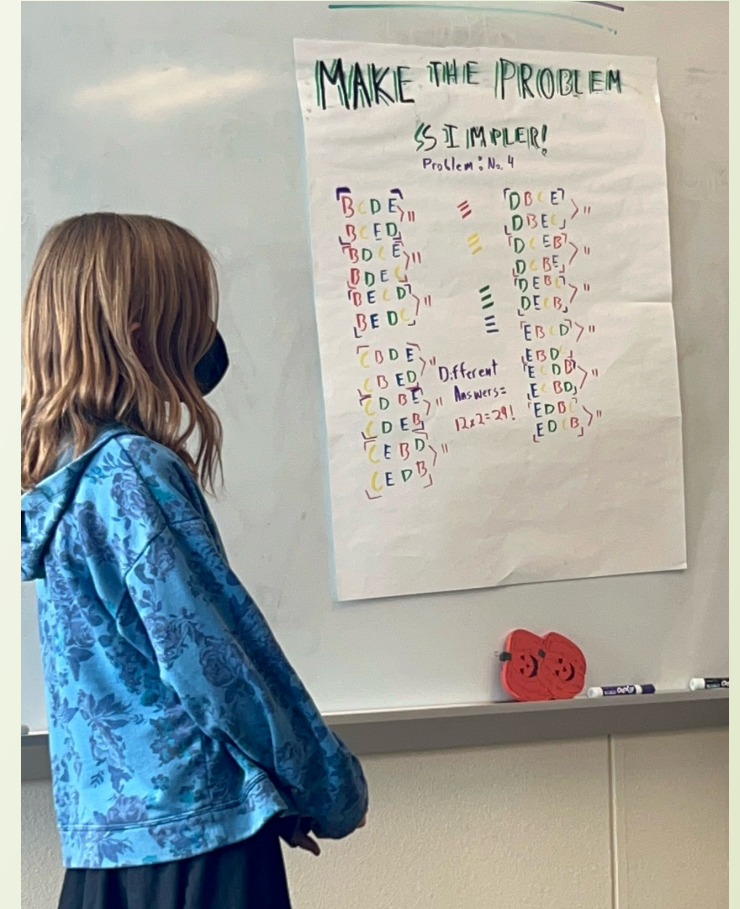


Pairing Math Competitions with Math Wrangles throughout a School District

Joint Mathematics Meetings 2024

4 January 2024

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MathAmigos

(mathamigos.org)

In spring of 2017 a small group of math education-enthusiastic Santa Feans started meeting in order to raise awareness and levels of mathematics understanding in Santa Fe, New Mexico public schools (SFPS). We initially focused on teacher professional development.

Since then, we have entered the classrooms, teaching over two hundred students each year and modeling our methods for the teachers in those classrooms.

Two years ago, we began greatly expanding the use of mathematics competitions—and adding *math wrangles*.



School District Context



- ❖ Total SFPS students: 12,875 in 30 schools
 - ❖ District **math proficiency average: 24%** (grades 3-8, MathAmigos' core grades)
 - ❖ The district's highest-rated schools (small schools in affluent neighborhoods) demonstrate proficiency in the 30%-45% range, with one at 58%.
- ❖ Working within primarily Title I schools (25 of 30 schools in Santa Fe, NM), most of which test around 12% math proficiency, we have been able help more than 200 upper elementary and middle school students a year develop positive math identities through district-sponsored math competitions and wrangles (math debates). We tripled participation in one competition in one year, and have increased participation again this year.
- ❖ More than half of the students come from underserved communities.
- ❖ Many of our students are English language learners



Elements of the New Program

- ❖ Weekly sessions in 9 schools
- ❖ Alternating weeks in 6 additional schools
- ❖ Grades 2 through 8
- ❖ SA²GE classes (Services for Advanced Academics and Gifted Education)
 - ❖ Note: Gifted does not necessarily mean strong in math or reading skills
 - ❖ Most of the SA²GE teachers do not have much background in mathematics
- ❖ Nearly all are Title I schools
- ❖ Program:
 - ❖ Begin with math circles (grades 4-8) for quarter 1, then sporadically throughout the year
 - ❖ Introduce problem-solving techniques through Olympiad (MOEMS*) problems to entire class
 - ❖ Ongoing student participation in MOEMS competition
 - ❖ Class-level problem solving morphs into two teams solving math wrangle sets in separate rooms
 - ❖ Wrangle practice
 - ❖ Wrangle discussion, debrief, exploration of alternate approaches
 - ❖ Interschool wrangles

* Math Olympiads for Elementary & Middle School Students

Some Math Circle Problems

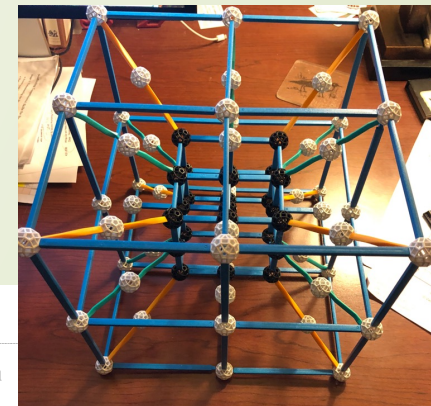
1. (Find the) Polite Numbers to 20

- ❖ A polite number can be written as the sum of two or more consecutive positive integers
- ❖ Only addition is needed, so accessible to all grades and skills
- ❖ Is about noticing patterns:
 - ❖ “Rude” or impolite numbers
 - ❖ Especially polite numbers (can be expressed as a sum of different numbers)
- ❖ Useful for introducing algebra at early (as early as 3rd) grades for generating the polite numbers that are the sum of 2, 3, or 4 (or more) successive numbers

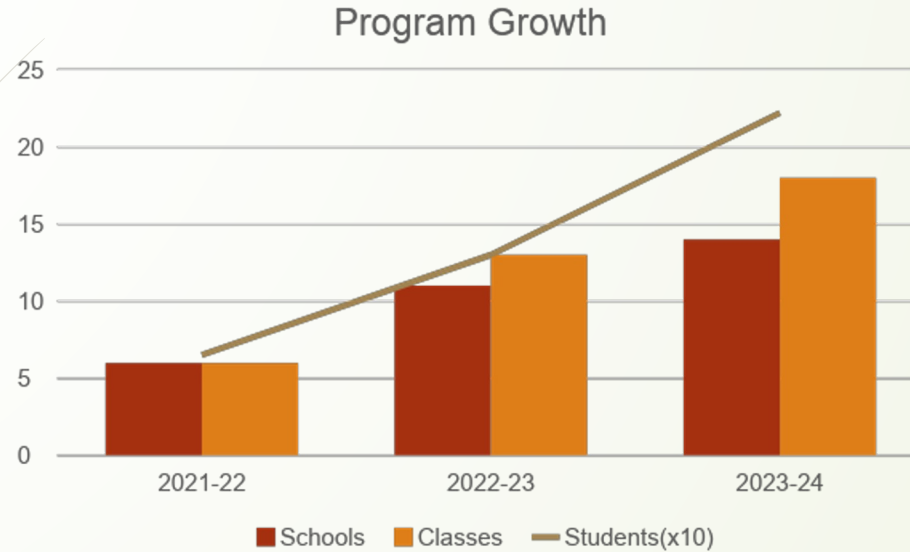
2. The Game of Set

- ❖ A game, so no overt math
- ❖ Introducing dimensions: 4D, 3D, 2D using Zometools.
And to understand 4D Set better, “make it small” by going to 2D
- ❖ How many sets in 2D? Students notice that playing 2D is like Tic-Tac-Toe, until someone finds a set that is not in a row
- ❖ 2D Set is Tic-Tac-Toe (students notice this right away), but on a torus
- ❖ Counting sets and a taste of combinatorics

	7	8	9	
3	1	2	3	1
6	4	5	6	4
9	7	8	9	7
	1	2	3	



School Program Growth 2021-2024



School Year	Schools	Classes	Students
2021-22	6	6	65
2022-23	11	13	130
2023-24	14	18	222

- ❖ School, teacher, and student numbers have been increasing steadily over 3 years
- ❖ Not all of these students are also participating in the MOEMS competition and wrangles



MOEMS[®] Competitions Overview

- ❖ Mathematical Olympiads for Elementary and Middle Schools is an international math competition program that provides 5 monthly problem-solving contests for elementary and middle school students
- ❖ There are two divisions:
 - ❖ The elementary division is for grades 4, 5 and 6
 - ❖ The middle school division for grades 6, 7 and 8
- ❖ 5 monthly contests, 5 questions each, of increasing difficulty
- ❖ Problem sets administered online each month from November through March. Online administration option became available during SY 20-21.
- ❖ Floor to ceiling range is quite high. Most students can get one correct answer, 2-3 correct with reasoning, and 4-5 with well-developed skills.
- ❖ Fully worked solutions and extension problems are available post-test

A Type of MOEMS* Competition Problem

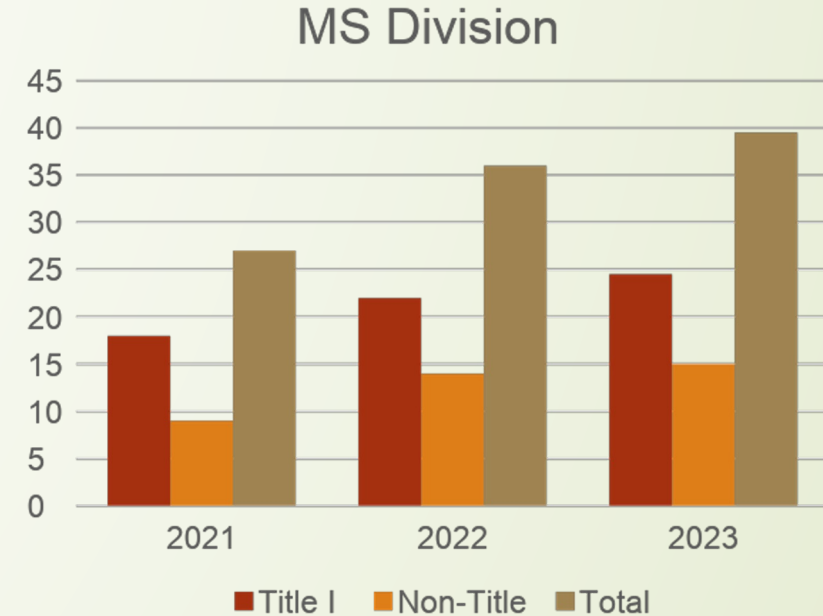
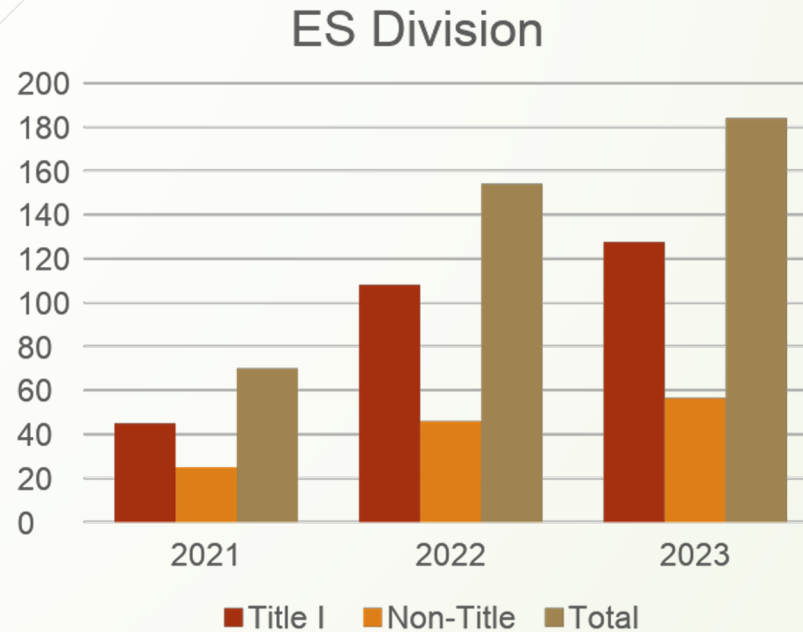
1. Cryptarithms: Each letter represents a different digit. What 4-digit number does TOUR represent?

- ❖ A puzzle that help develop number sense
- ❖ AI apps such as ChatGPT fail to solve these, despite “recognizing” them or solve using brute force substitutions for all possible letters

TWO
<u>+TWO</u>
FOUR

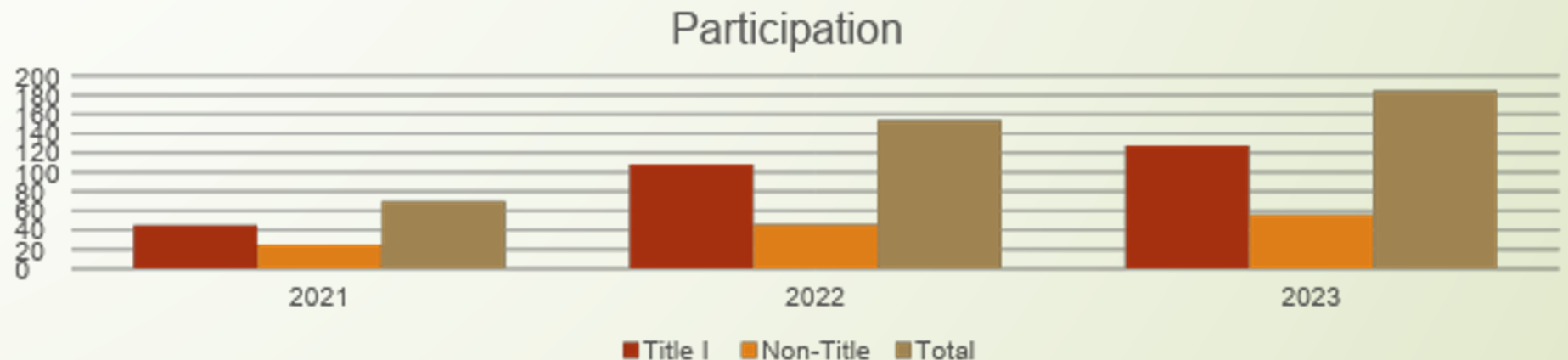
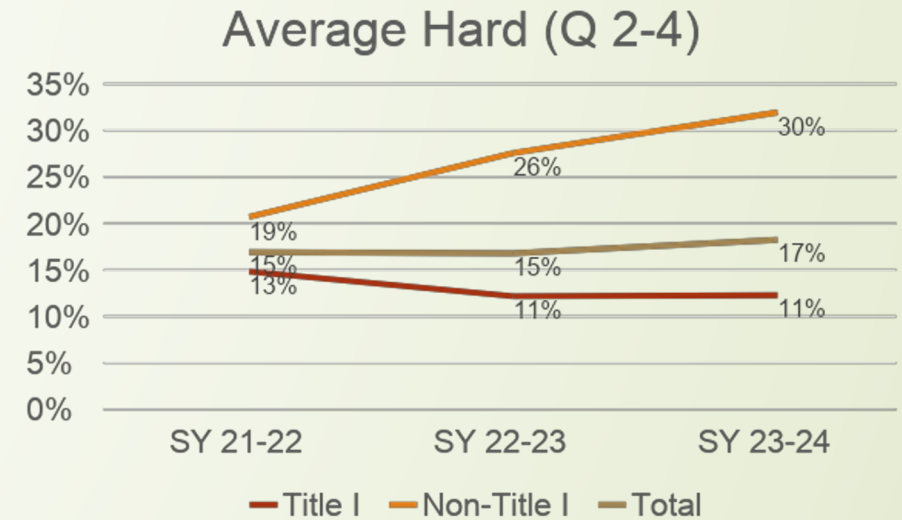
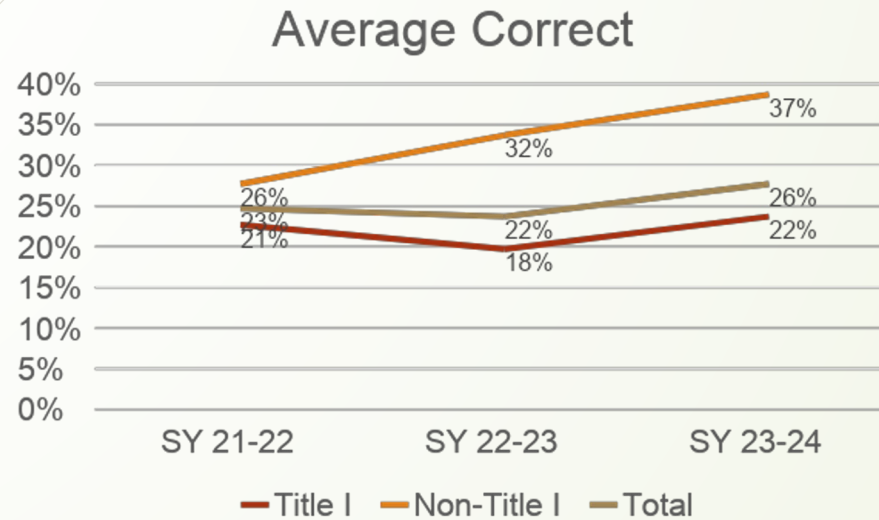
* <https://moems.org>

MOEMS Competitions Growth 2021-2024



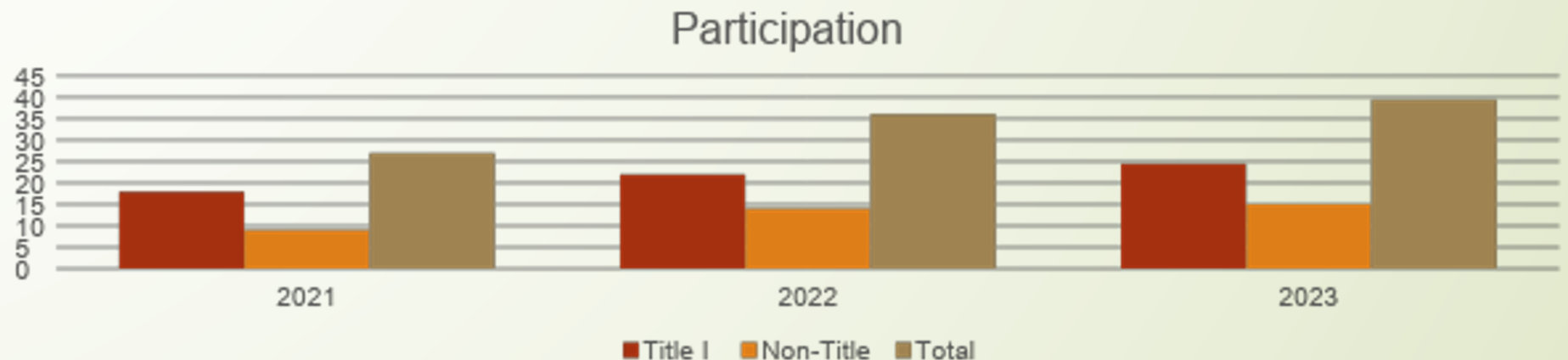
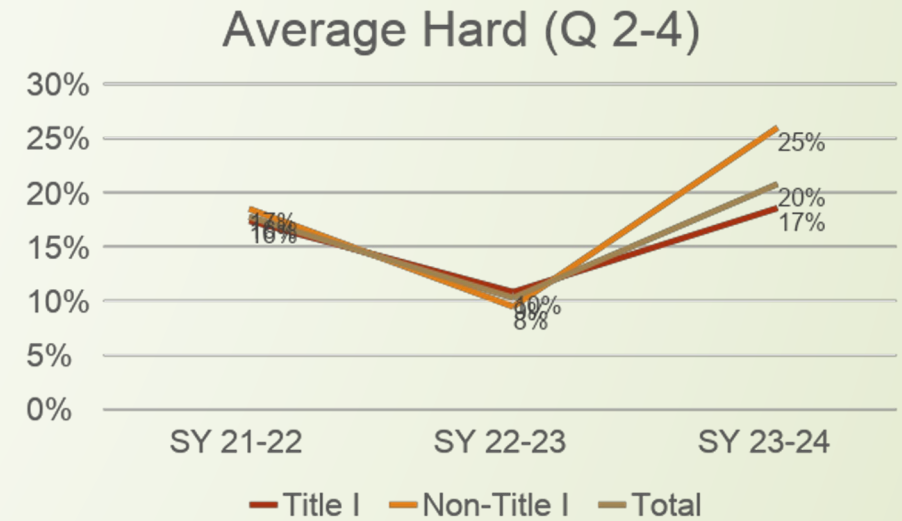
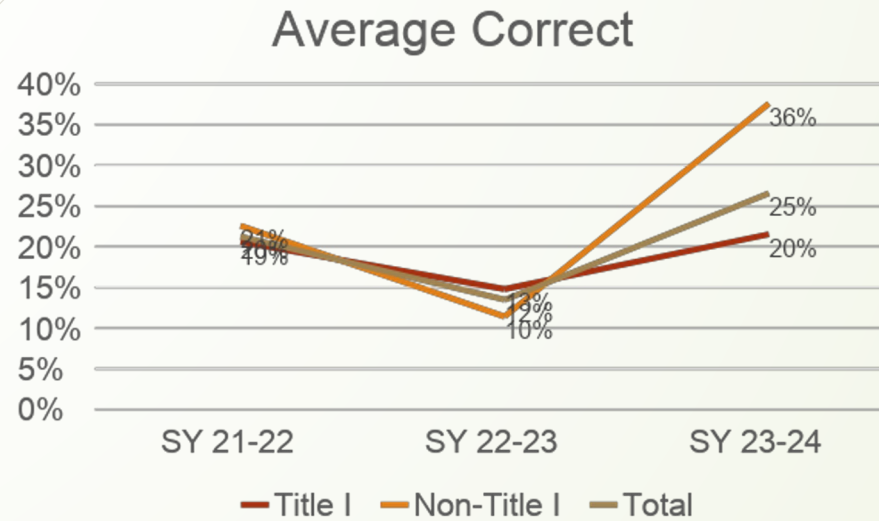
- ❖ School, teacher, and student numbers have been steadily over 3 years
- ❖ Not all of these students are also participating in the circles and wrangles
- ❖ Mixture of students identified for gifted education and interested/motivated others.

MOEMS Elementary (Grades 4-6)



23-24 reports average from first two contests

MOEMS Middle (Grades 7-8 – 6th omitted)



23-24 reports average from first two contests



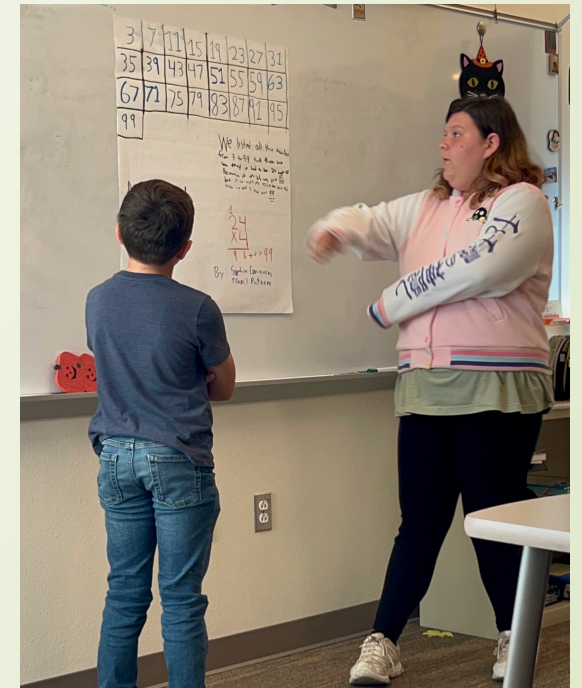
Newest Effort: Competitions & Wrangles

- ❖ In partnership with the district's SA²GE* program, which also runs the math competitions for the district, since 2021 we have been dramatically increasing the numbers of grades 3-8 students taking the MOEMS** tests. We tripled the numbers in 2022-23—to 160—from the previous year, and now have 21 schools participating, most of them being Title I schools (most students qualifying for free or reduced lunch).
- ❖ Several students ranked nationally or internationally in year 2 (2022-23)
- ❖ The number of students performing above average has increased, but the more important change may have to do with their attitude towards math. A majority of the students surveyed after the 2022-23 school year expressed positive feelings about “hard problems,” and about two-thirds reported positive changes to the ways they felt about and approached difficult math.
- ❖ In May of 2023 the SFPS hosted its first district-wide math wrangle, one elementary level and one middle school level wrangle.
- ❖ We believe this is the first such district-wide wrangle in the US.

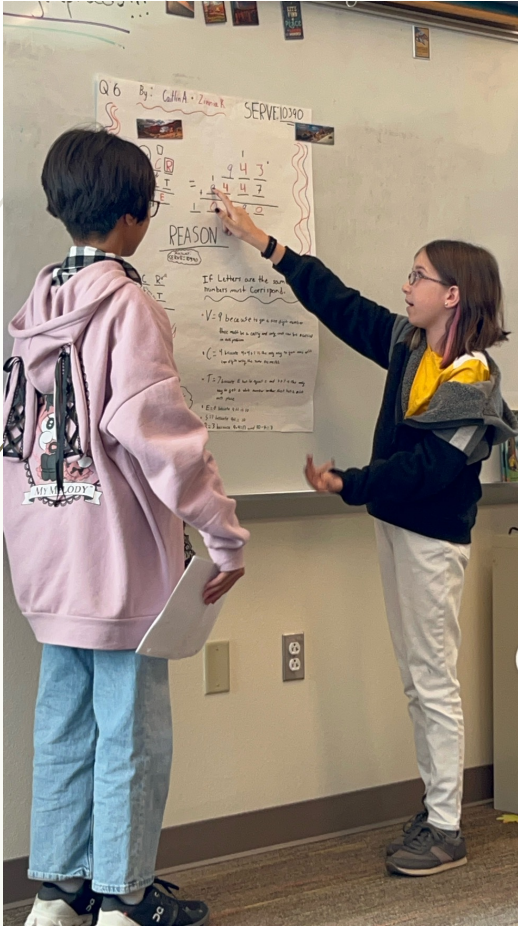
* SA²GE = Services for Advanced Academics and Gifted Education ** Math Olympiads for Elementary & Middle School Students

Benefits of the Competitions to Wrangles Project

- ❖ For the students
 - ❖ Competition Prep: Teamwork, strategic thinking, conversing about math
 - ❖ Competitions: Problem solving skills, resilience, math, confidence
 - ❖ Wrangles: Presenting mathematical reasoning, conversations, confidence, public speaking, debating skills and strategies, logic, teamwork, use of clear and proper language, respectful critiquing of presentations, etc.
- ❖ For the teachers
 - ❖ Confidence with hard math
 - ❖ Independence in training competition and wrangle teams
- ❖ For the schools:
 - ❖ Visibility and pride



How do you feel about hard math problems?



❖ 1st year Students

- ❖ I feel pretty good about them when I'm solving them, happy or sad when I know they're right or wrong, and eager to know what I scored on them.
- ❖ Hard math problems are fun (and brain frying).
- ❖ MAD AND CONFUSED
- ❖ I like to experiment with hard math problems over time.

❖ Multi-Year Students

- ❖ I have mixed feelings about hard to harder math problems. One side of me thinks hard math problems are enjoyable while the other thinks they are frustrating.
- ❖ I can sometimes feel stressed or confused but I do love challenges and I think it can be really fun.
- ❖ I LOVE them, they are so much fun.
- ❖ I honestly feel stressed and a little bit rushed, when I'm stuck on a problem and everyone is done.



How have your feelings and approaches to hard problems changed over the course of this Math Olympiad season?

❖ 1st Year Students

- ❖ "My feelings have changed because originally, I thought Math Problems were either too hard or too easy, and I like the Math Olympiad problems because they feel right."
- ❖ "I went from scared to fine with the math."
- ❖ "At first I would just try one way to solve it and if the solution seemed correct, but now I try a lot of things to make it work."
- ❖ "I've felt more frustrated."

❖ Multi-Year Students

- ❖ "I've learned that rather than using my energy to be angry at a problem, I can use it to solve the problem. Sometimes."
- ❖ "My feelings and approaches to hard problems have changed because instead of just knowing all of the answers like I normally do, I really have to think more about what the question is asking me so I can know what my answer will be."
- ❖ "I do it in more creative ways"
- ❖ "I have found out more ways to use the strategies I have been taught but I still am not that good at complex problems."



Competitions & Resilience



- ❖ Late in the 2022-23 school year I asked a teacher in one of my weekly classes whether regular practice with competition problems and wrangles had improved his students' math performance in their regular classes.
- ❖ He said that these (accelerated) students normally did well in those classes and so he saw no significant improvement there, but...
- ❖ ...when they *had* encountered hard problems, tears were often the result.
- ❖ But now, no tears. Now they were accustomed to hard problems.
- ❖ We hope to measure resilience in the coming school year

District Math Wrangle Overview

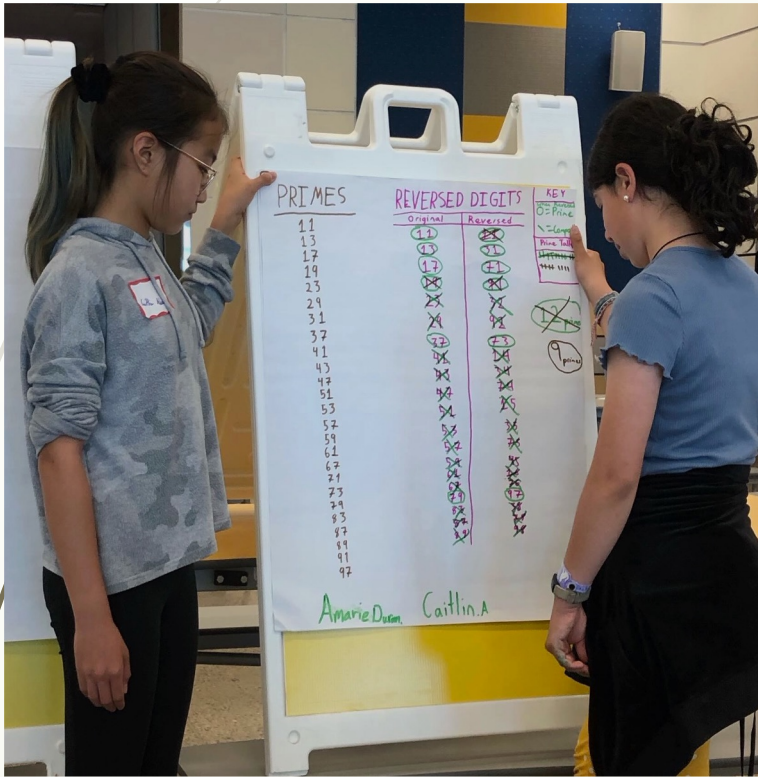
- ❖ A math wrangle is a mathematics debate, where the two teams go to separate rooms and try to solve a set of hard problems. Solutions are written on large pads.
- ❖ Pairing: Our wrangle used MOEMS-style problems of the type that the students had seen during the year for wrangle prep, critical especially when they'd had little wrangle practice.
- ❖ The Math Wrangle:
 - ❖ A coin toss, and one team challenges the other to solve one of the problems
 - ❖ The challenged team presents and explains a complete solution, not just an answer
 - ❖ The other team may challenge (rebut) the result, and judges assign points. And so on.
 - ❖ No student or student pair may present or rebut more than once
- ❖ May 2023 district-wide wrangle
 - ❖ 58 students from 12 district schools participated in wrangles or small math circles
 - ❖ Students not on the 4 wrangle teams formed small math circles around one of our school-year mentored teachers and worked on the wrangle problems so that they would be ready to understand the wrangle session
 - ❖ About 20 teachers worked as wrangle proctors or math circle leaders
 - ❖ About 20 parents attended and helped transport the students that school day
 - ❖ We surveyed all participants



Wrangles: Responses & Surveys

- ❖ When the wrangle was done, I asked everyone if they might want to do this again, and there was a resounding “YES!” from the participants
 - ❖ Students reported that they:
 - ❖ felt challenged, had fun and cared more about the hard math problems
 - ❖ grown in confidence, comfort, concentration and determination
 - ❖ had learned to use better strategies, find patterns and think creatively rather than assume they should already have answers
 - ❖ found the problems “are very challenging, but once you find the pattern or the secret to the problem it becomes really easy, which makes me happy.”
 - ❖ felt the event helped them gain confidence, with one writing the wrangle, “only solidified that I truly love math.”
 - ❖ And one middle school student said it was the best thing he’d done all year
 - ❖ Parents noted students’ enjoyment at working together, writing, “the students looked so happy throughout.”
 - ❖ In 2023-24 we are planning wrangles between pairs of school throughout the year
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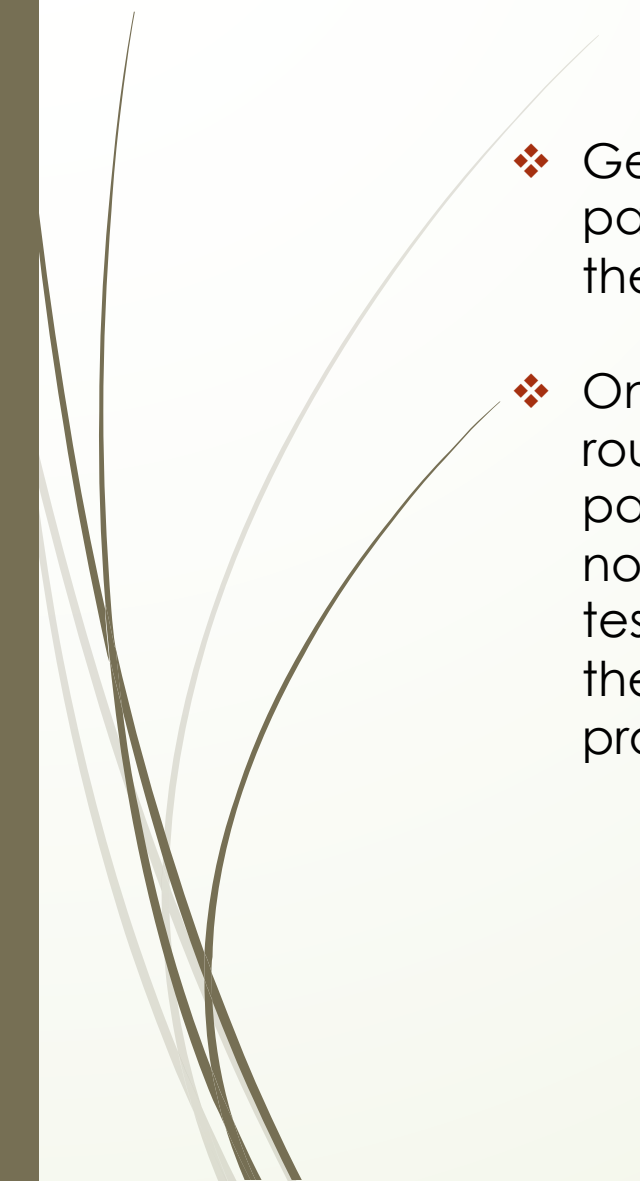
Impact on Mathematical Motivation



- ❖ 5th and 6th grader SA²GE students begged to be permitted to bring their classmates to their math circles and competition class--and then fetched them. (Dual-Bilingual School)
- ❖ When told at the end of a math circle session in a general education class that the teacher had some bad news, students shouted “oh no! you’re not coming back”. But no, the bad news was that they were only almost correct about their solutions. They showed visible relief. (Average Elementary)

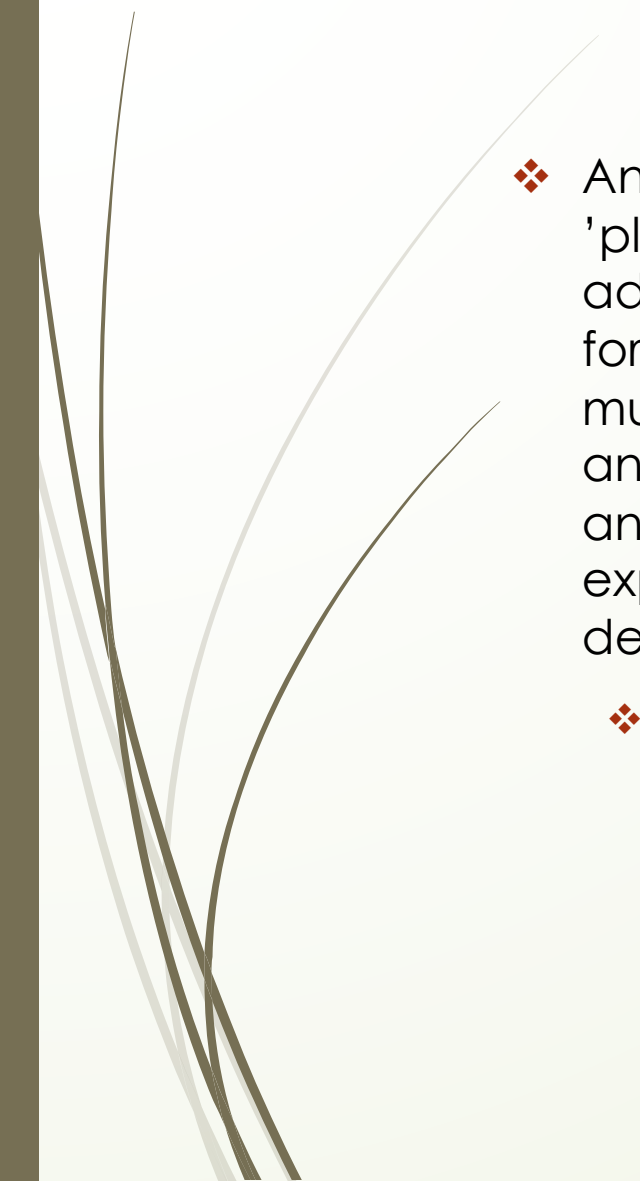


Impact on Number Sense

- ❖ General education fourth grade students at one school have been participating in Math Circles and their teachers have commented on some of the positive impacts they have observed to their SA²GE teacher partner.
 - ❖ One teacher saw that the students were able to grasp the concept of rounding numbers more quickly and easily after those students had explored patterns/sequences in a Math Circle activity. This impact was more noticeable in the group of students who had lower scores on standardized tests. Providing students the opportunity to explore the numbers “in-between” the tens gave them insights into the process of rounding and the general properties of counting numbers.
- 



Impact on Playful Mathematical Affect

- 
- ❖ Another teacher noted that these students do not often have the chance to 'play' with numbers, and this exploration through simple counting and addition was beneficial to their number sense. Most of the math experiences for these students is rigid and formulaic, when they work with numbers it's much more a tedious task coupled with fear and shame of getting the wrong answer. On the other hand, Math Circles provide an opportunity to freely play and explore with numbers without as many constraints or consequences. The exploration and "playing" with numbers is an important process in the development of mathematical thinking.
 - ❖ SA²GE teacher connecting with partner 4th Grade General Ed Teacher



Impact on one SA²GE Teacher

"I never saw myself as a strong math student. After a certain point, it just didn't make sense to me. I didn't understand fractions until I started baking in high school. I paid my sister to do my geometry work in high school. My brother (who was in 4th grade at the time) helped me with pre algebra in 8th grade...I felt stupid anytime I couldn't understand what I was supposed to do."

"I never wanted to teach math past the 3rd grade level-I didn't know how I could teach it and explain it when I wasn't understanding it myself. When I started teaching gifted ed, I realized I had to teach math. I avoided it as much as possible, never really getting into complicated math concepts-just having the kids do number and critical thinking puzzles."



Impact on one SA²GE Teacher



“Math Circles, Math Amigos, MOEMs changed everything. I enjoy teaching math/creative problem solving now. I prefer it over teaching ELA. I learned how playful and creative math can be. I realized I can do the work right alongside my students and share my problem solving strategies and not worry if I didn't get the correct answer. I love modeling problem solving for them, talking them through challenges they may face with a MOEMs problem.”

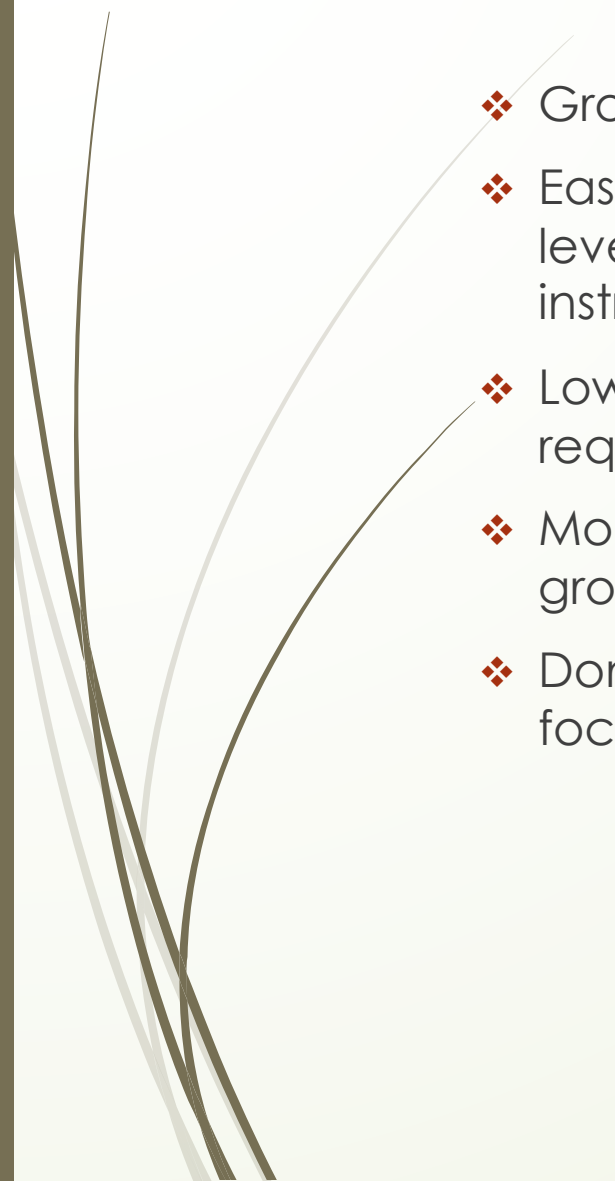
“I love learning from my students-often times they are able to solve problems in completely different ways than what I thought of. I am able to see patterns, find shortcuts and make some pretty cool connections in math now. I have enjoyed watching my youngest students work on problem solving along with the older students. My students see my SA²GE math time as a time to play, ask questions, discover new things and to learn from their mistakes.”

Program Leaders: Lessons Learned (James)

- ❖ Grade levels & ages:
 - ❖ Early work more grades 6-9. Shifted to more earlier grades, 2-6, with some 7 & 8
 - ❖ Cultivating a culture of mathematical inquiry, weeks 2 to 8: respectful conversation (with the teacher, classmates, teacher of record), challenging the teacher, "leaping to the board"
 - ❖ Coping with wide range of grade levels/skills in math
 - ❖ Need to focus on patterns and early grade level math circles
 - ❖ Experience with the above giving insight and motivation to...
- ❖ ...branch out year 3 into general education classes at two schools
 - ❖ Teaching an entire grade level
 - ❖ 3-4 grade levels of math skill in a single class
 - ❖ Requires low threshold/high ceiling play and pattern exploration benefitting all students
 - ❖ Teachers have requested materials to follow up on our weekly sessions
- ❖ Seeing the uses and value of competition problems, especially with group solving leading into wrangles
- ❖ Gratifying levels of teacher growth as mathematical thinkers and doers, leading to **independence** with the materials we use. Many have been studying and using AMS Math Circle Library books, Art of Problem Solving materials, and more, on their own.



Program Leaders: Lessons Learned (Geoffrey)

- 
- ❖ Growth across a horizon is not linear.
 - ❖ Easily available data such as grades, teacher observations, and grade-level achievement can lead to narrowing of focus in math content instruction. That may boomerang.
 - ❖ Low mathematical self-efficacy is culturally self-reinforcing. Changing that requires work with teachers, students, and the community.
 - ❖ More reasoning-focused problems support more heterogeneous student grouping.
 - ❖ Domain-specific identification of mathematical talent can lead to too little focus on language development.

Retention Questions

Years In MOEMS	N
≥3	28
2	83
1	140

- ❖ Who stays?
- ❖ Who leaves?
- ❖ Why?
 - ❖ What impacts retention?
 - ❖ What is the year one experience like?



More Questions

- ❖ How does student mathematical identity and sense of belonging change over time?
- ❖ To what extent do circle-competition-wrangles change mathematical problem solving abilities and temperament?
- ❖ How does the programming affect other teachers?
- ❖ What skills are/can be developed to improve contest performance?
- ❖ Does the program have effects on mathematical creativity?
- ❖ Does the program have effects on self-management?
- ❖ How far and in what ways can we expand the program to more broadly affect the school district and broader community?
- ❖ How can this program be used by the schools to trumpet some successes in math?
- ❖ What should we be studying further? Resilience? Detailed pre- and post-measures of attitudes?



Thanks!

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