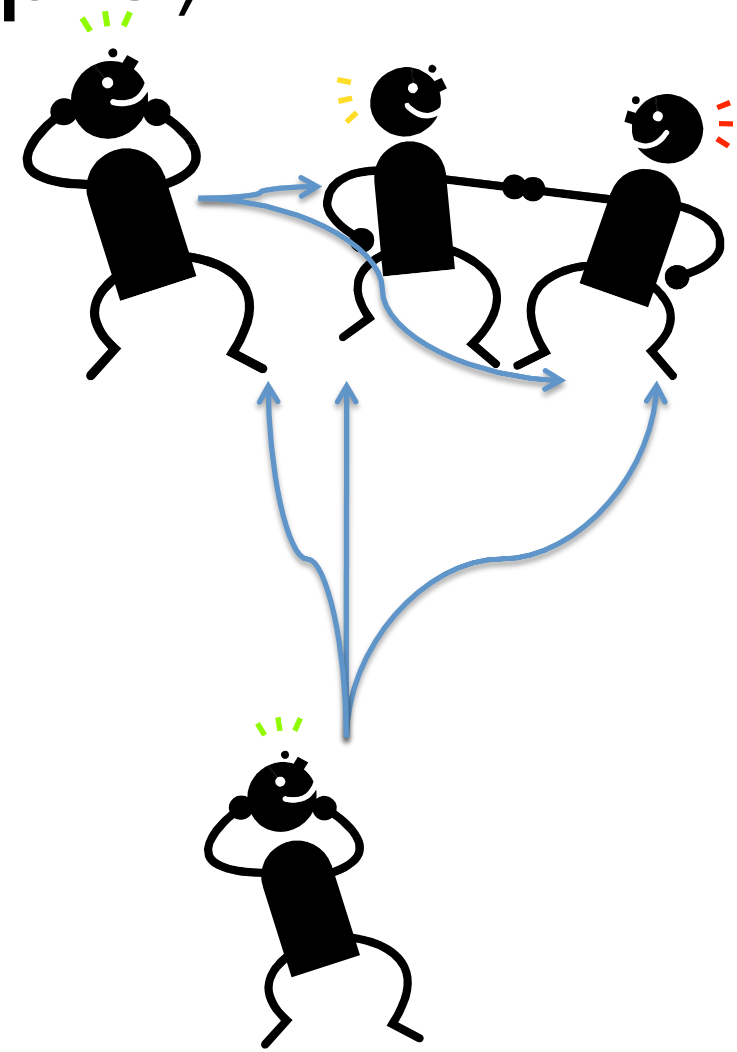
If there are 3 people, how many handshakes are there?

3



If there are 4 people, how many handshakes are there?

6



If we add another person so that there are 5 people, how many handshakes are there?

10

So far, this is our data:

People	Number of handshakes	Pattern
2	1	
3	3	3 = 1 + 2
4	6	6 = 3 + 3
5	10	10 = 6 + 4

Looking at the pattern:

When a new person joins the group, that person shakes hands with everyone in the group.

- If there are 5 people in the group and a 6^{th} person joins, that adds 5 handshakes to the total.
- If there are 6 people in the group and a 7^{th} person joins, that adds 6 handshakes to the total.

Can you predict how many handshakes there would be if there are 10 people in the room?

What if there were 100 people in the room?

If there are 10 people in the room, there will be

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$ handshakes

If there are 100 people in the room, there will be

1 + 2 + 3 + 4 + … + 96 + 97 + 98 + 99 = ?????? handshakes

100 people:

$1 + 2 + 3 + 4 + ... + 96 + 97 + 98 + 99$

$= (1 + 99) + (2 + 98) + (3 + 97)$
$+ (4 + 96) + \dots + (49 + 51)$
$+ 50$

$= 49(100) + 50$

$= 4950$ handshakes.

Handshake Theorem: If there are N people in a room, the total number of handshakes will be $\frac{1}{2} N(N - 1)$

Why? We use the pattern to see there are this many handshakes:

$1 + 2 + 3 + \ldots + (N - 3) + (N - 2) + (N - 1)$

$= [1 + (N - 1)] + [2 + (N - 2)] + [3 + (N - 3)] + \ldots$

$+ [\frac{1}{2}(N - 1) + \frac{1}{2}(N + 1)] + \frac{1}{2}N$

$= \frac{1}{2} N(N - 1).$