

A transition from geometry
To number theory
And then....????

SONA DIAGRAMS

MATHFEST 2022

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This book has a great SONA section

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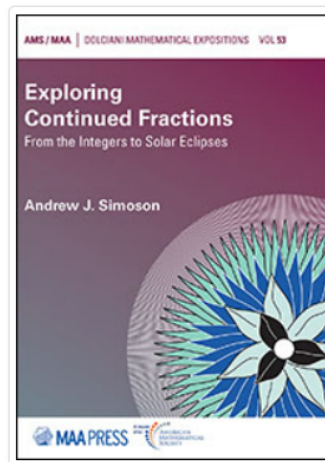
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Dolciani Mathematical Expositions
Volume: 53; 2019; 480 pp; Softcover

Exploring Continued Fractions: From the Integers to Solar Eclipses

[Andrew J. Simoson](#): King University, Bristol, TN

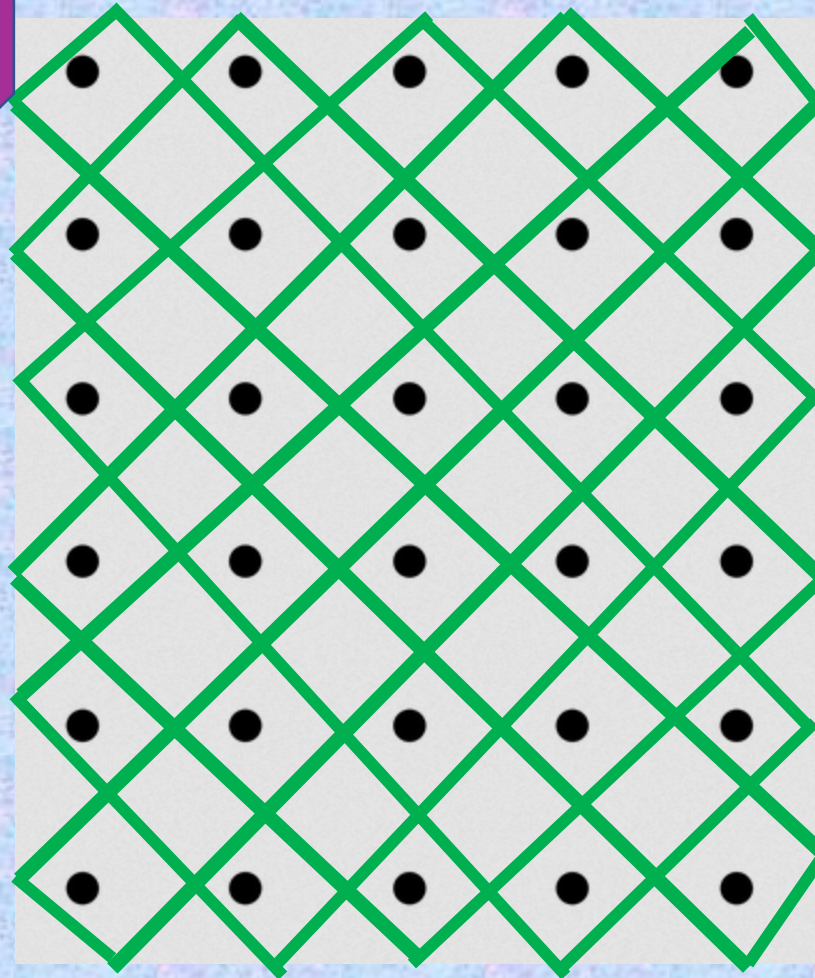
MAA Press: An Imprint of the American Mathematical Society

There is a nineteen-year recurrence in the apparent position of the sun and moon against the background of the stars, a pattern observed long ago by the Babylonians. In the course of those nineteen years the Earth experiences 235 lunar cycles. Suppose we calculate the ratio of Earth's period about the sun to the moon's period about Earth. That ratio has $235/19$ as one of its early continued fraction convergents, which explains the apparent periodicity.

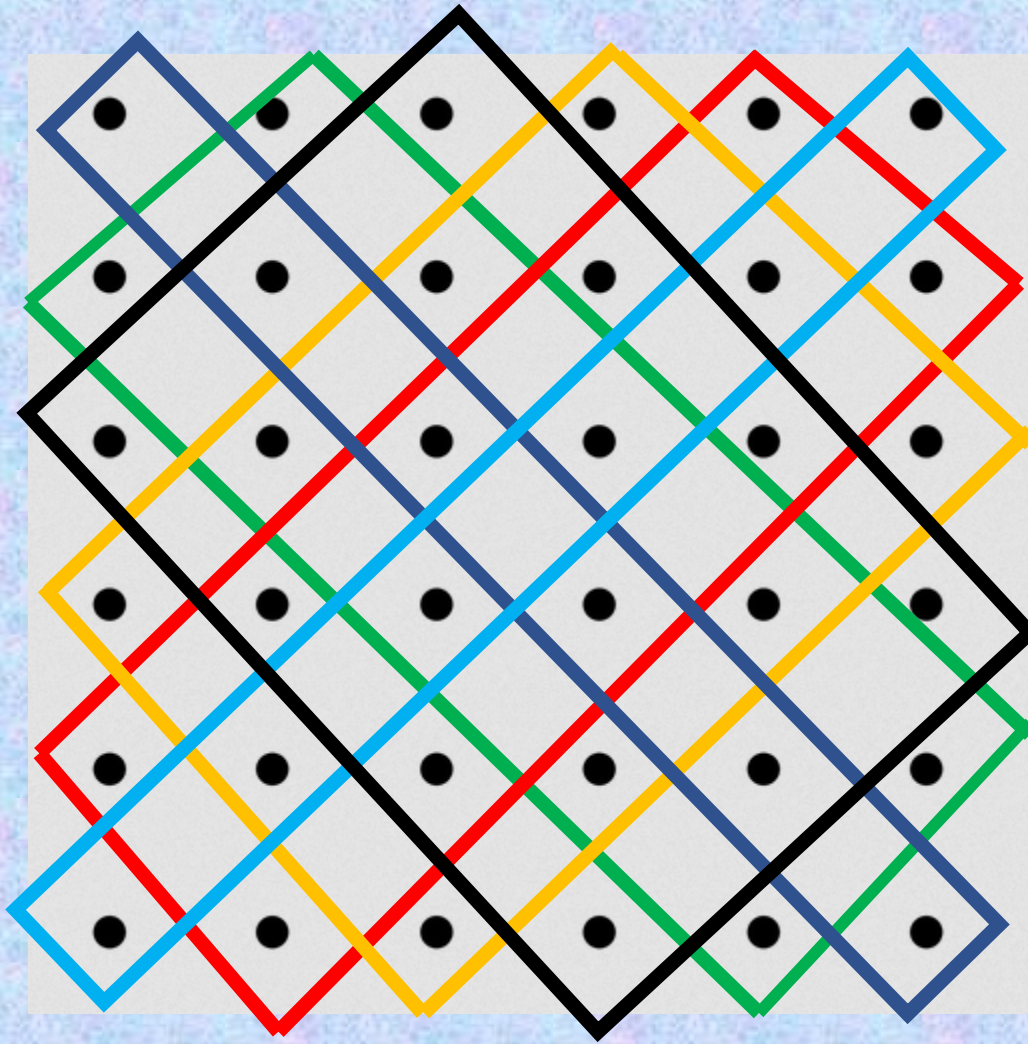
Exploring Continued Fractions explains this and other recurrent phenomena—astronomical transits and conjunctions, lifecycles of cicadas, eclipses—by way of continued fraction expansions. The deeper purpose is to find patterns, solve puzzles, and discover some appealing number theory. The reader will explore several algorithms for computing continued fractions, including some new to the literature. He or she will also explore the surprisingly large portion of number theory connected to continued fractions: Pythagorean triples, Diophantine equations, the Stern-Brocot tree, and a number of combinatorial sequences.

The Sona Diagrams of the Chokwa People

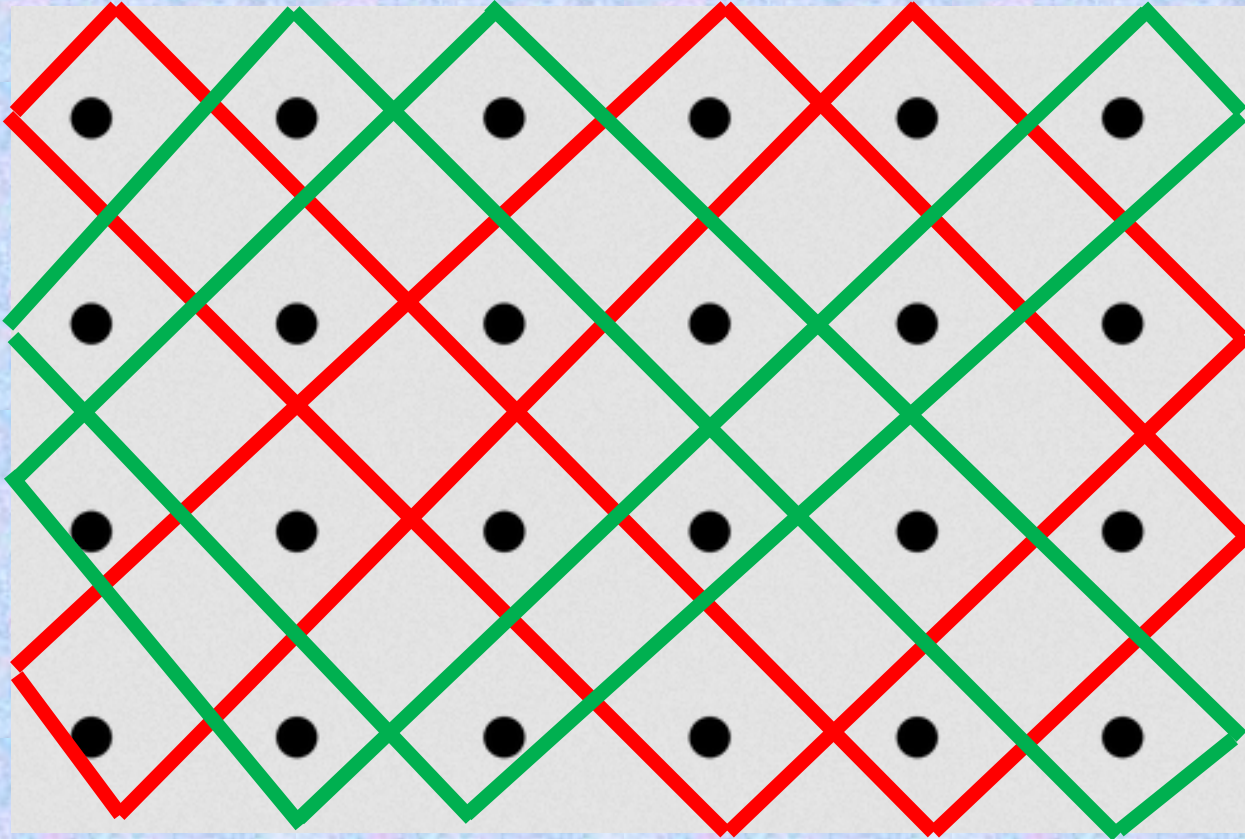
1 LOOP 5 x 6



6 SEPARATE LOOPS 6 x 6



2 SEPARATE LOOPS 4 x 6



Notice all line segments have slope 1 or -1

In an $m \times n$ grid there will
Be $\text{GCD}(m,n)$ loops!

The chocolate bar algorithm

$$\text{GCD}(N,M)=\text{GCD}(N-M,M)$$

Always Eat the biggest Square

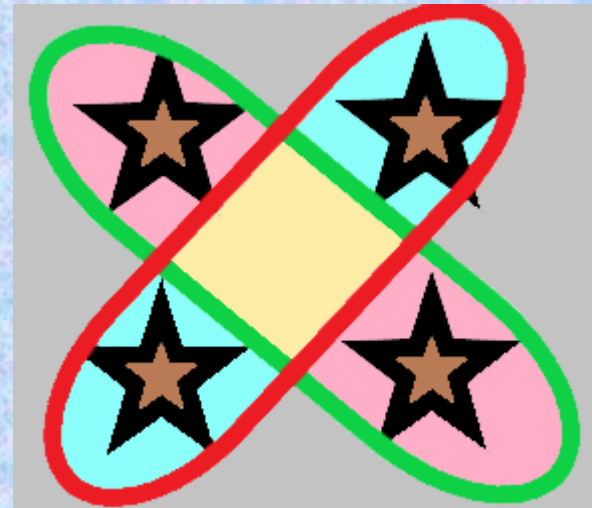


$$\text{GCD}(10,6)=\text{GCD}(10-6,6)=\text{GCD}(4,6)$$

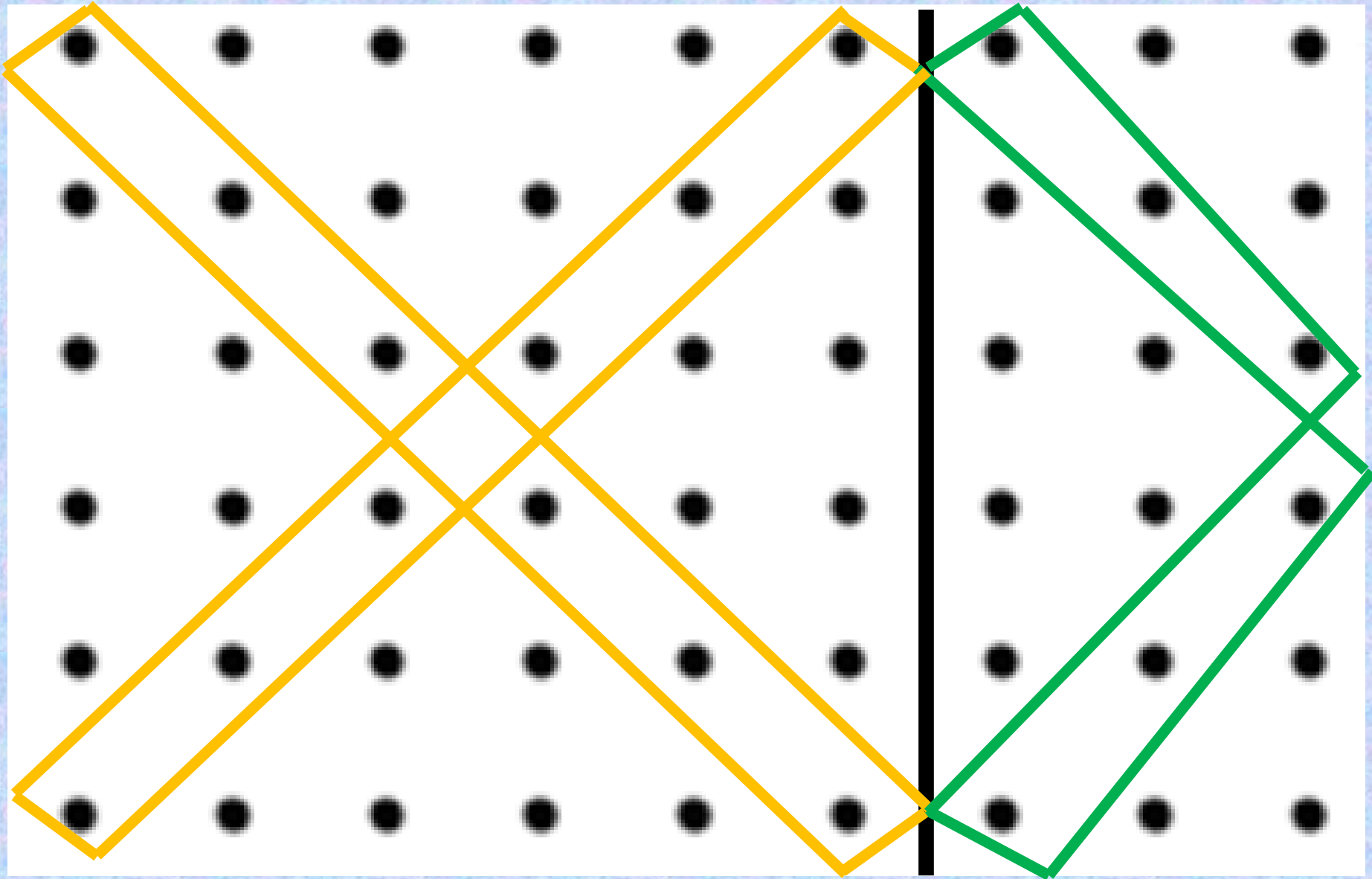
$$\text{GCD}(4,6)=\text{GCD}(4,6-4)=\text{GCD}(4,2)$$

$$\text{GCD}(4,2)=\text{GCD}(4-2,2)=\text{GCD}(2,2)$$

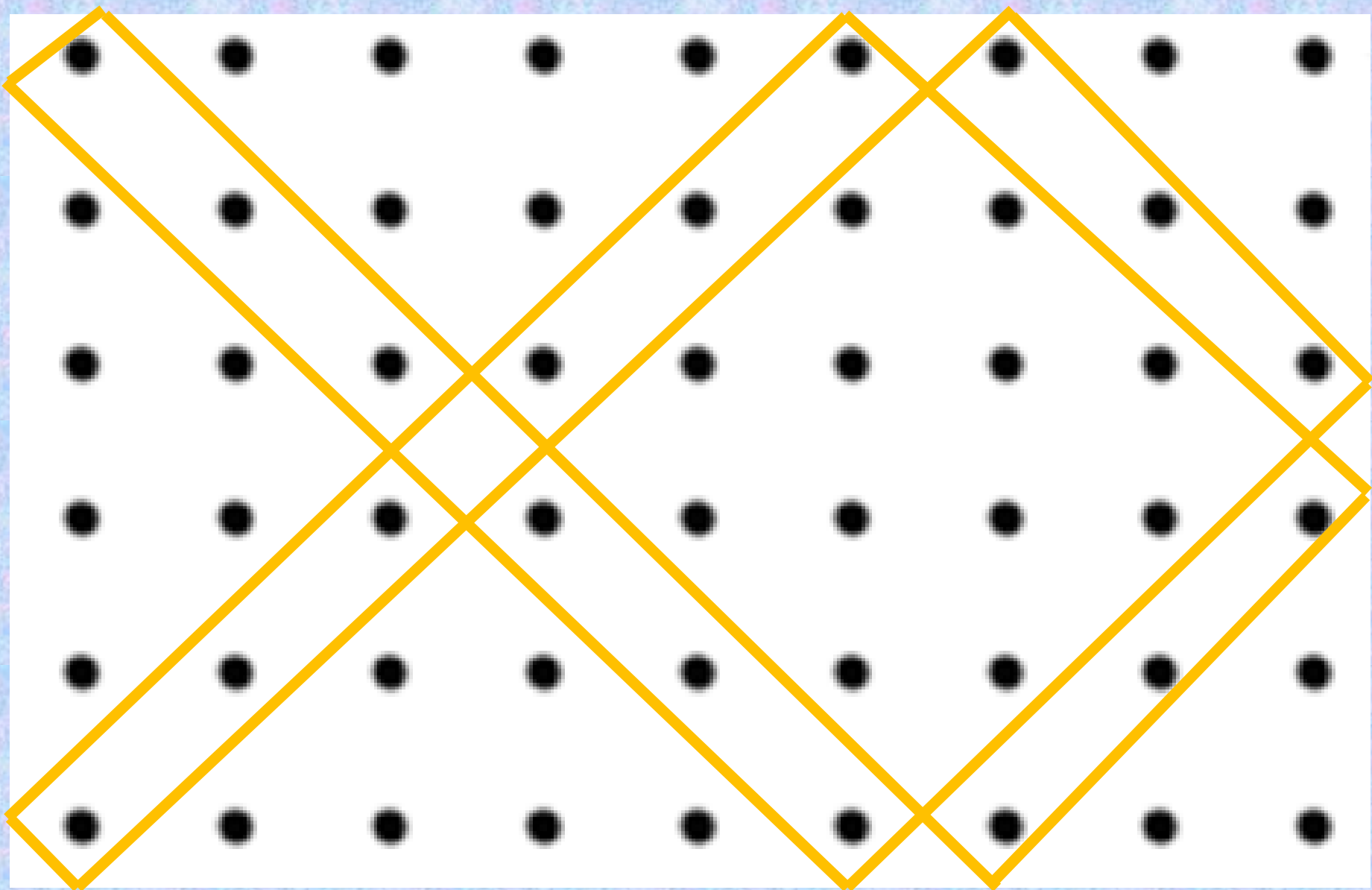
$$= 2$$

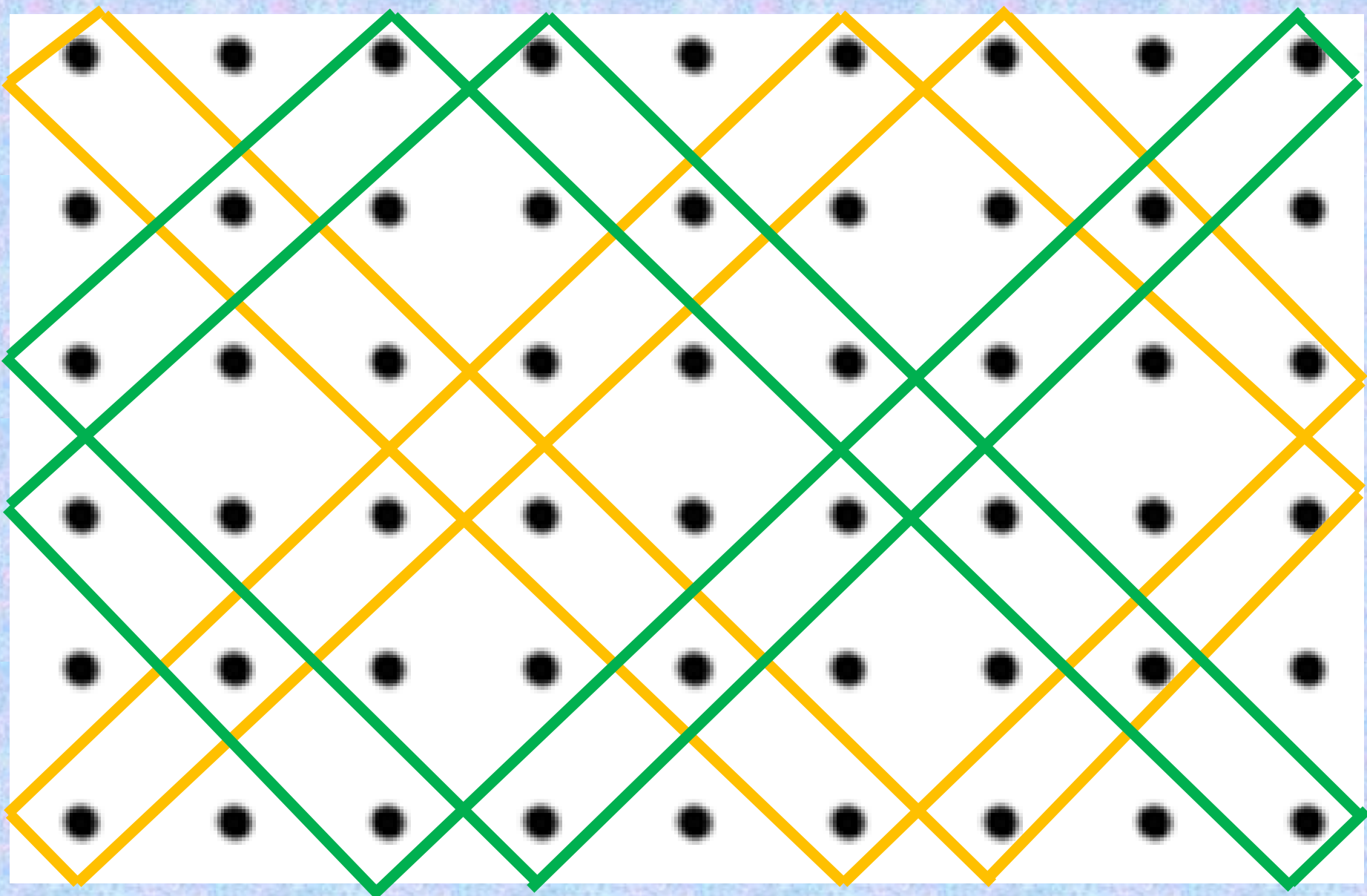


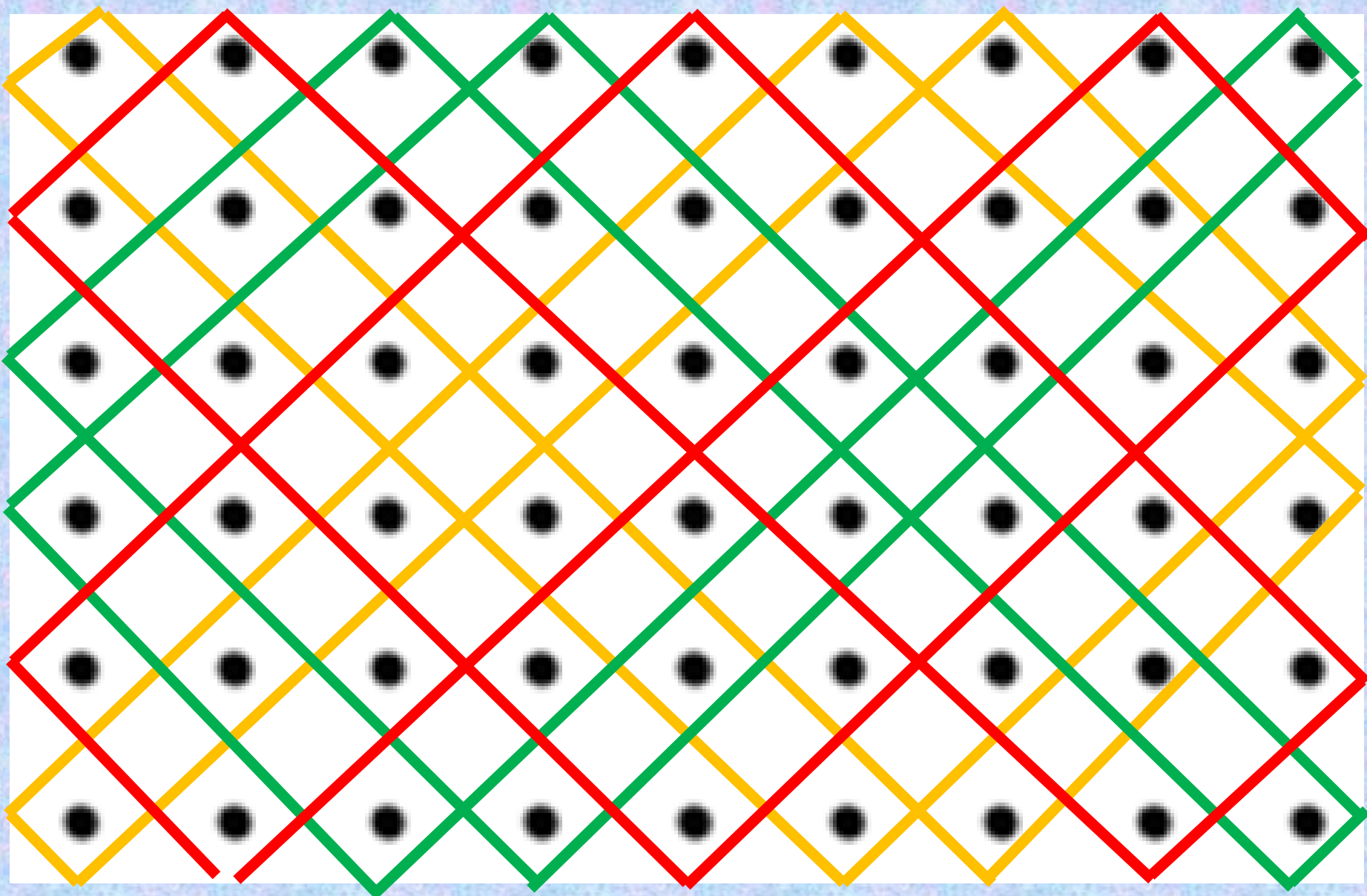
The total loop could be a loop in right side



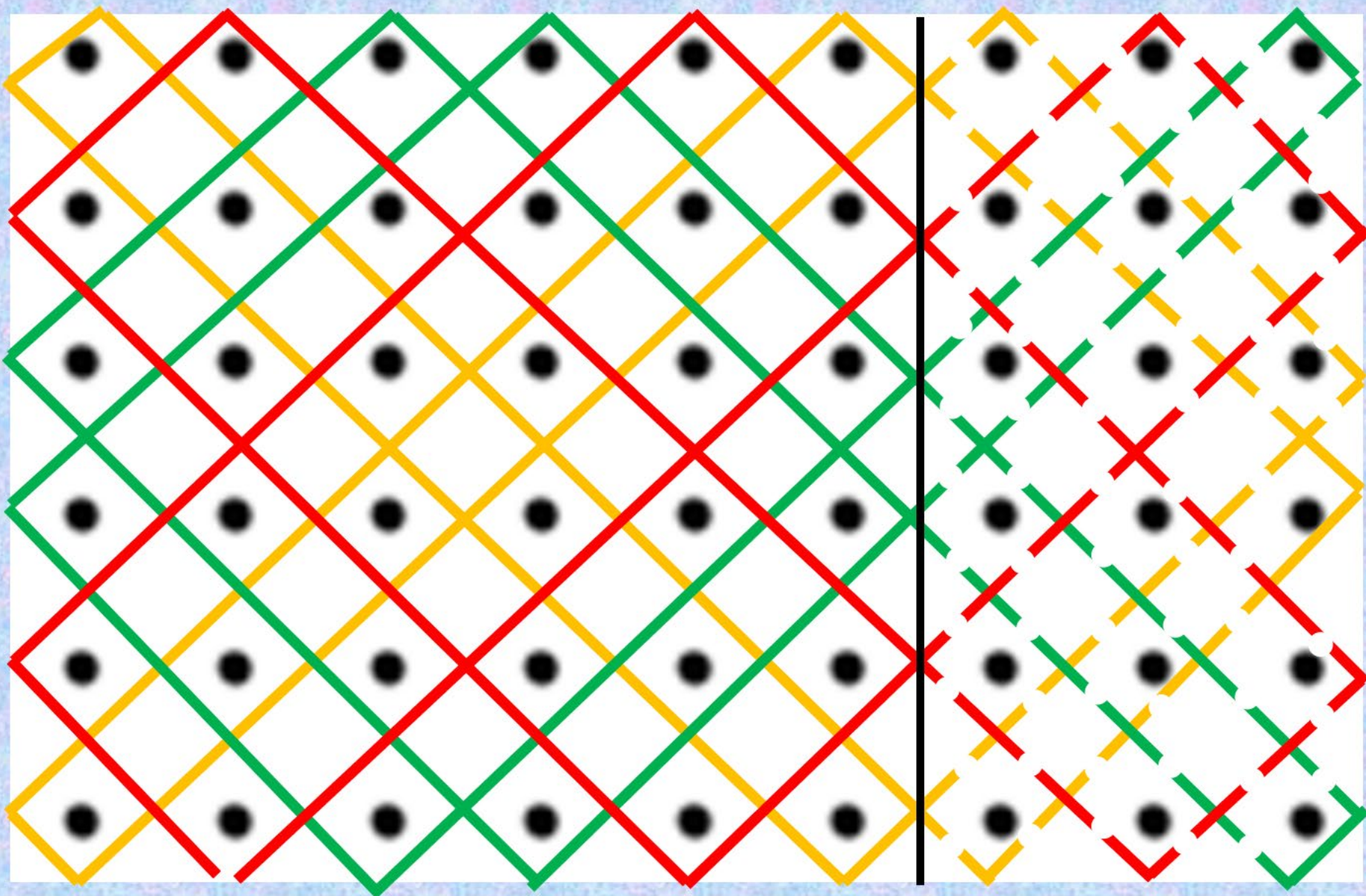
So 6 by 9 has as many loops as the 3 by 6.





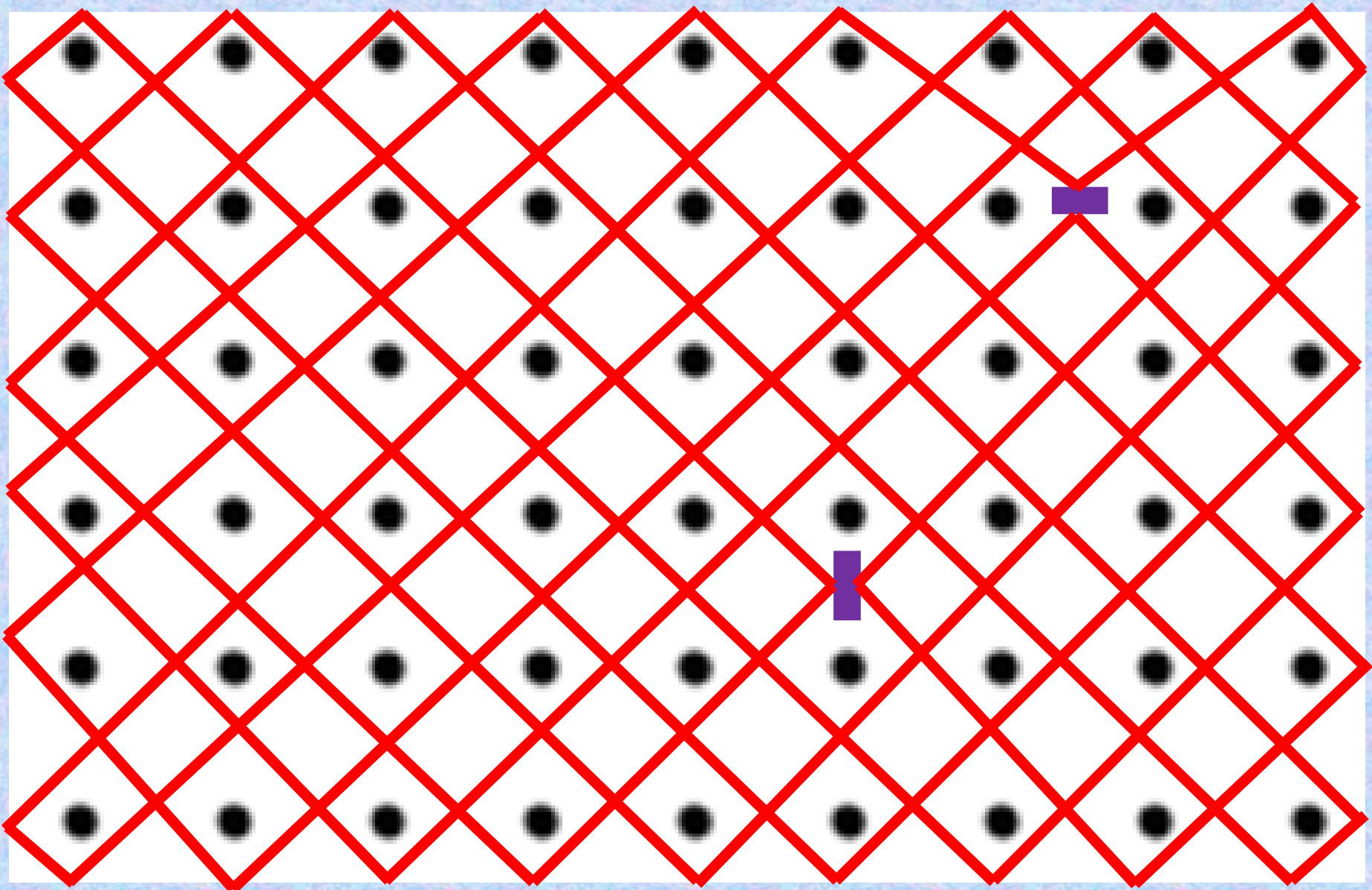


3 LOOPS!

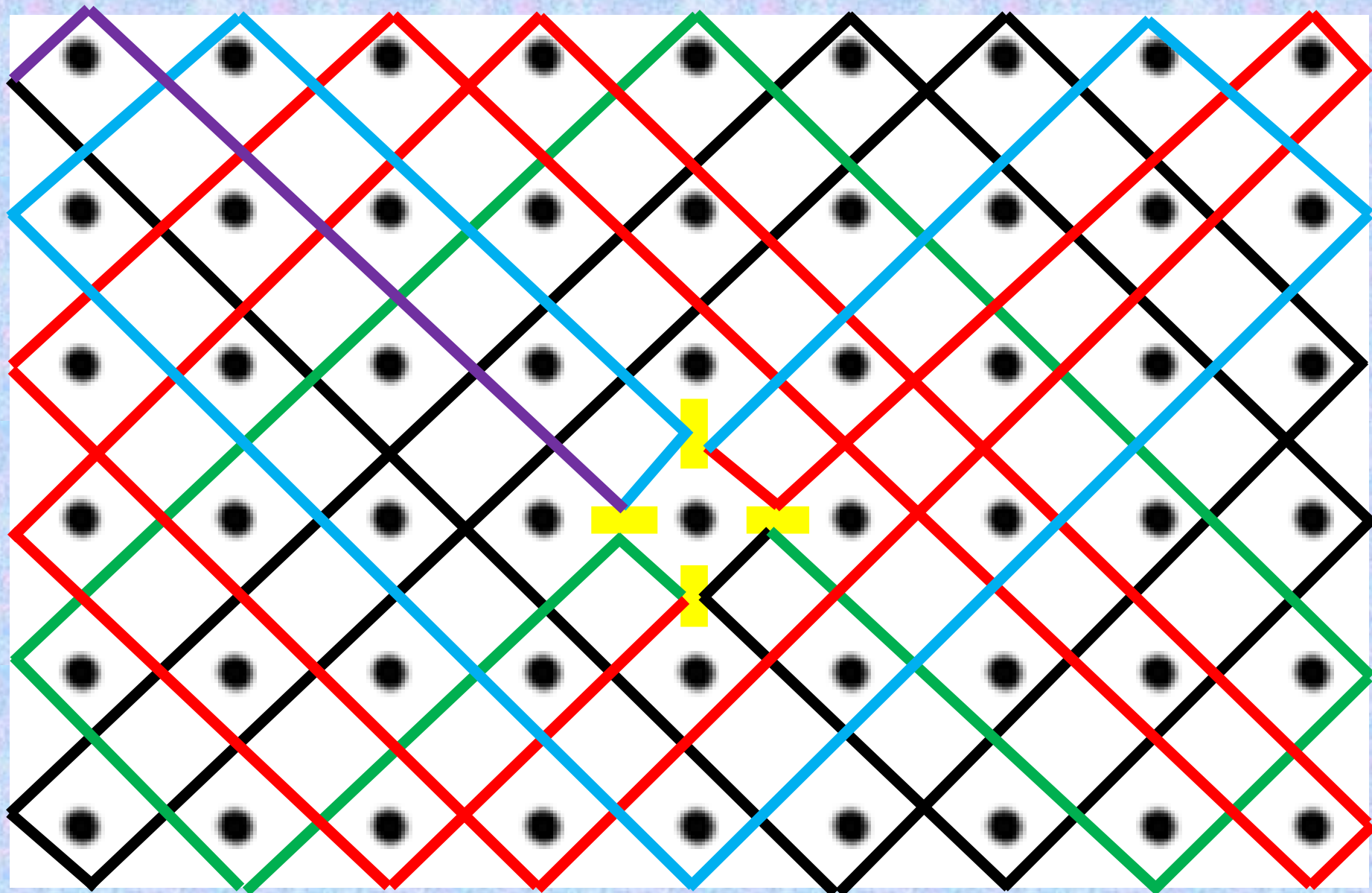


3 LOOPS!

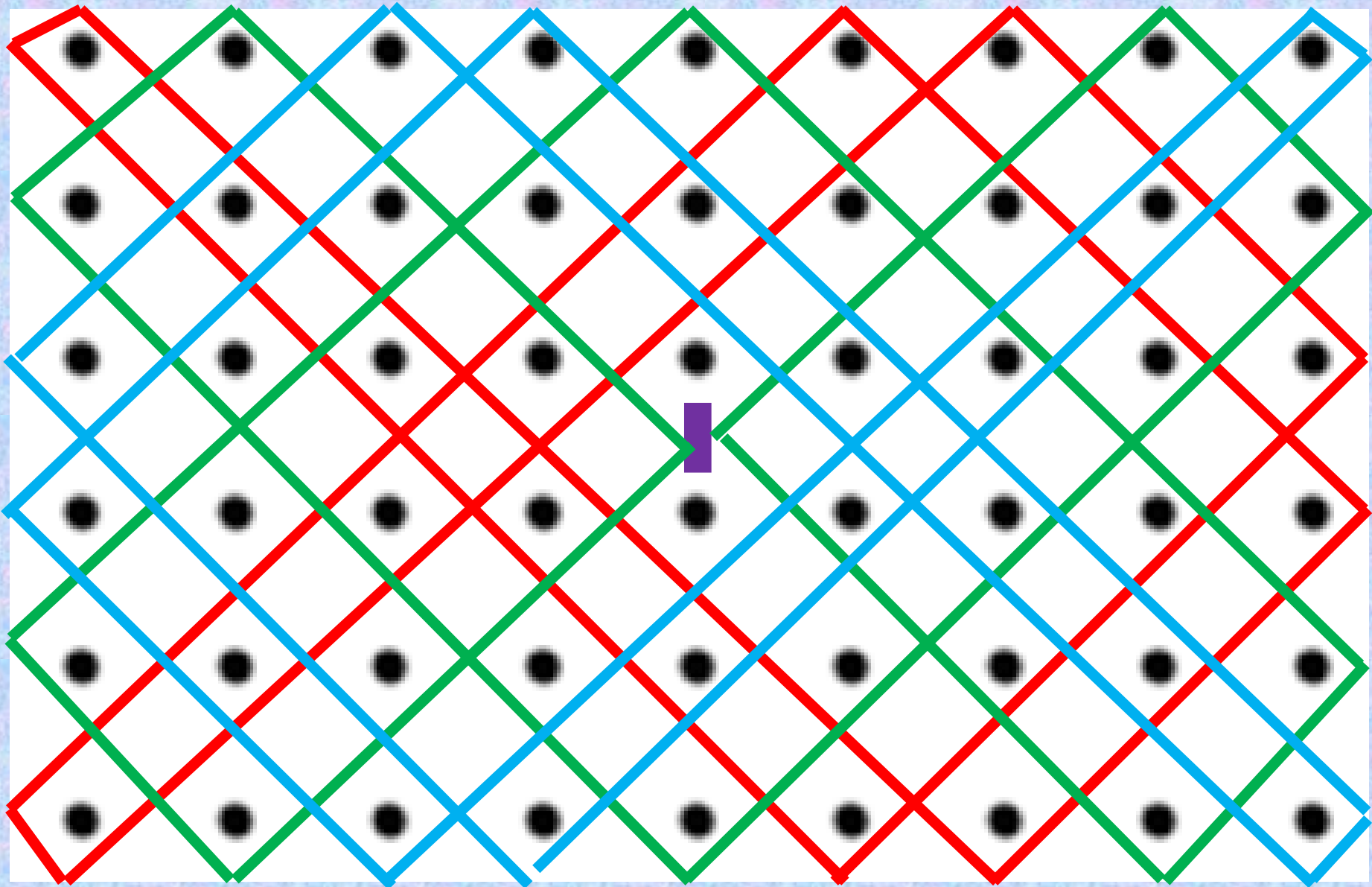
BARRIERS



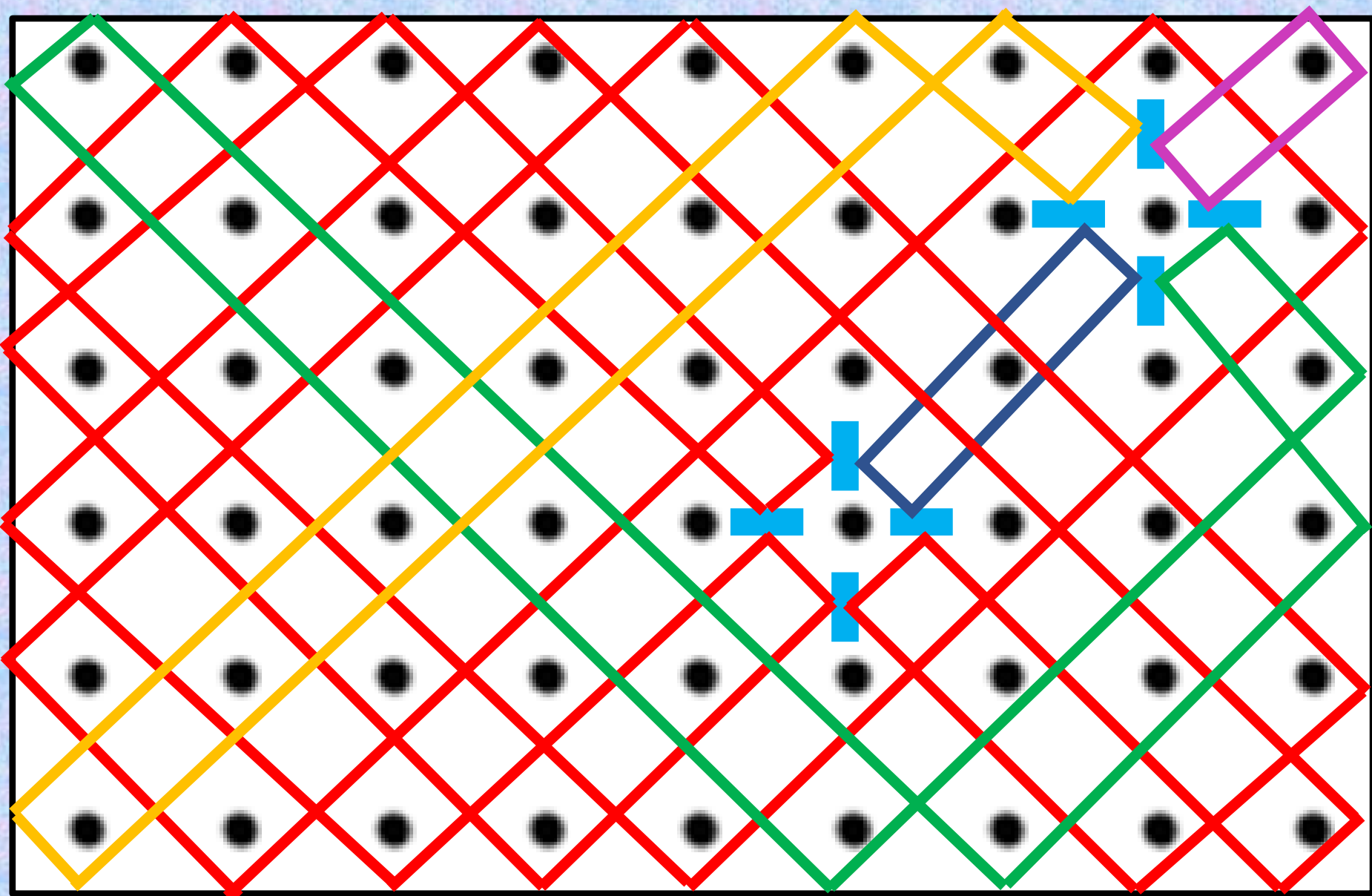
NOW ONLY ONE LOOP!



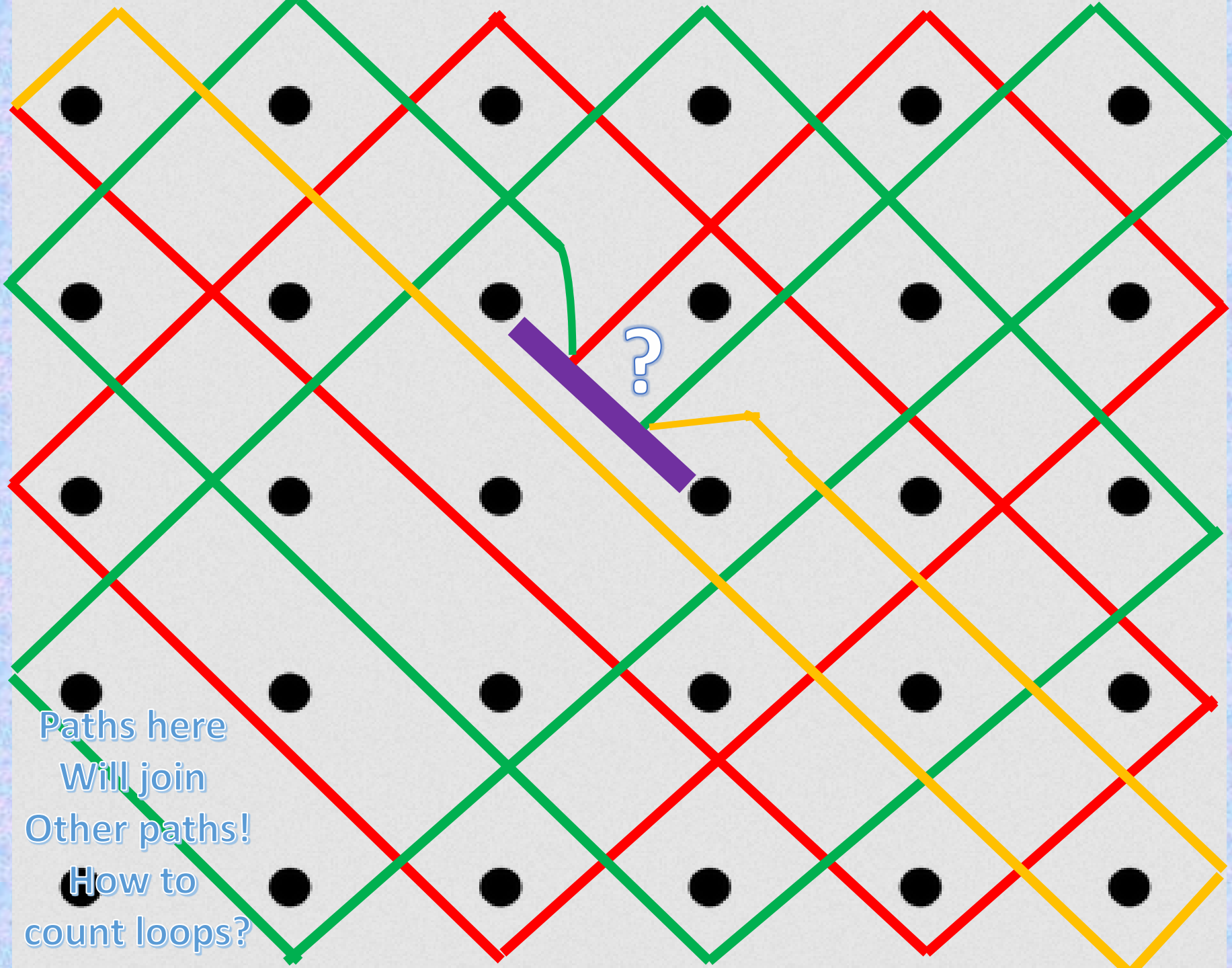
STILL ONLY ONE LOOP!



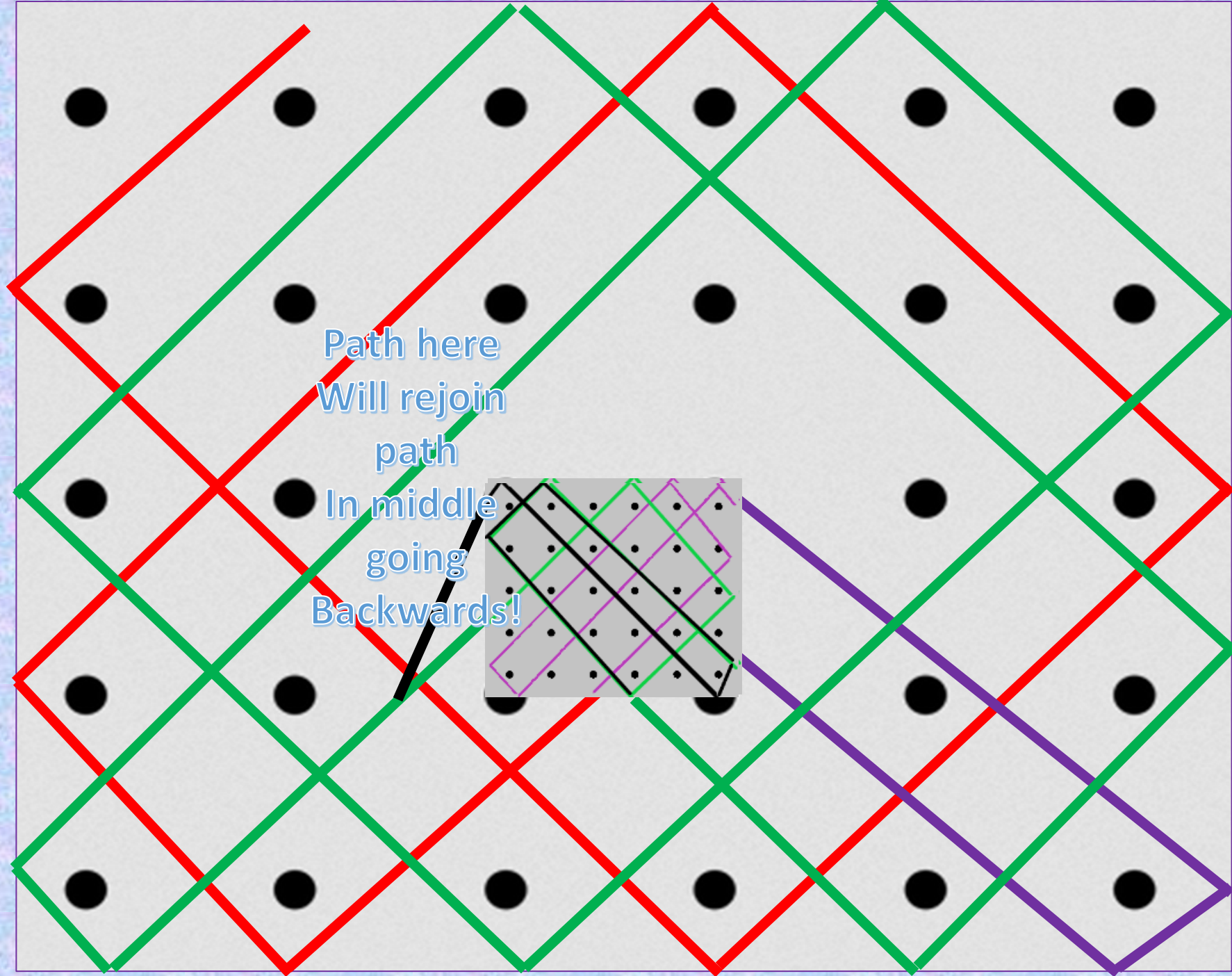
STILL 3 LOOPS!



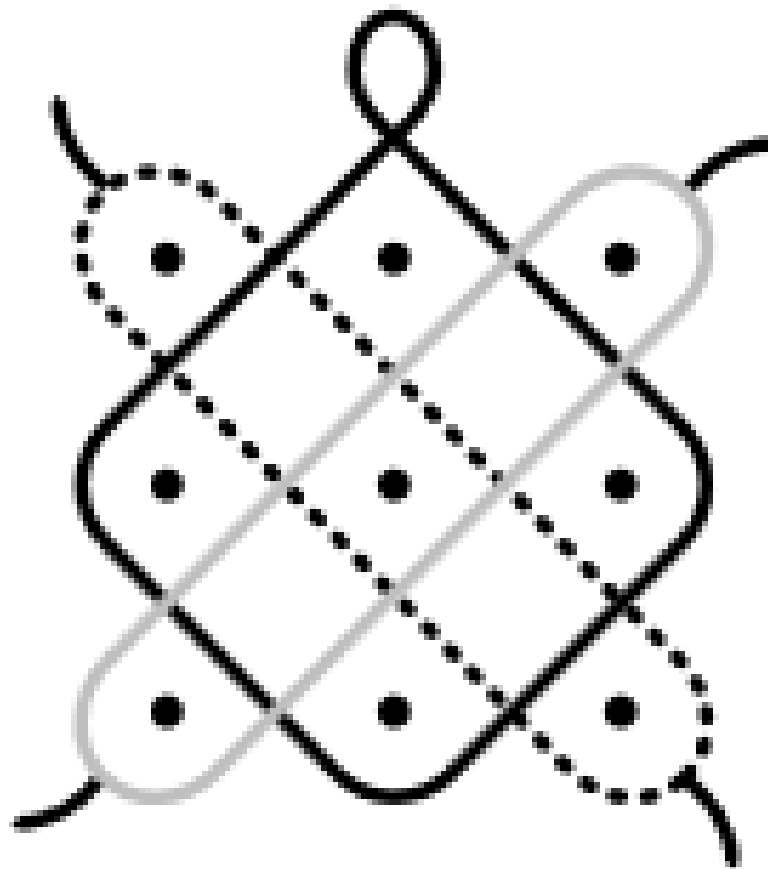
FIVE LOOPS!



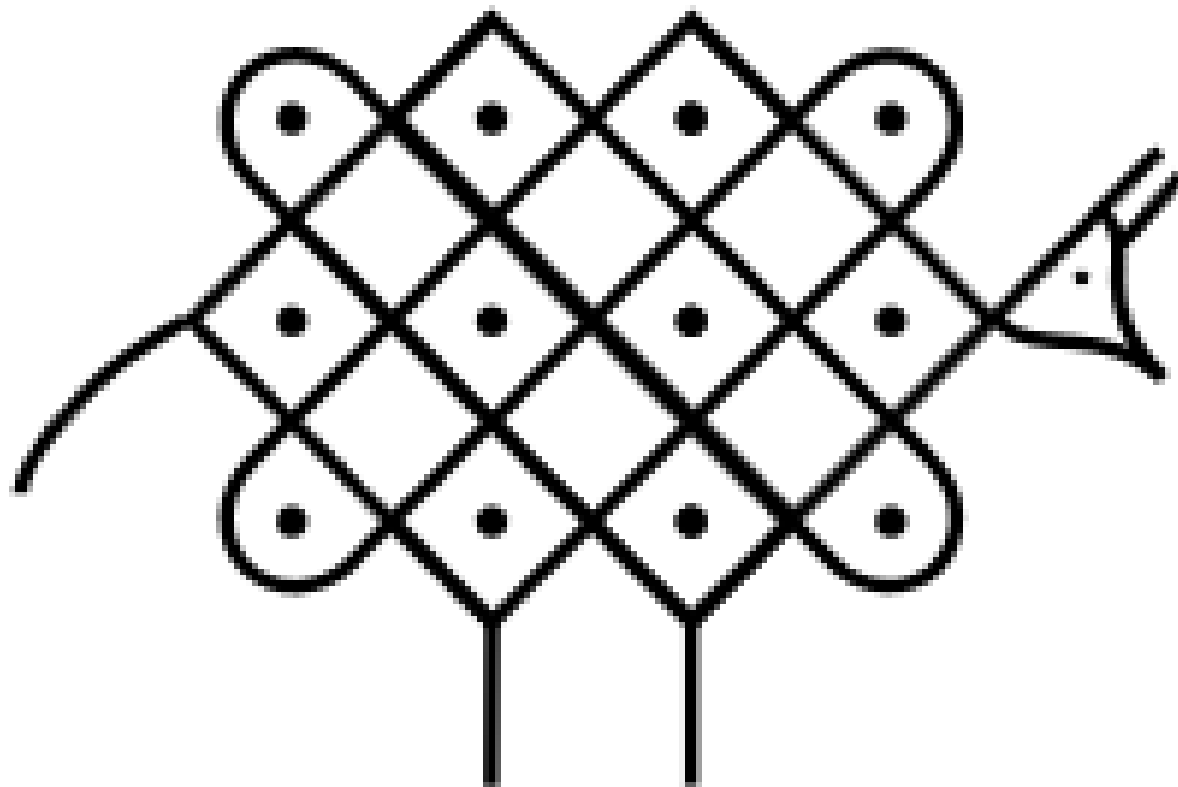
Paths here
Will join
Other paths!
How to
count loops?



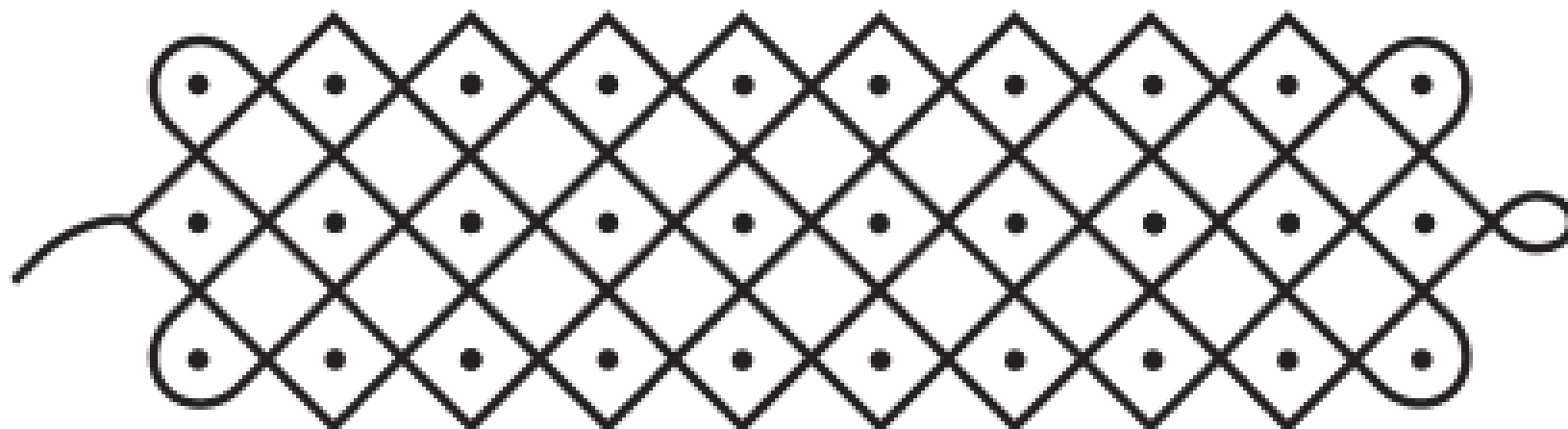
THE TURTLE



THE ANTELOPE

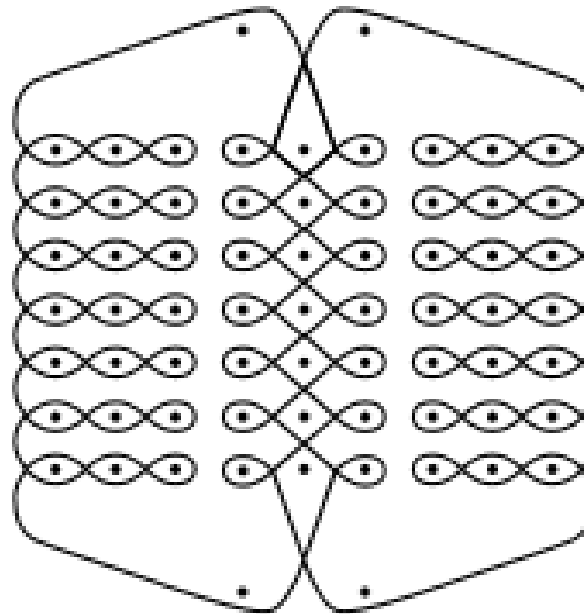


THE LIONESS

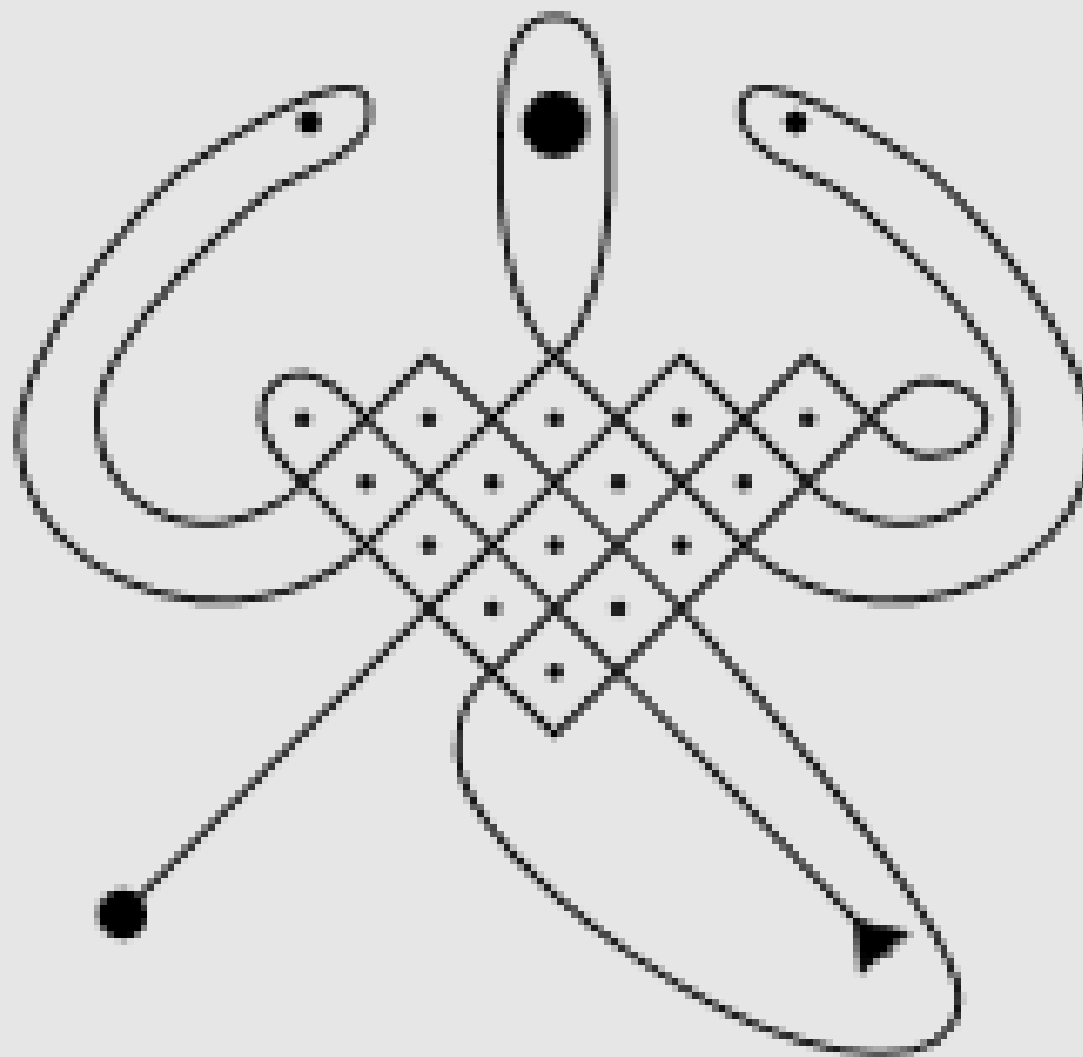


The Stork and the Leopard

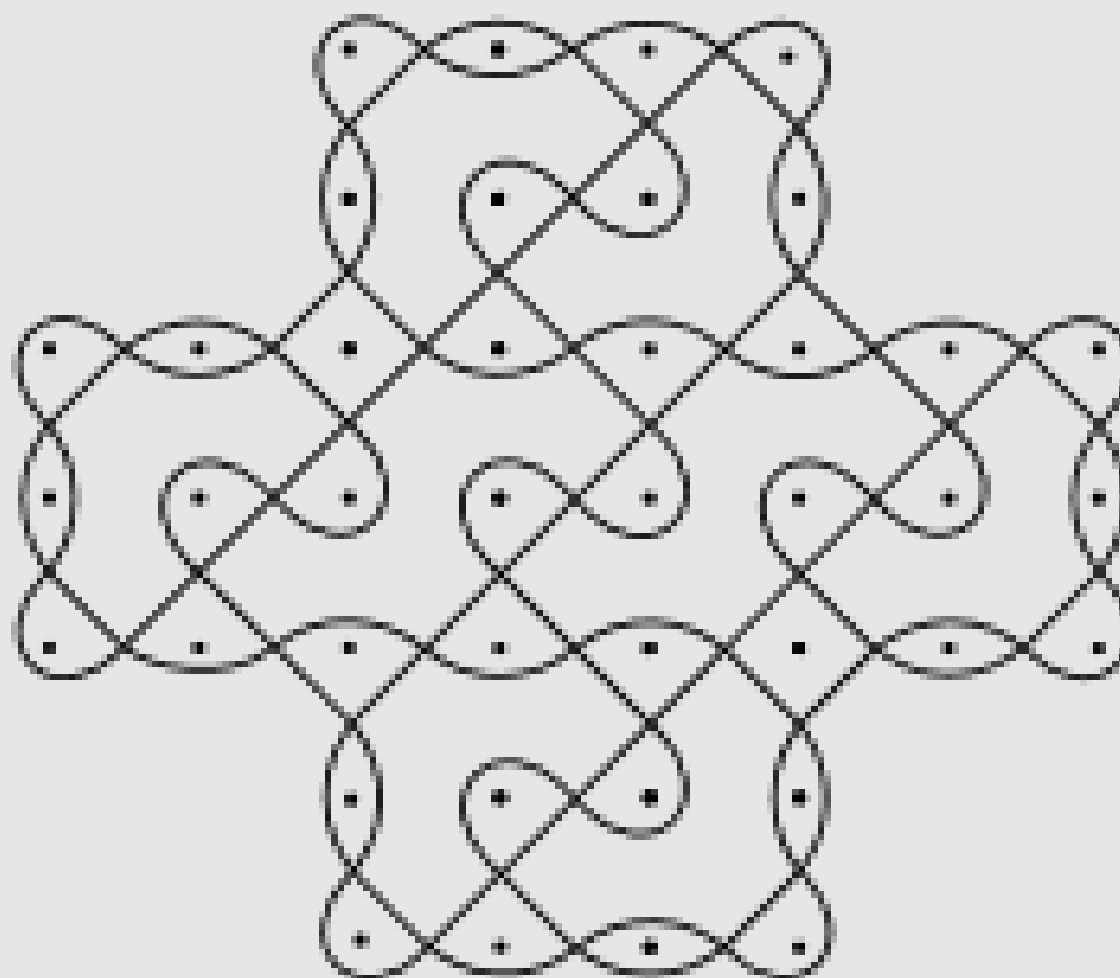
Once the leopard Kajama asked the stork Kumbi for some feathers to line his den. Some time later, the stork asked the leopard for a piece of his skin. When Kajama granted the stork's request, he died. Kajama's sons tried to take revenge, but Kumbi, who knew the swamp very well, was able to escape.



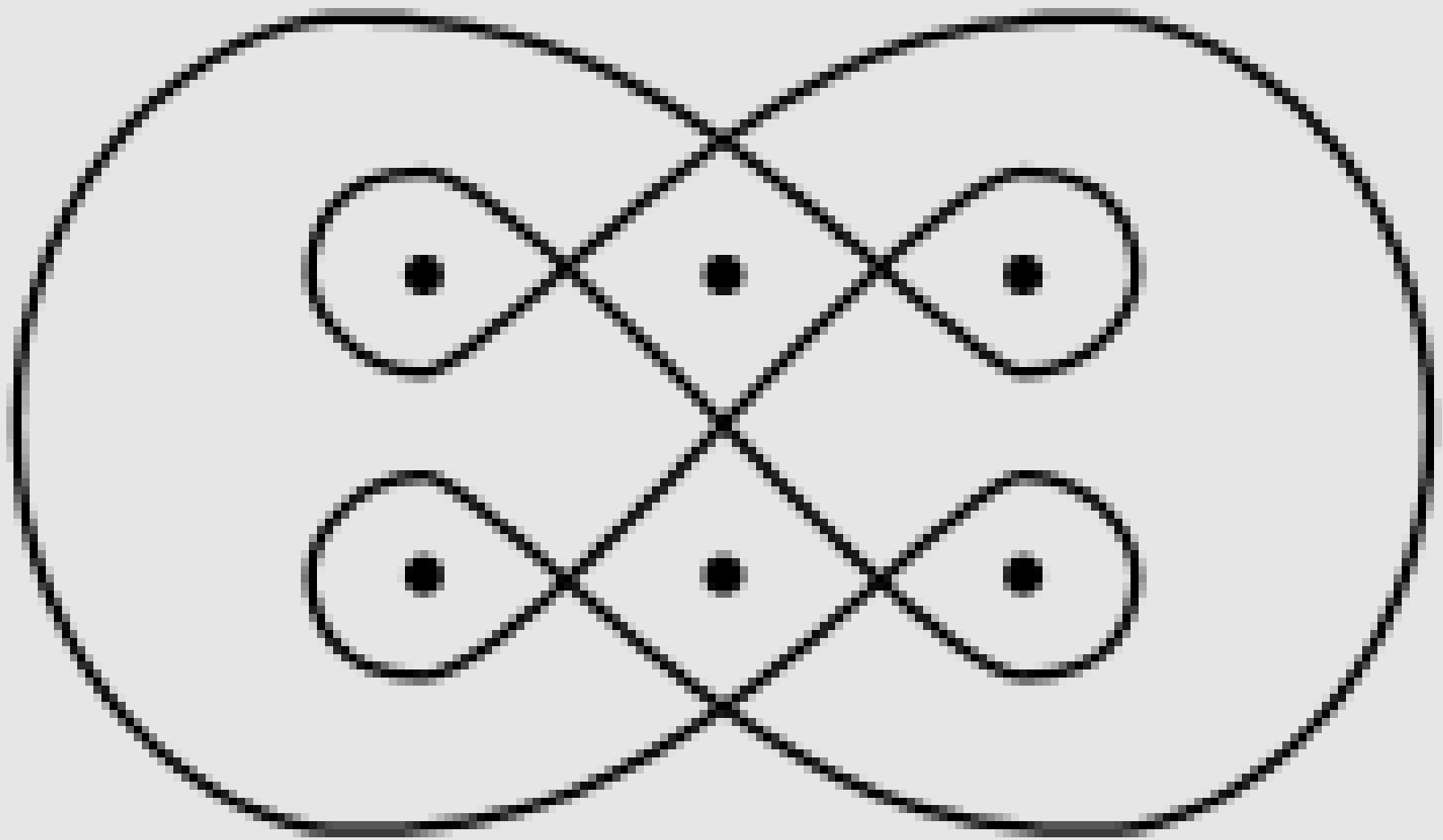
IN THIS DRAWING, THE WINDING LINE IS THE PATH OF THE FLEEING STORK, KUMBI. THE POINTS REPRESENT THE SWAMP THROUGH WHICH KUMBI MAKES HIS ESCAPE. THE DRAWING ACTUALLY CONSISTS OF TWO INTERTWINING CURVED LINES. YOU MAY WANT TO TRACE OVER THE ESCAPE ROUTE BY FOLLOWING IT WITH TWO DIFFERENT COLORED PENCILS ON A PHOTOCOPY OF THIS DRAWING.



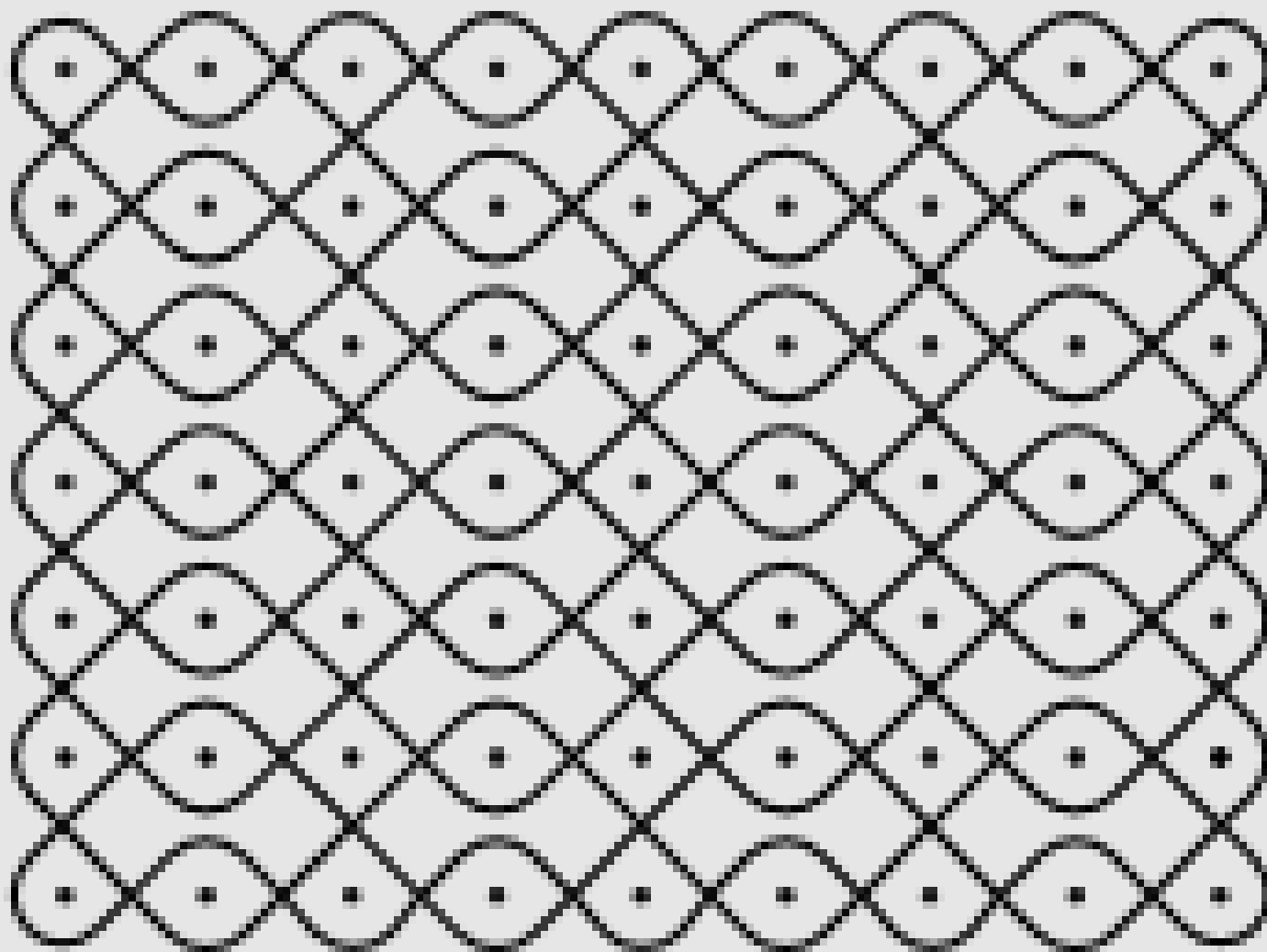
FLYING BIRD



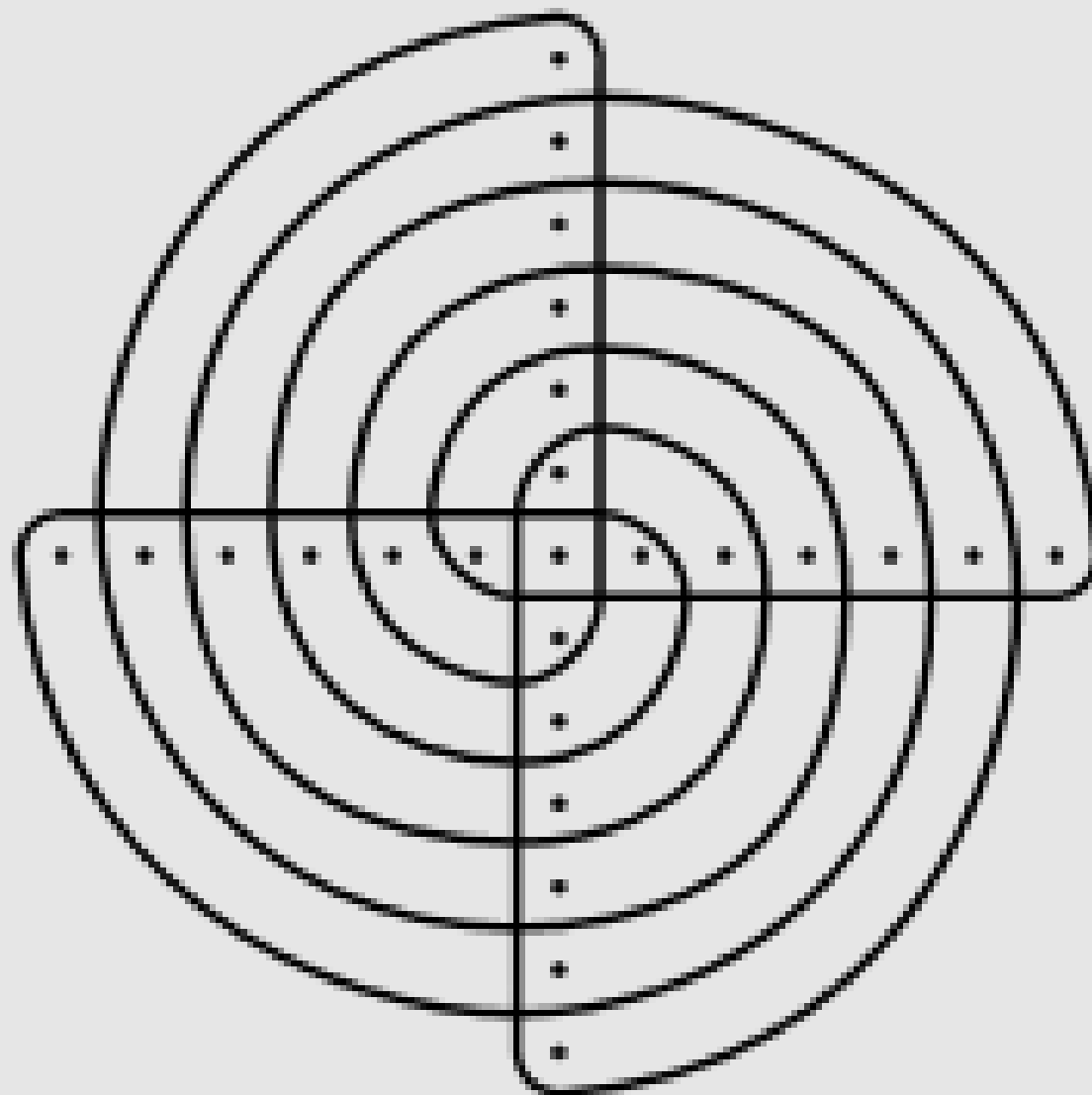
CHASED CHICKEN'S PATH



ANTELOPE FOOTPRINT



LARGE LION'S STOMACH



SPIDER IN ITS WEB