ROLLING DICE

Materials: One die, and one piece of grid paper so that one face of the die fits perfectly on a fundamental domain of the grid paper.

Put a coordinate system on the grid paper and define square $S(n, m)$ to be $[n, n+1] \times [m, m+1]$.

To describe the orientation of the die once it sits in a standard fundamental domain it suffices to specify the number on the top of the die and the number on the front of the die.

(1) Start with the die in square $S(0, 0)$ with 1 on top and 2 in front. Is it possible to roll the die along some path on the grid paper so that it ends in square $S(0, 2)$ with 5 on top and 6 in front?
(2) Describe all orientations that the die can land in after following a loop that starts and ends in square $S(0, 0)$.
(3) Describe several loops that do nothing to the orientation of the cube.
(4) What happens when a tetrahedron is rolled on a triangular grid?
(5) What happens when an octahedron is rolled on a triangular grid?
(6) What happens when a $1 \times 1 \times 2$ cube is rolled on a square grid?
(7) What happens in any of these rolling problems, when only certain portions of the grid are allowed?

Comments: Of course the presentation of these questions can be adjusted according to the audience. (The game cuboid is based on this idea.) There are many generalizations. This is related to the following concepts: parallel transport, holonomy, angle defect, curvature, categories and goupoids, and the Euler characteristic.