

Quads: A SET-like game with a Twist

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EVENQUADS



NOTABLE WOMEN IN MATHEMATICS PLAYING CARDS

One deck, five new games

Support this project now!

<https://www.kickstarter.com/projects/evenquads/evenquads>



EVENQUADS

CELEBRATING WOMEN IN MATH



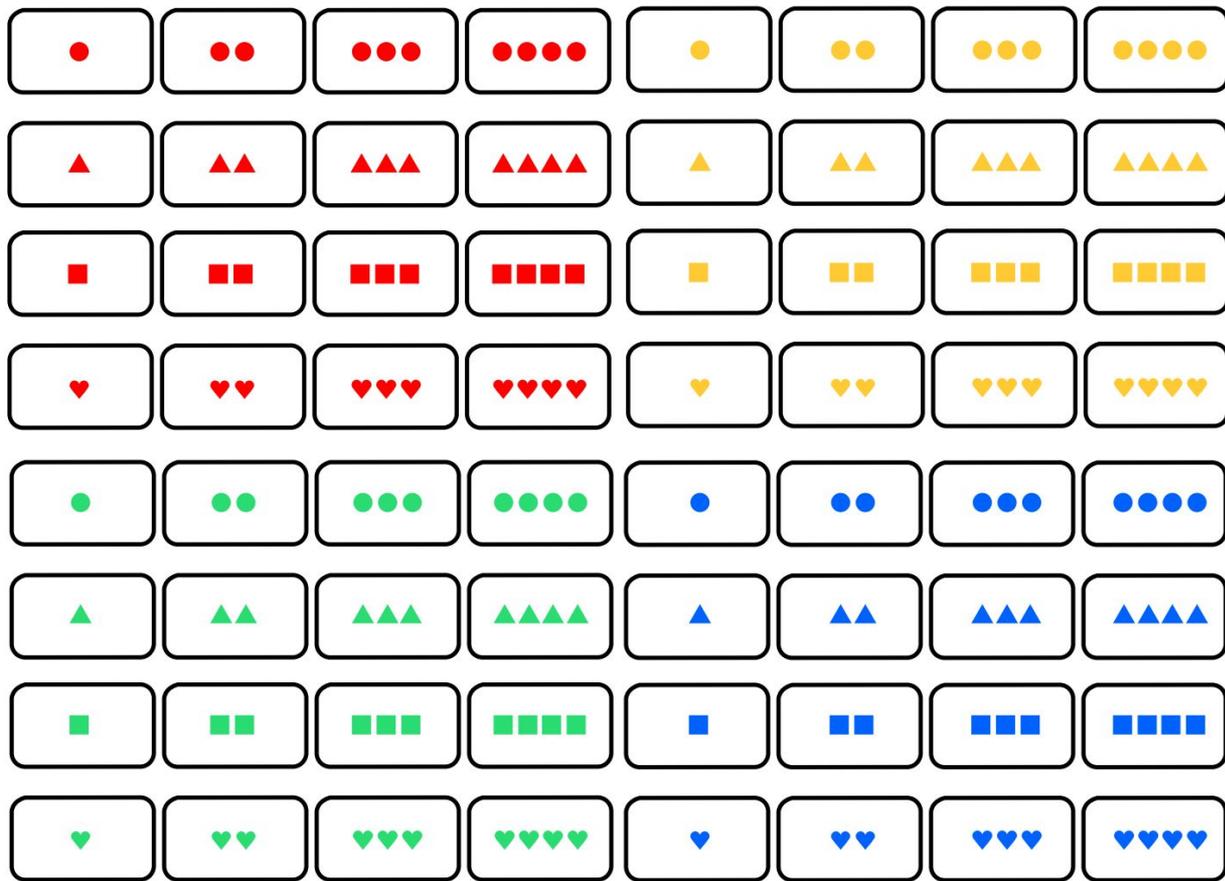
Quads

64 cards

3 attributes

- Number
- Shape
- Color

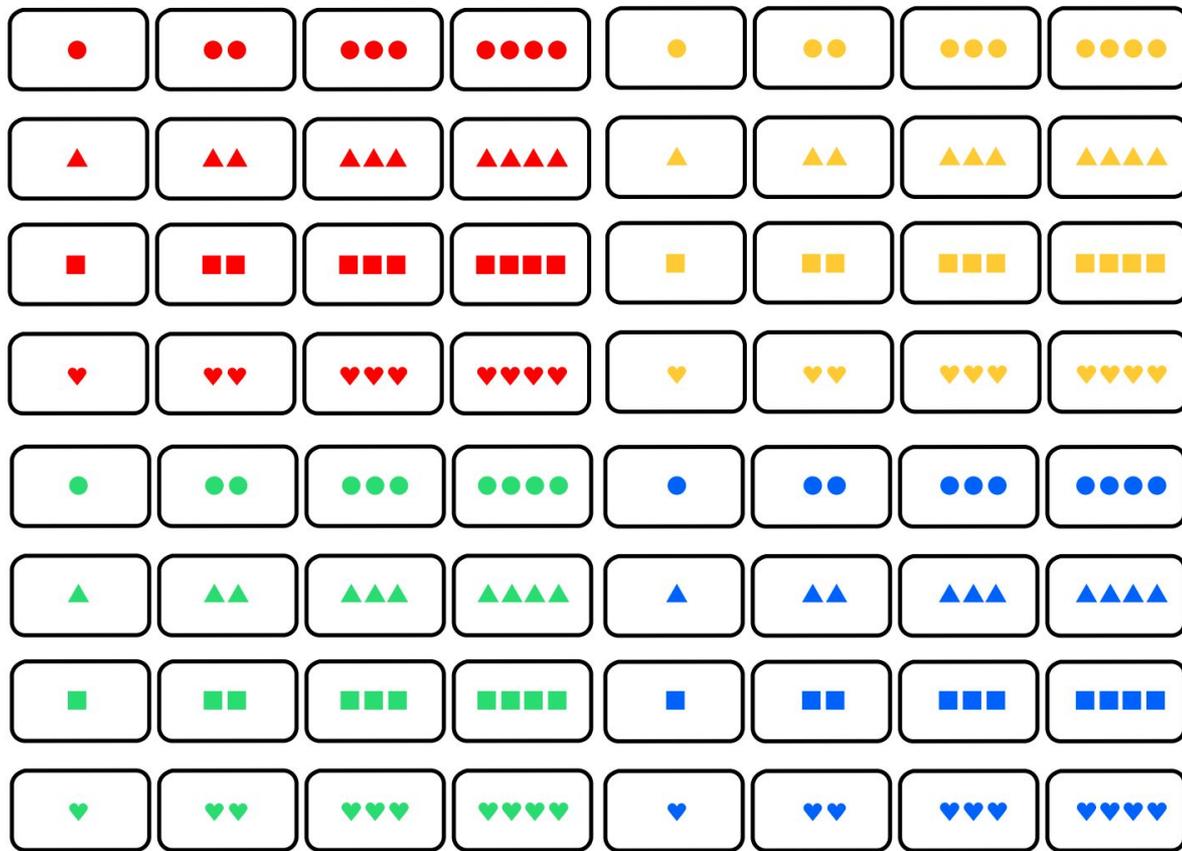
Each attribute
has 4 states.



Quads

A QUAD is 4 cards that in each attribute are one of the following:

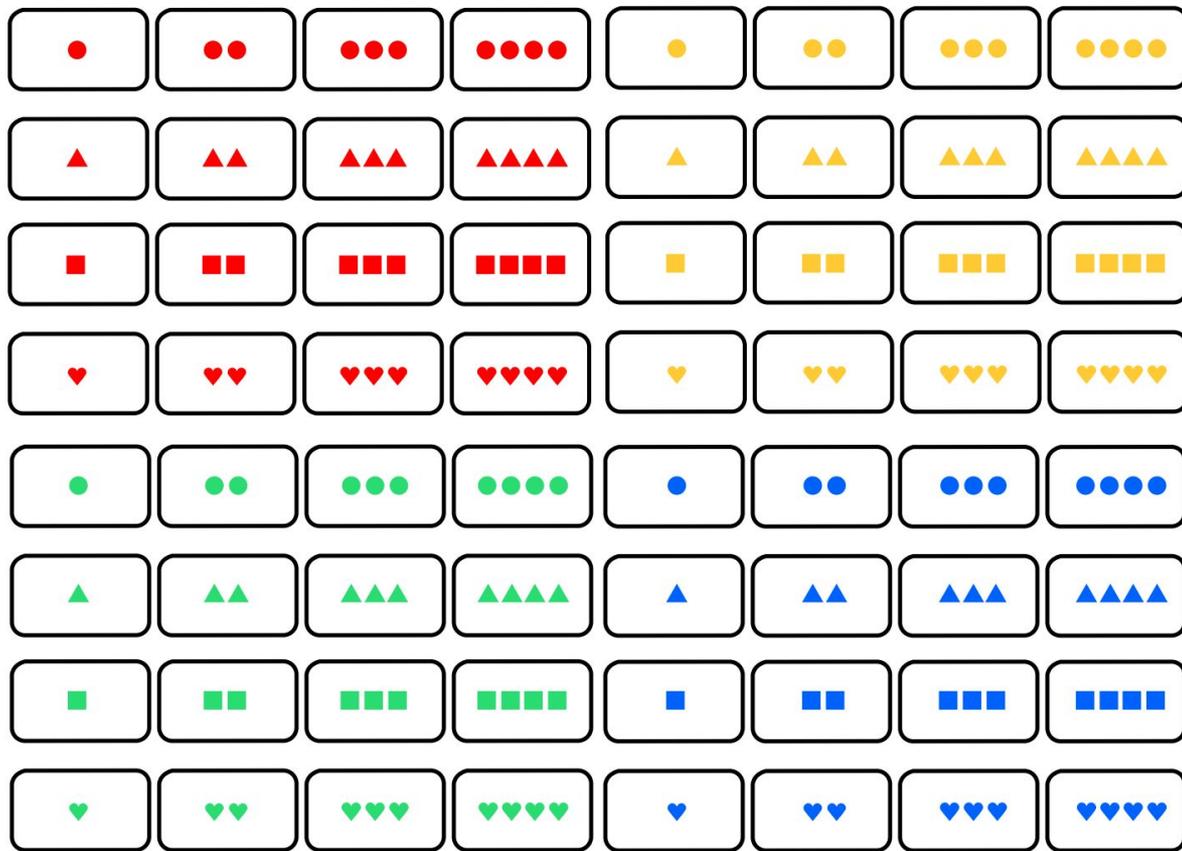
1. All the same
2. All different



Quads

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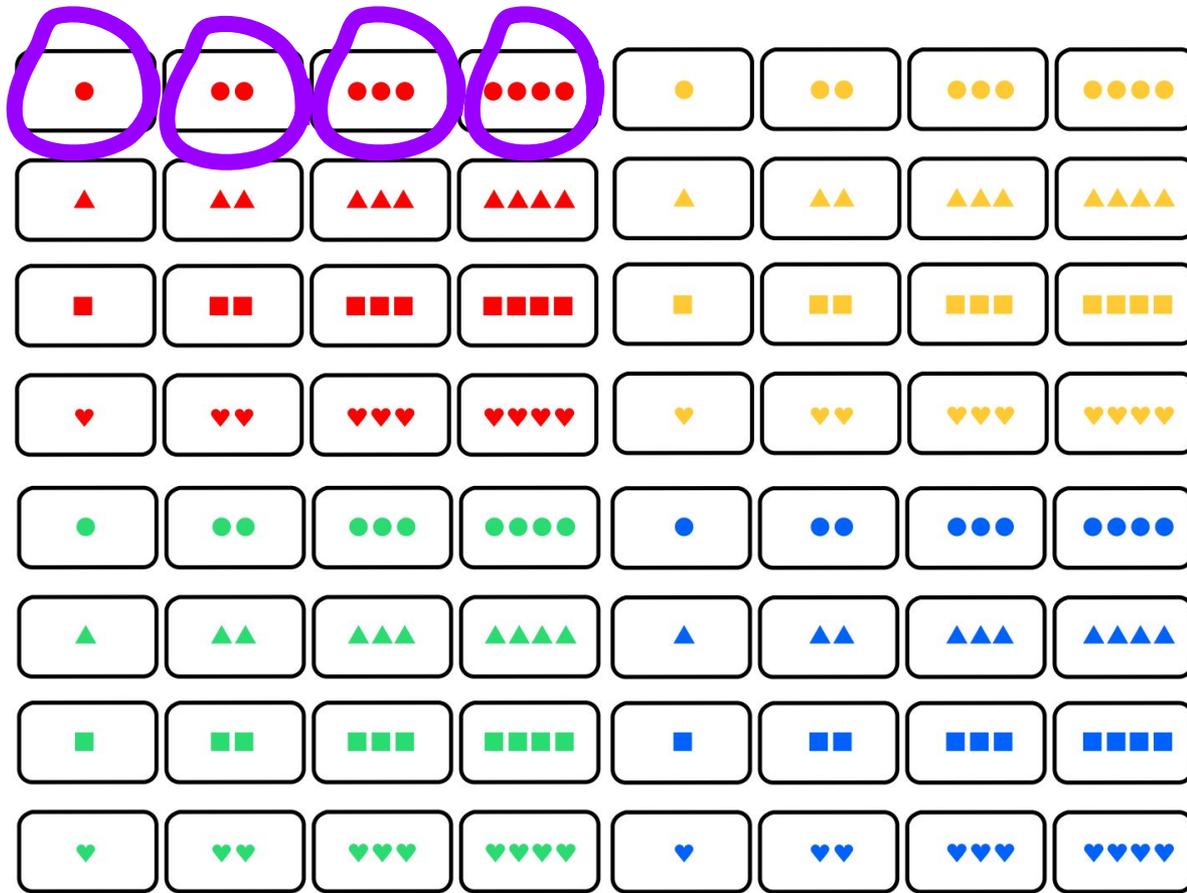
1. All the same
2. All different
3. 2 pairs



Quads

Examples

1. Diff numbers, same shape, same color.

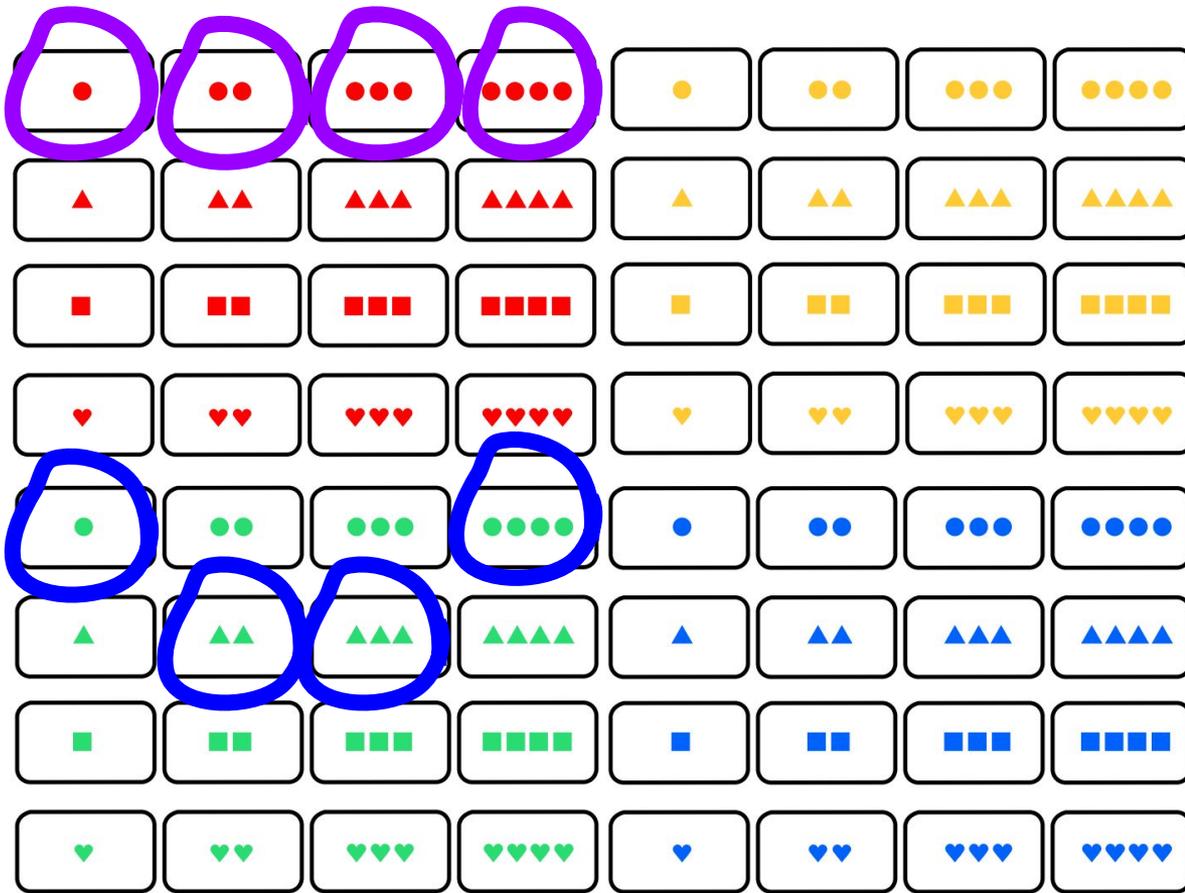


Quads

Examples

1. Diff numbers, same shape, same color.

2. Diff numbers, 2 pairs of shapes, same color.



Quads

Examples

1. Diff numbers, same shape, same color.

2. Diff numbers, 2 pairs of shapes, same color.

3. 2 numbers, 2 shapes, 2 colors



Quads

Fundamental Theorem
of Quads:

Theorem 1: For any 3
cards, there is a unique
4th that makes a Quad.

Q: Which card completes
the **pink** Quad?



Quads

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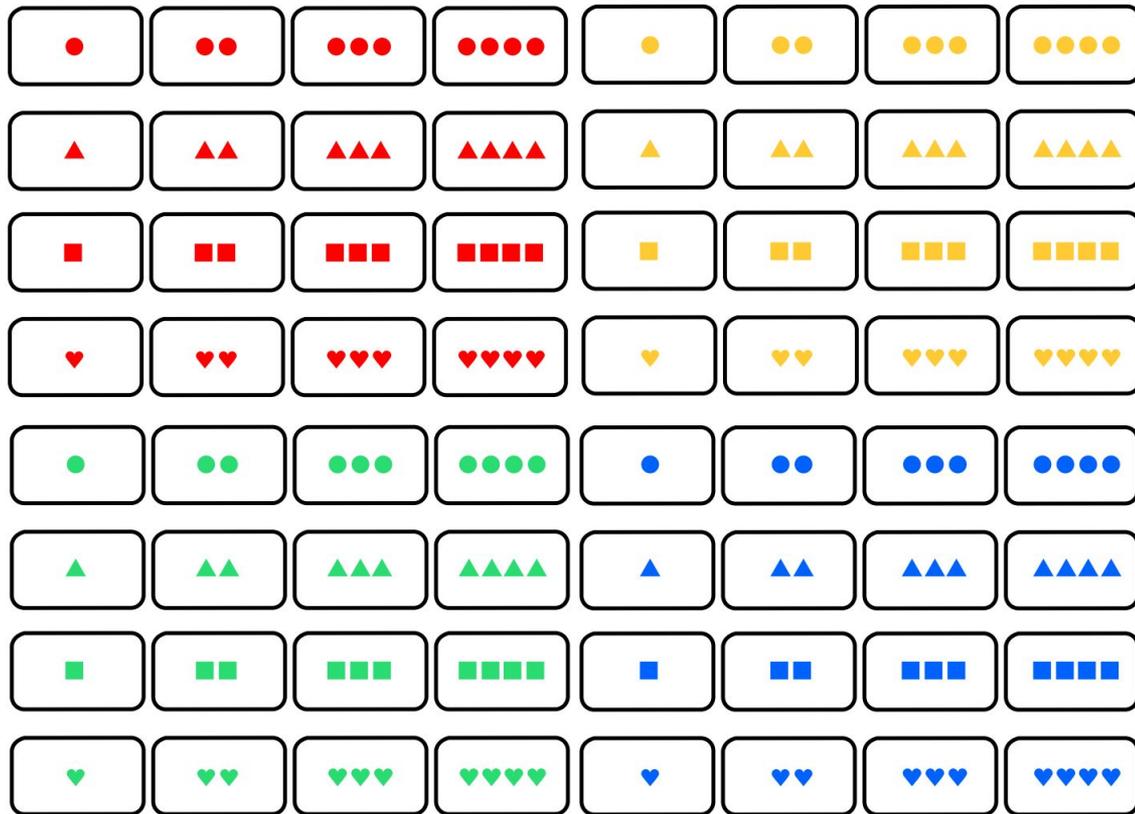
Q: Which card completes
the **pink** Quad?



Quads

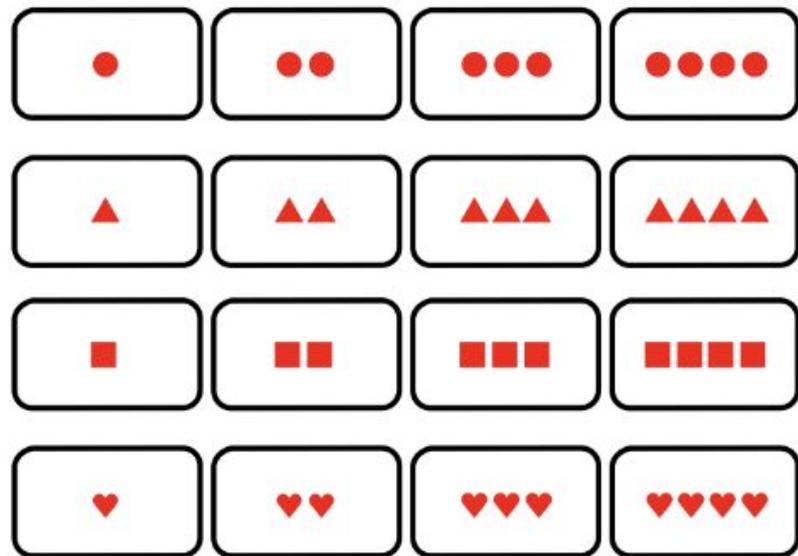
Can be used to
introduce topics in:

- Combinatorics
- Probability
- Abstract Algebra
- Linear Algebra
- Finite Geometry



Quads and Combinatorics

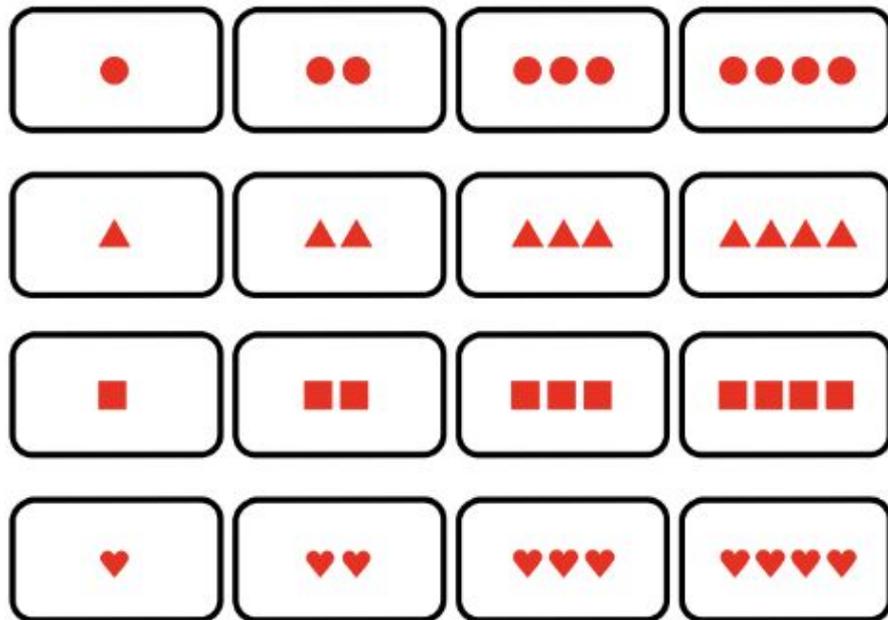
1. How many pairs of cards are both hearts?
2. How many QUADS are there?
3. How many QUADS are different in each attribute?
4. How many QUADS include the card **one-red-triangle**?



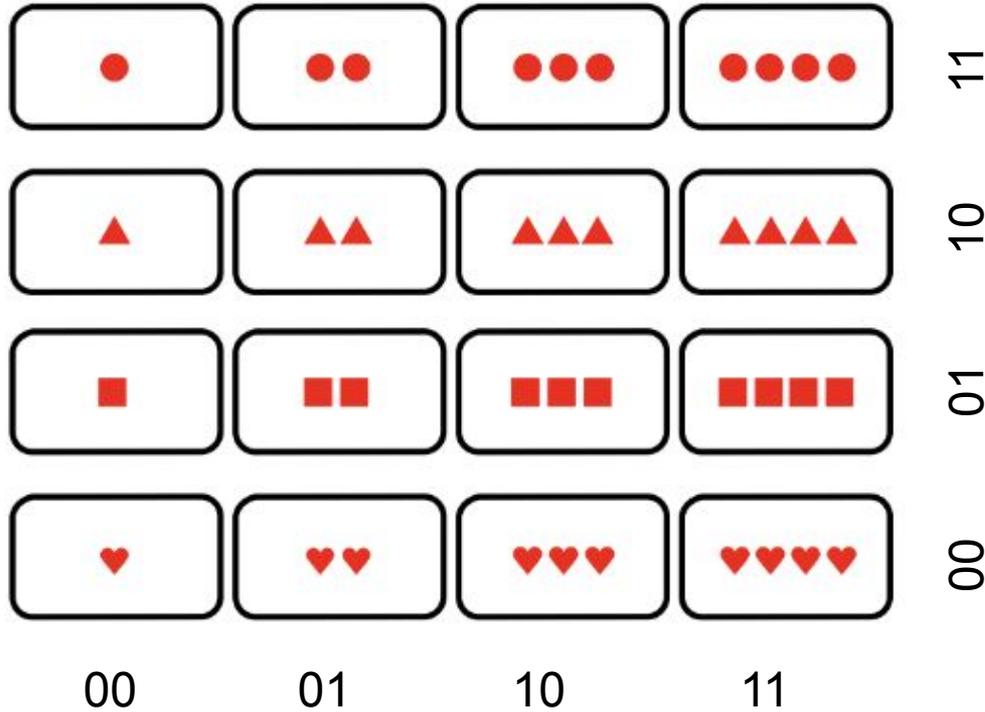
Quads and Probability

1. Pick 4 cards at random. What is the probability they form a QUAD?
2. What is the probability that a k -card layout contains a QUAD?
3. What is the expected value of the number of QUADS in a k -card layout?

Quads and Finite Geometry

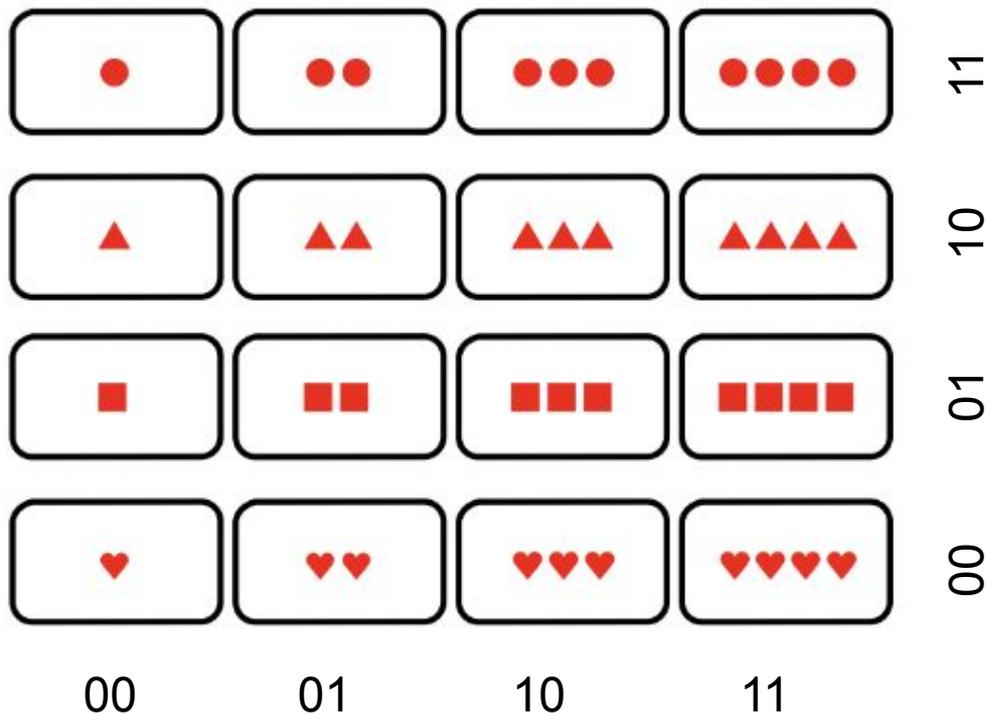


Quads and Finite Geometry



★ Code states of attributes with bit strings of length 2

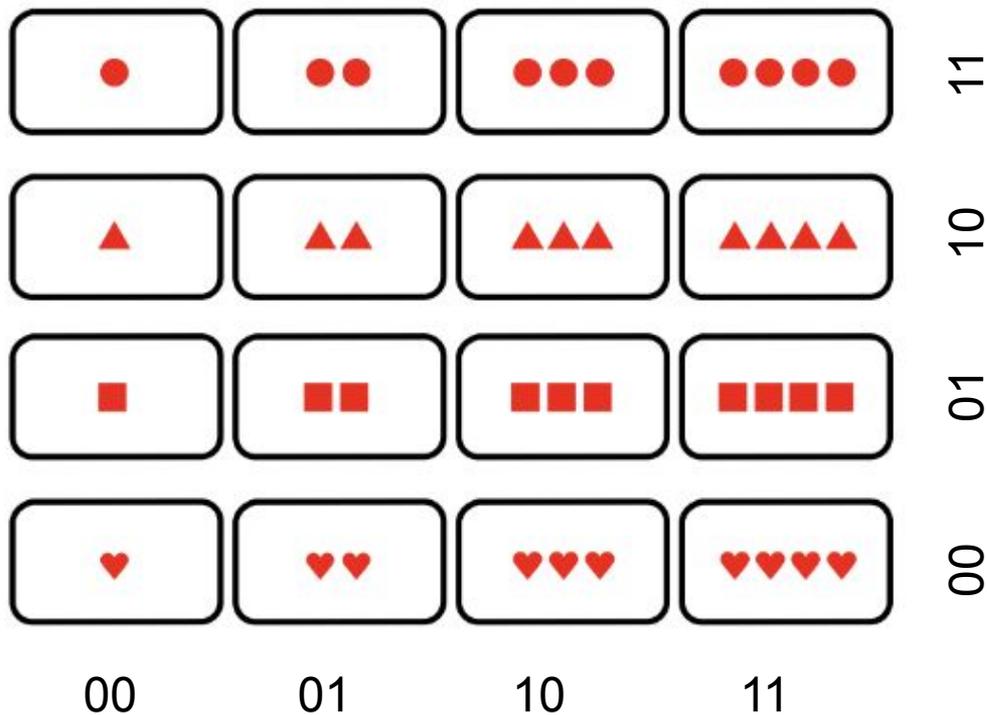
Quads and Finite Geometry



★ Code states of attributes with bit strings of length 2

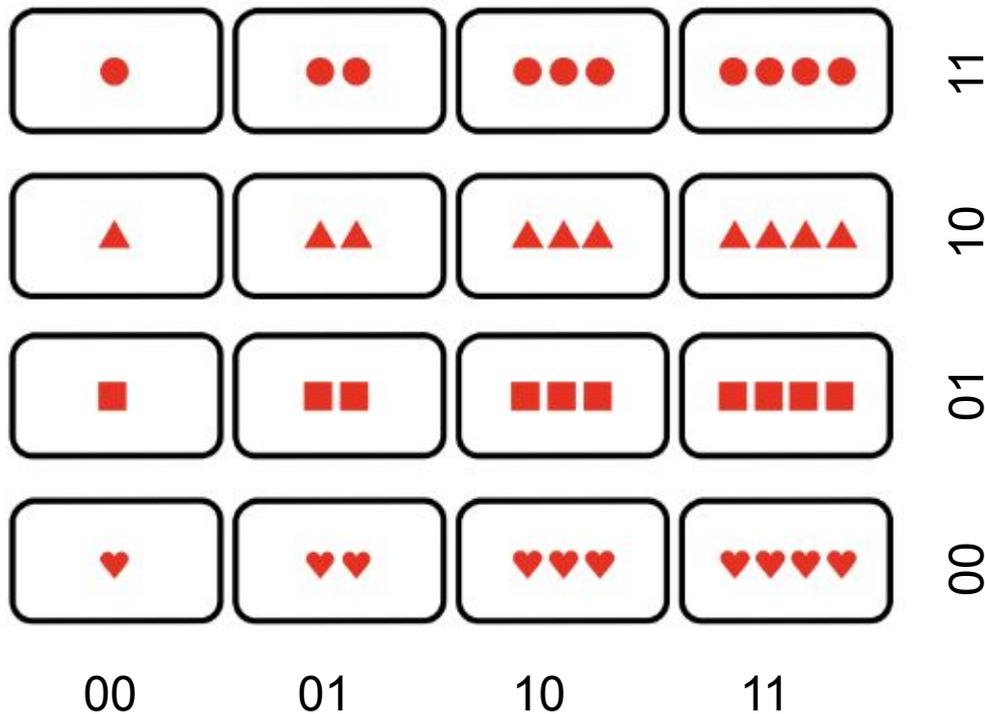
★ Code cards with 2 bits per attribute

Quads and Finite Geometry



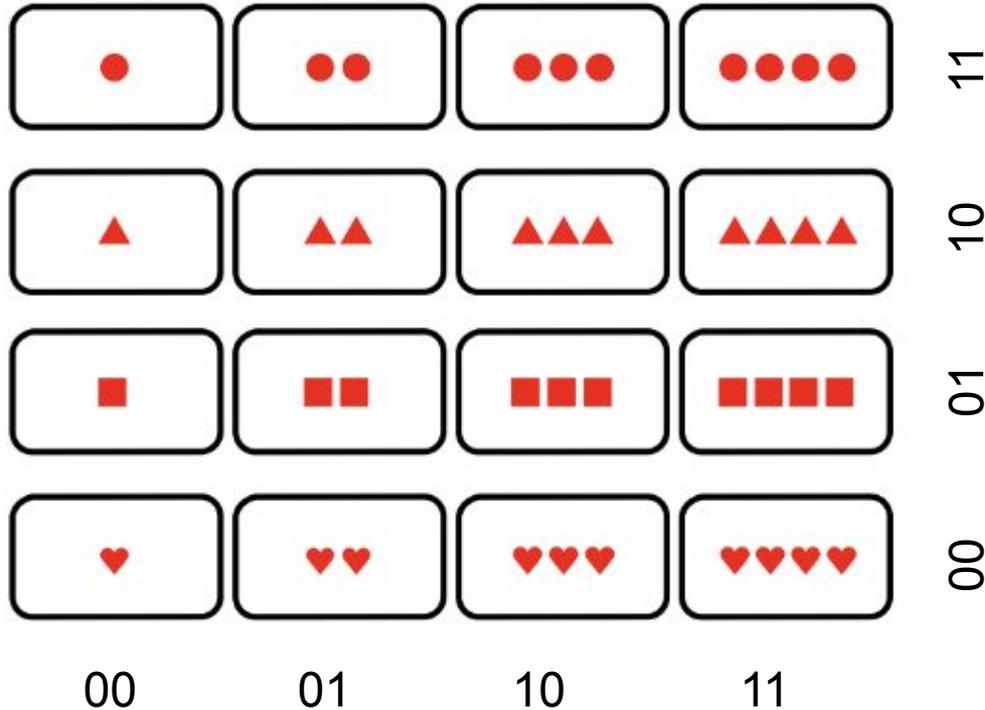
- ★ Code states of attributes with bit strings of length 2
- ★ Code cards with 2 bits per attribute
- ★ 2-red-circles = 0111 or $(0,1,1,1)$.
- ★ Red Quad cards are vectors in \mathbf{Z}_2^4

Quads and Finite Geometry



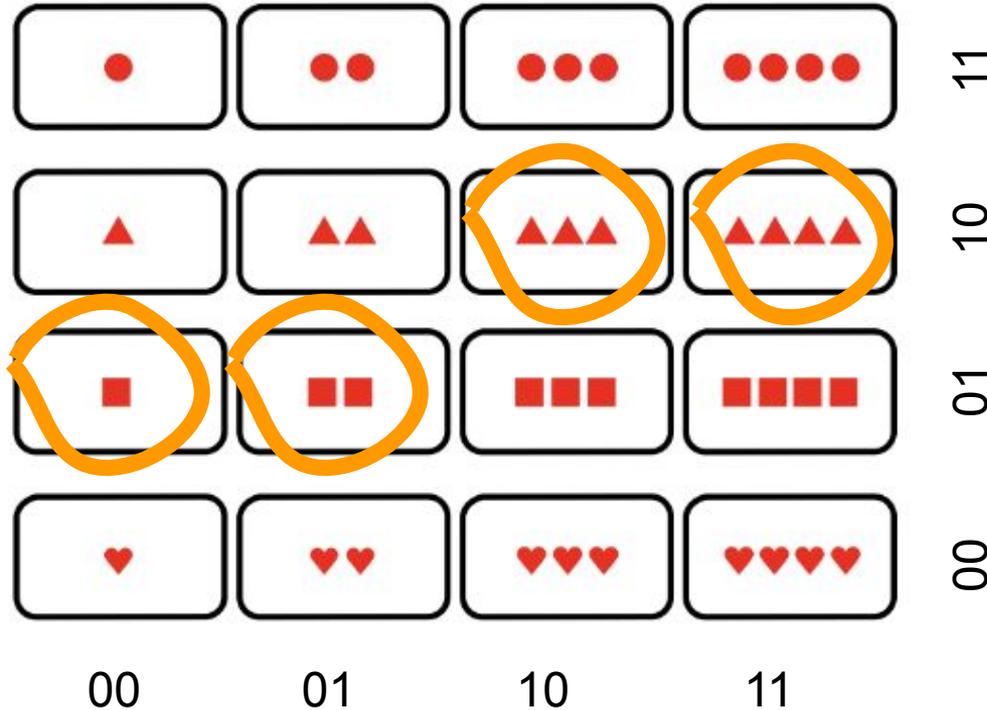
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- ★ Code cards with 2 bits per attribute
- ★ 2-red-circles = 0111 or (0,1,1,1).
- ★ Red Quad cards are vectors in \mathbf{Z}_2^4
- ★ Red cards are a model for the finite geometry AG(4,2)
- ★ Quads are planes in this geometry!

Quads and Algebra



- ❖ Find a Quad
- ❖ Add up the bit strings mod 2 in each component.

Quads and Algebra



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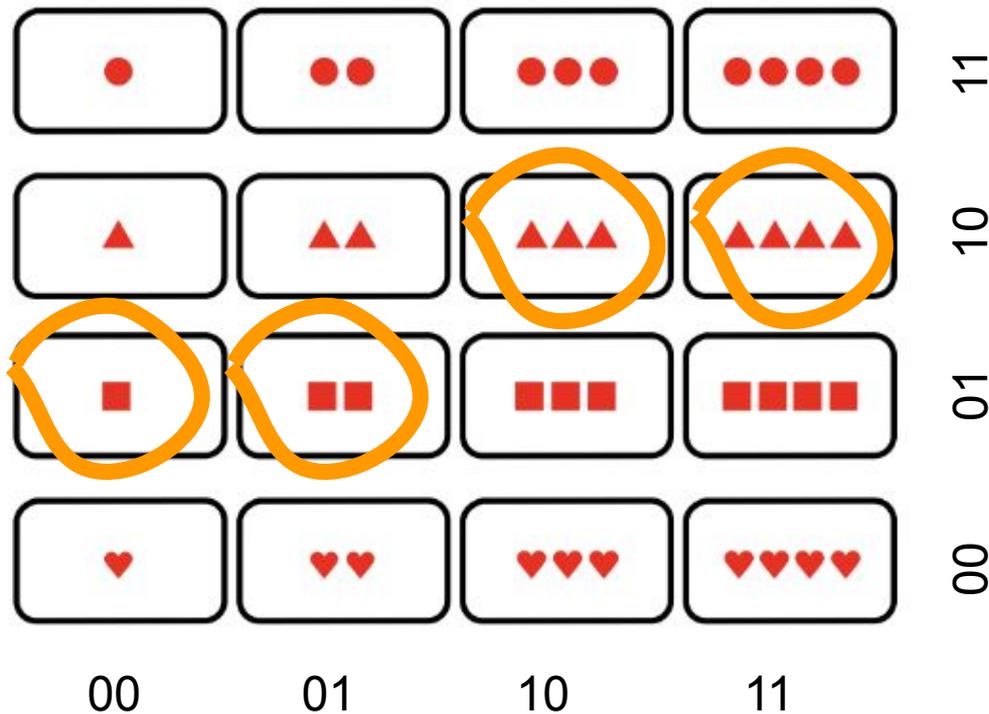
0001

0101

1010

1110

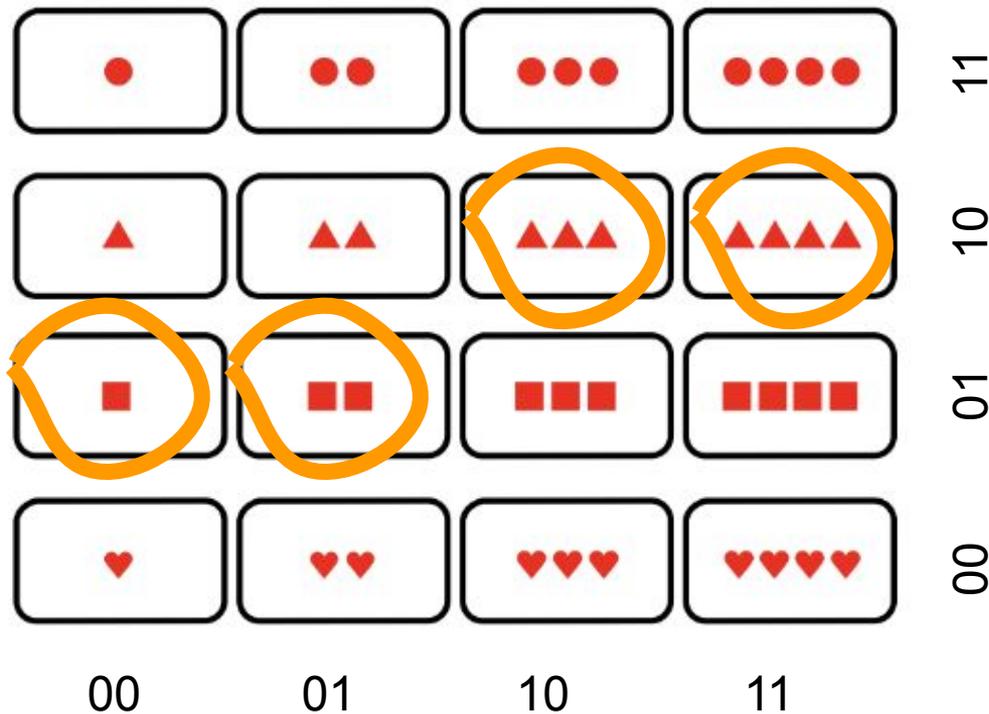
Quads and Algebra



- ❖ Find a Quad
- ❖ Add up the bit strings mod 2 in each component.

$$\begin{array}{r} 0001 \\ 0101 \\ 1010 \\ 1110 \\ \hline 0000 \end{array}$$

Quads and Algebra



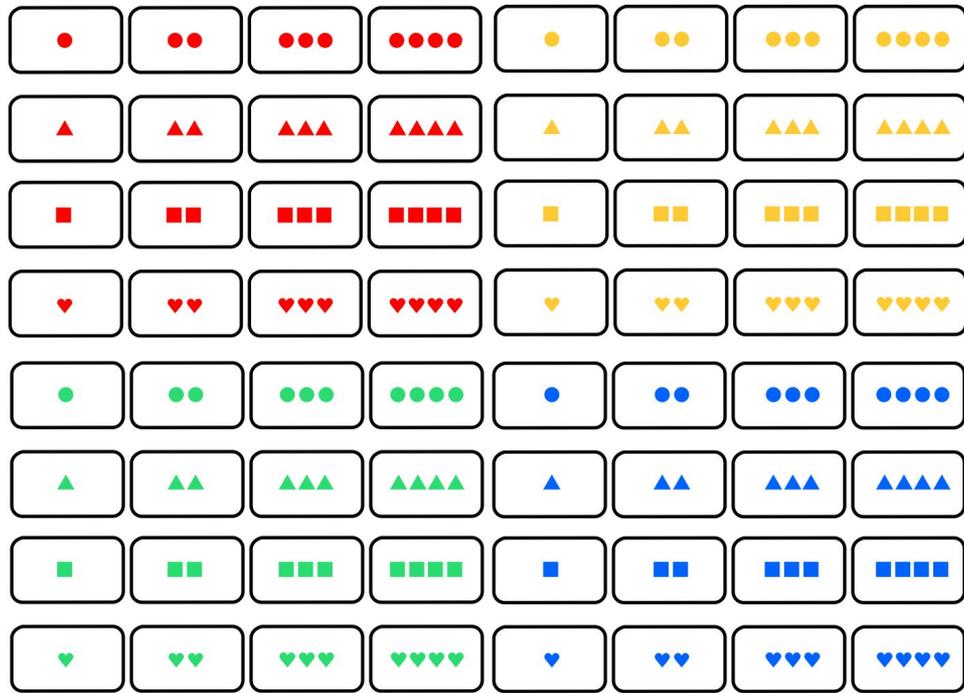
Quads

Theorem:

A, B, C, D are a Quad if and only if

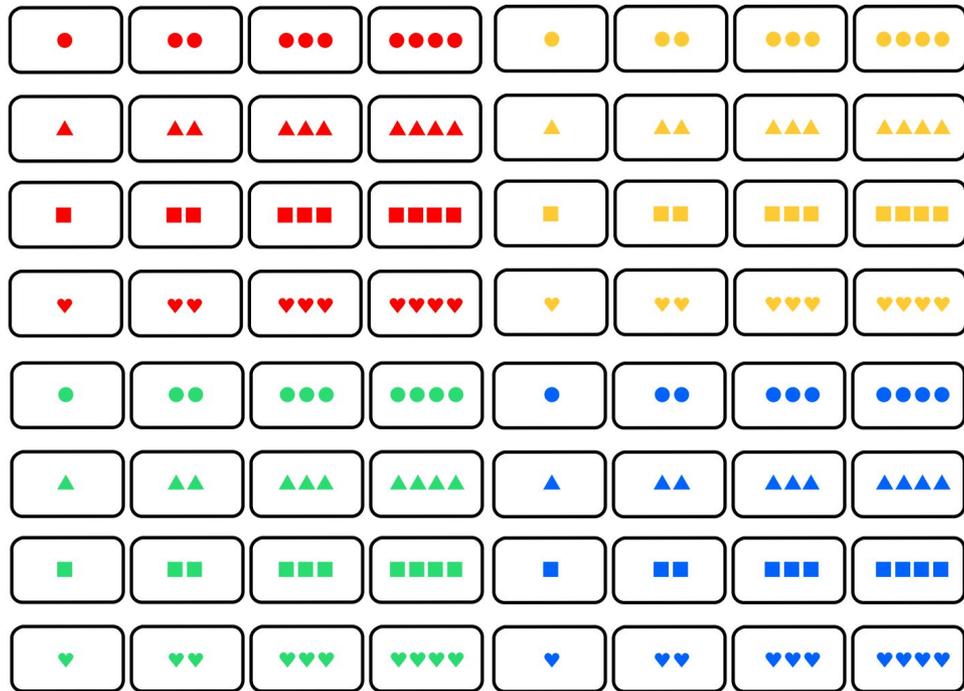
$$A+B+C+D = 0 \pmod{2}$$

Quads and Abstract Algebra



★ A Quads Deck is the group: \mathbf{Z}_2^6

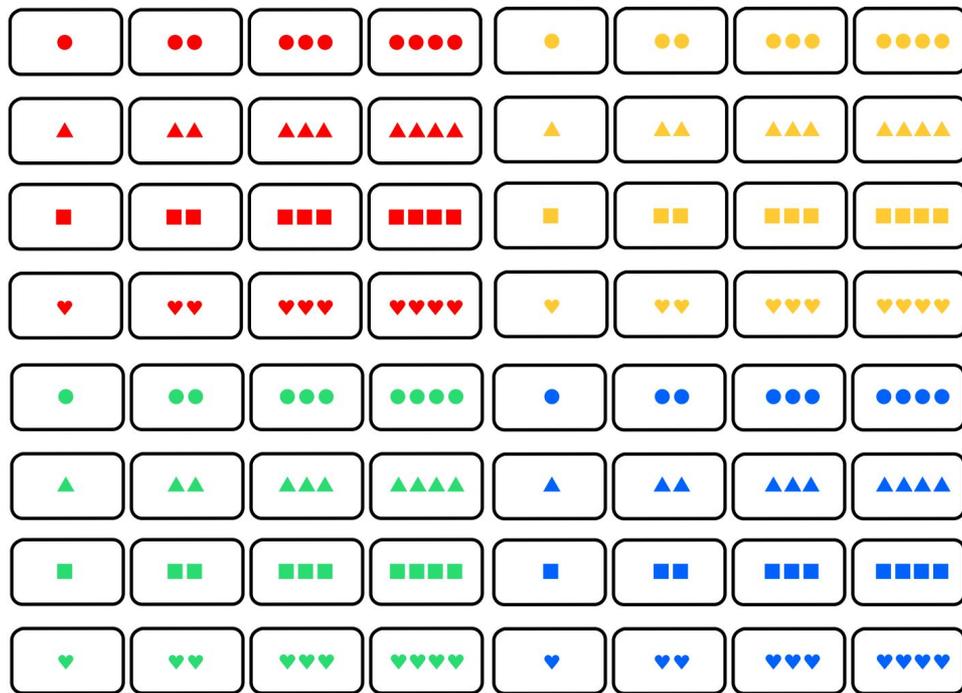
Quads and Abstract Algebra



★ A Quads Deck is the group: \mathbf{Z}_2^6

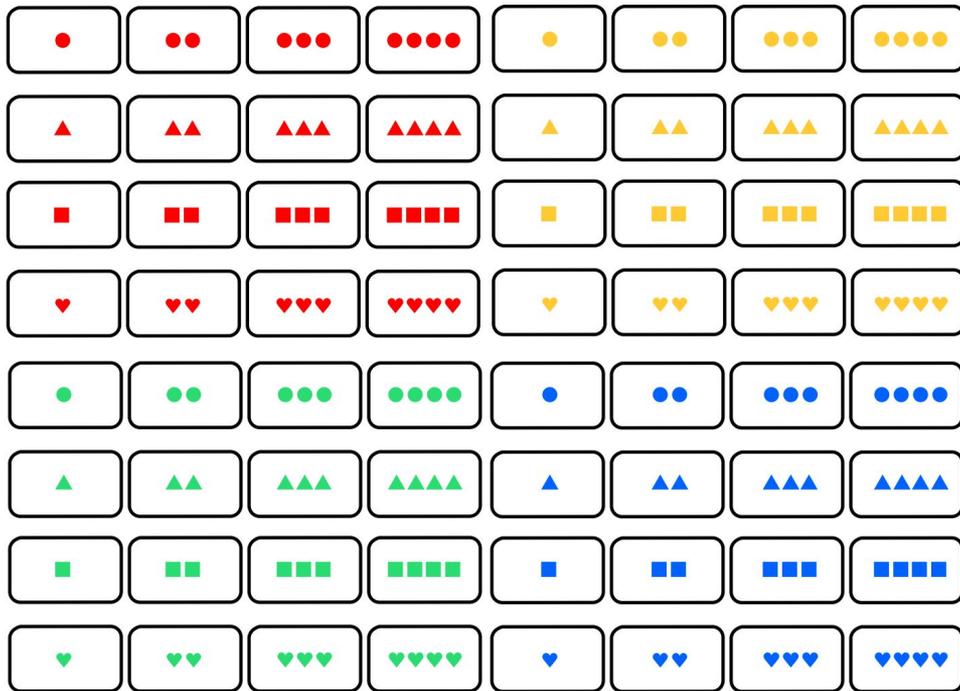
★ Quads containing 0000 are subgroups!

Quads and Abstract Algebra



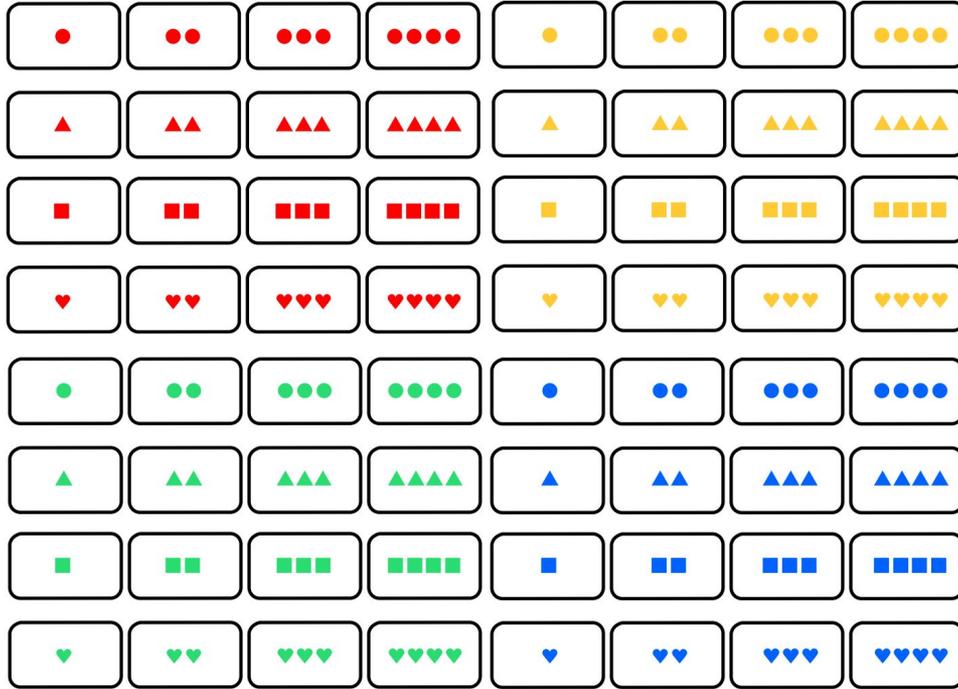
- ★ A Quads Deck is the group: \mathbf{Z}_2^6
- ★ Quads containing 0000 are subgroups!
- ★ What else is a subgroup?

Quads and Abstract Algebra



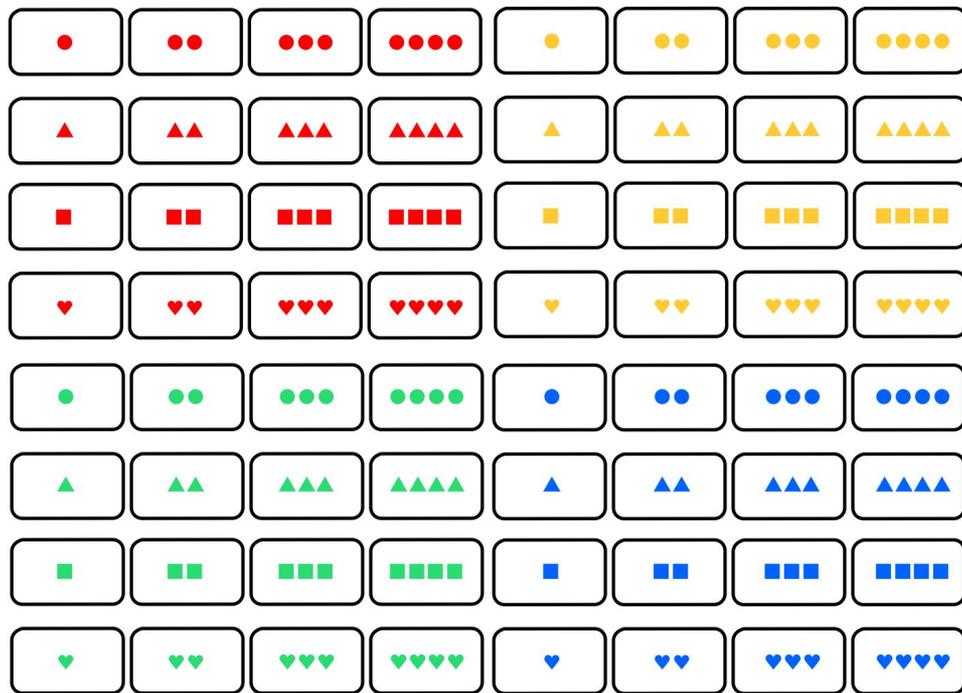
- ★ A Quads Deck is the group: \mathbf{Z}_2^6
- ★ Quads containing 0000 are subgroups!
- ★ The Red cards are a subgroup, if 0000 is Red.

Quads and Abstract Algebra



- ★ A Quads Deck is the group: $G = \mathbf{Z}_2^6$
- ★ Quads containing 0000 are subgroups!
- ★ The Red cards are a subgroup R .
- ★ The cosets of G/R are the Red, Yellow, Green and Blue cards!

Quads and Linear Algebra



- ★ A Quads Deck is the vector space $\mathbf{V} = \mathbf{Z}_2^6$, over the field \mathbf{Z}_2 .
- ★ Quads containing 0000 are subspaces.
- ★ The Red cards are a subspace R .
- ★ The translates of R are the other colors.

- ❖ Go to <https://laurenrose.org/research/> for packet to use in college classes or high school math circles.
- ❖ Go to <https://tinyurl.com/evenquadsapp> to play Quads online.
- ❖ Research Questions: Find “caps”, sets of cards with no Quads, in particular “maximal caps”: Friday 8am, student talks.

- ❖ **EvenQuads** is for sale at the AWM booth in the Exhibition Center, or on their website.
- ❖ All proceeds go to AWM.