OPERATION COOKIE JAR

(presented by Gabriella Pinter, gapinter@uwm.edu)

There are 15 cookie jars, numbered consecutively from 1 to 15. The number of cookies in each jar is equal to the number of the jar. A "move" consists of choosing one or more jars, then removing one or more cookies from the chosen jars—but the same number of cookies from each jar … adapted from *The Inquisitive Problem Solver* by P. Vaderlind, R. Guy and L. Larson, MAA, 2002. P34. Page 7.

What are good questions to ask? Let's explore them!

A natural question is that you try to find the minimum number of moves that are necessary to empty all the jars.

Take an inventory of the different strategies used to empty the jars.
Explore how your strategy works if the number of jars is changed.
What if the number of cookies in the jars change as well?

Let's play with smaller numbers:
What if the jars contain cookies: a, a+1, a+2 ? Two or three moves?
Can we fill three jars so that 1, 2 or 3 moves will be needed to empty them? What about four jars?
Can n jars be filled in a way that n steps are needed to empty them?

What if the jars contain {1,5,33,36} or {2,6,7,13}, {33, 34, 36, 40, 48} or {1,2,4,11,16,17} cookies? Which strategies are the most efficient in these cases? Is there a procedure that will always work to empty the jars in the fewest moves?

What if 'negative cookies' can also be taken, i.e., cookies can be added to certain jars, but again the same number of cookies to all the chosen jars? How many moves does it take to empty jars {1, 2, 4, 8, 16} now?

Let's move to 2D – arrange our jars in a rectangular array

What kind of questions could we ask now?

Let's say that a "move" consists of removing the same number of cookies from all jars in all columns – BUT consecutive jars from the left and from the right can be blocked in each row. How can we empty all the jars in our rectangular array?

Try the following example:

5	5	3	3
2	2	5	2
5	3	3	2
2	5	5	3

To read more about this and similar problems:

M. Develin. Optimal subset representations of integer sets. Journal of Number Theory, 89:212–221, 2001.

Moulton, David Petrie. Representing powers of numbers as subset sums of small sets. J. Number Theory, 89 (2001), no. 2, 193-211.

D. Mills, Some observations on subset sum representations, Electronic Journal of Combinatirial Number Theory, 6 (2006), #A25

M.J. Collins, D. Kempe, J. Saia, and M. Young. Nonnegative integral subset representations of integer sets. Inform. Process. Lett., 101, 129–133, 2007.

S. Luan, J. Saia and M. Young, Approximation algorithms for minimizing segments in radiation therapy, Inform. Process. Lett., 101, 239–244, 2007.

C. Engelbeen and S. Fiorini. Constrained decompositions of integer matrices and their applications to intensity modulated radiation therapy. Networks, 2009. DOI 10.1002/net.20324.

C. Engelbeen, The Segmentation Problem in Radiation Therapy, PhD thesis, 2010.