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Math Club for Young Women

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Project 2

Platonic and Archimedean Solids

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Content

Objectives

- Learn about the Platonic solids and some of their interesting properties.
- Learn about their associated truncated polyhedra.
- Learn where some of these solids can be found in the real world.
- Use this knowledge to make a truncated tetrahedron ornament and learn how you can make your own "math" jewelry.

2.1. Polygons

A **polygon** (or a **polygonal closed curve**) is a plane geometric figure consisting of a finite number of straight line segments connected at the ends to form a closed shape.

The name is derived from the Greek words:

- poly – "many" and
- gon – "angle"

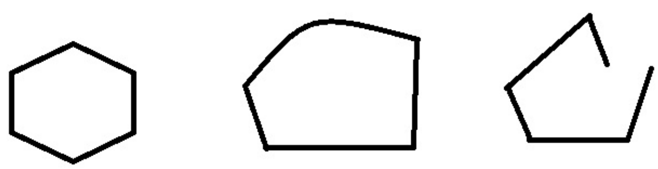
The line segments are called **edges** of the polygon.

The points where two edges meet are called **vertices** of the polygon.

Question: What kinds of polygons have you seen before?

Answer: ____

Question: Which of the figures below are polygons? Explain.

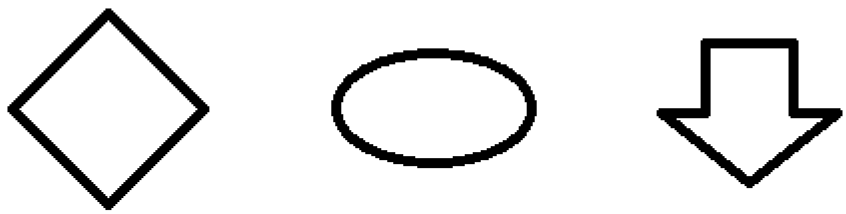


Answer: ___________ ___________ ___________

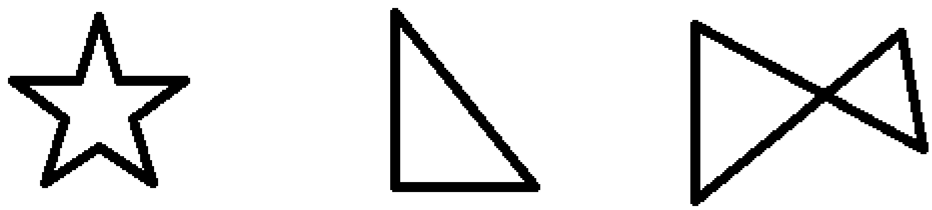
Types of polygons:

Regular/Irregular	A polygon that has all of its sides and angles equal is called a **regular polygon**. Otherwise it is called an **irregular polygon**.
Convex/Non-convex	A polygon with the property that all line segments connecting two points on the boundary only contain points of the boundary or interior points of the polygon is called a **convex polygon**. Otherwise it is called a **non-convex polygon**.
Simple/Complex	A **simple polygon** is a polygon that does not intersect itself. A **complex polygon** intersects itself. Note that each convex polygon is simple.

Exercise 1: For each of the figures below decide if they are polygons or not. If they are polygons decide if they are regular/convex/simple polygons. Explain.



a) *b)* *c)*



d) *e)* *f)*

Answer: *Regular polygons* ______________________

Convex polygons ______________________

Simple polygons ______________________

Question: How many different regular, convex polygons exist? Explain.

Answer: ____

2.2. Platonic Solids

A **polyhedron** is a closed, connected geometric figure in three dimensional space that consists of a finite number of flat polygonal regions (called **faces**). The faces meet along common line segments (called **edges**). And the edges meet at points (called **vertices**). Polyhedra are the generalization of the polygons in three dimensions.

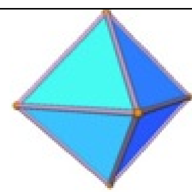
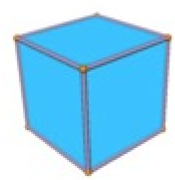
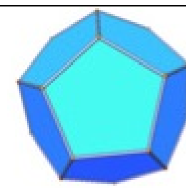
A **Platonic solid** is a polyhedron that is

- regular (all faces are congruent regular polygons, and the same number of faces meet at each vertex), and
- convex (every line segment connecting two points within the polyhedron is also within the polyhedron).

These polyhedra bear the name of the Greek philosopher Plato (427 -347 BC) who mentioned them in his *Timaeus*, but there is evidence that they had been discovered much earlier.

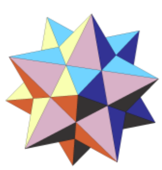
Exercise 2: Different polyhedra are given in the table below. Are they Platonic solids? Explain.

Answer: ____

Polyhedra[1]					
Type of polygonal faces	triangle	triangle	triangle	square	pentagon
Number of faces meeting at a vertex	3	5	4	3	3
Number of edges meeting at each vertex	3	5	4	3	3
Number of faces	4	20	8	6	12
Name	tetrahedron	icosahedron	octahedron	Cube (hexahedron)	dodecahedron

The name of each of these solids is obtained from the Greek word for the number of its faces.

Question: Is the polyhedron[2] on the right a Platonic solid? Explain.



*Answer:*____

Question: How many different Platonic solids exist? Explain.

Answer: ____

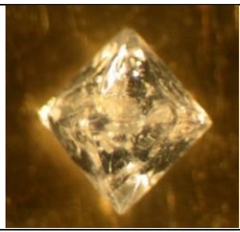
[1] http://www.pbs.org/wgbh/nova/physics/blog/2011/12/beautiful-losers-platos-geometry-of-elements/

[2] http://upload.wikimedia.org/wikipedia/commons/7/75/First_stellation_of_dodecahedron.png

Platonic solids in the world around us:

Question: Do you know where these Platonic solids can be found in everyday life?

Answer: ____

Mineral crystals Appear in form of tetrahedron, cube and octahedron (mineral diamond of fluoride crystal)) Beta Quartz crystal shown on the right[3].	
"Fool's Gold," or iron pyrite Forms crystals much like the dodecahedron[4], but their pentagonal faces are not regular.	
Dice[5], Rubik's cube (and other games and puzzles) Can be found in all five forms.	

Properties of the Platonic solids:

1. **Symmetry**

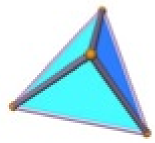
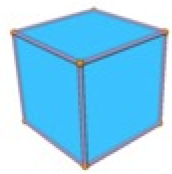
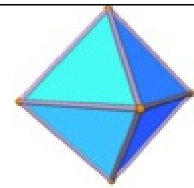
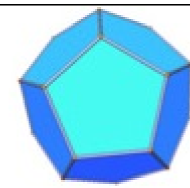
Question: Are the Platonic solids symmetric? Explain.

Answer: ____

[3] http://paradigmsearch.hubpages.com/hub/Crystal-Crystals-Crystalline-Rocks-and-Minerals-for-Rock-Hounds#slide4764301
[4] http://www.mindat.org/photo-250186.html
[5] http://upload.wikimedia.org/wikipedia/commons/c/c7/BluePlatonicDice.jpg

2. **Euler's polyhedron formula**

Exercise 3: Fill in the table below.

Polyhedron					
F = # of faces	4	6	8	12	20
V = # of vertices			6	20	12
E = # of edges			12	30	30
F+V-E					

Question: What can you say about the quantity F+V-E?

Answer: ____

The formula you've just discovered is called Euler's polyhedron formula:

For any convex polyhedron, the number of faces plus the number of vertices minus the number of edges is a constant (equal to 2), i.e.

$$F+V-E=2$$

This formula was independently discovered by Euler and Descartes.

Question: Using the table above, compare the number of faces and vertices of the Platonic solids. What can you observe?

Answer: ____

This is another interesting property of the Platonic solids:

3. The Platonic solids with the property you just discussed are called **dual** to each other. Hence, every Platonic solid has its **dual** that is also a Platonic solid with vertices that correspond to the faces of the original Platonic solid and vice versa.

Exercise 4: Draw the dual of the tetrahedron[6] following the steps listed below:

a. Draw the center of each face (these are the vertices of the dual);

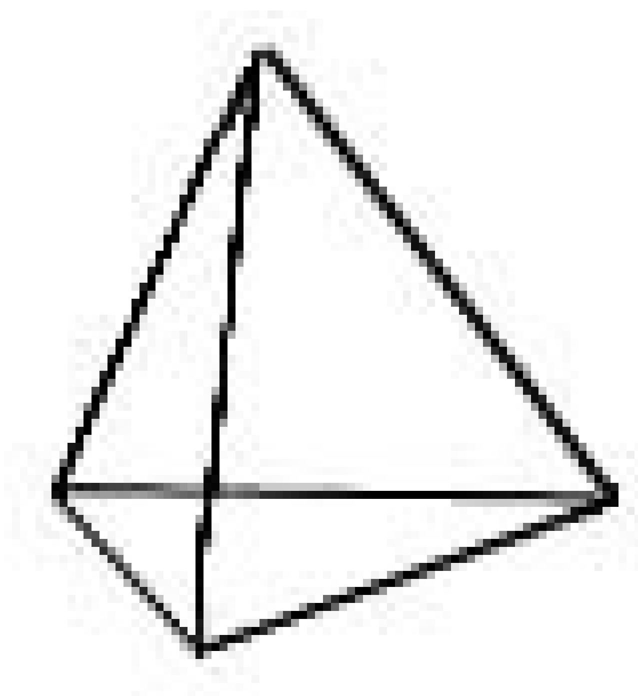
b. Draw segments connecting any pair of centers (these are the edges of the dual).

The polyhedron you just drew is the dual of the tetrahedron.

Question: Which Platonic solid is the dual of the tetrahedron?

*Answer:*________________________

The tetrahedron is called ***self-dual*** since its dual is also a tetrahedron.



[6] http://classicalastrologer.me/2013/06/30/venus-the-cube-in-arabian-cosmology/

Example 5: Draw the dual of the cube[7] following the steps listed below:

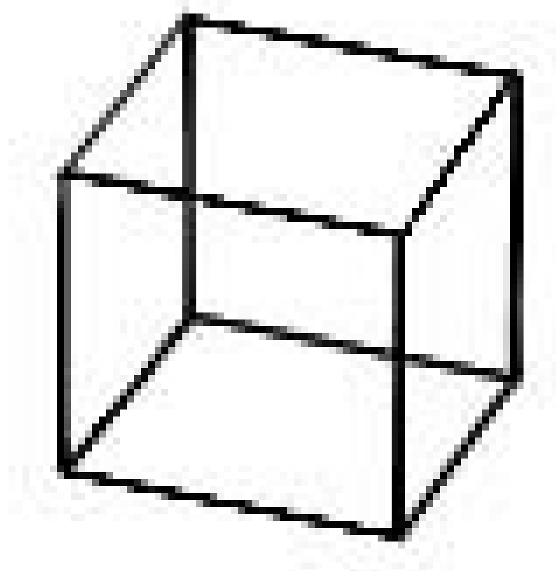
a. Draw the center of each face (these are the vertices of the dual);

b. Draw segments connecting the centers of adjacent faces (these are the edges of the dual).

The polyhedron you just drew is the dual of the cube.

Question: Which Platonic solid is the dual of the cube?

*Answer:*________________________



Exercise 6: Draw the dual of the octahedron [8]following the steps listed below:

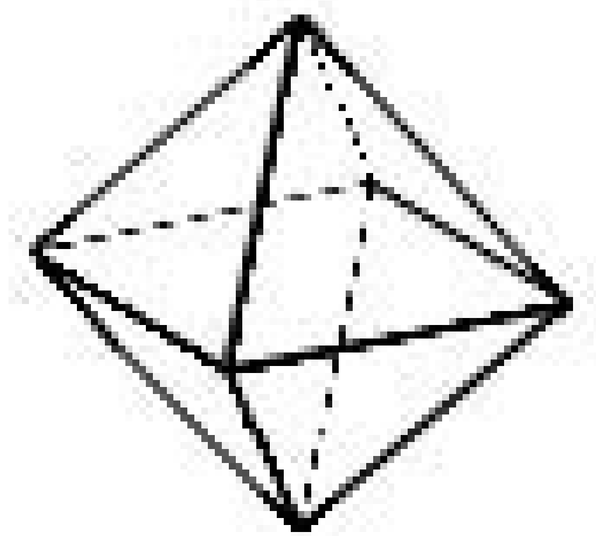
a. Draw the center of each face;

b. Draw segments connecting the centers of adjacent faces.

The polyhedron you just drew is the dual of the octahedron.

Question: Which Platonic solid is the dual of the octahedron?

*Answer:*________________________



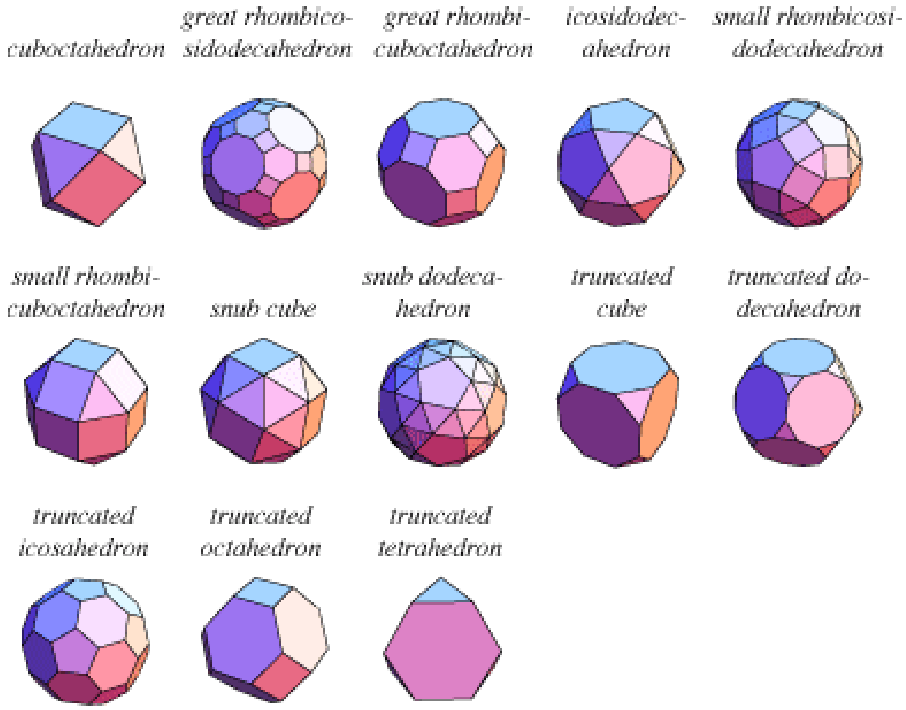
Question: Which polyhedron is the dual of the dodecahedron? What is the dual of the icosahedron?

Answer: ____

[7] http://classicalastrologer.me/2013/06/30/venus-the-cube-in-arabian-cosmology/
[8] http://classicalastrologer.me/2013/06/30/venus-the-cube-in-arabian-cosmology/

2.3. Archimedean Solids

Archimedean solids are *semi-regular polyhedra.* They have faces that are regular polygons of more than one type and identical vertices. There are a total of 13 Archimedean solids[9] (shown below).



They are attributed to Archimedes, even though it appears that the first person to describe all of them was Kepler.

Question: Are the Archimedean solids symmetric?

Answer: ____

Some of the Archimedean solids can be obtained by truncation of the Platonic solids.

A **truncation of a polyhedron** is an operation that cuts portions of the polyhedron around its vertices, creating a new face in place of each vertex.

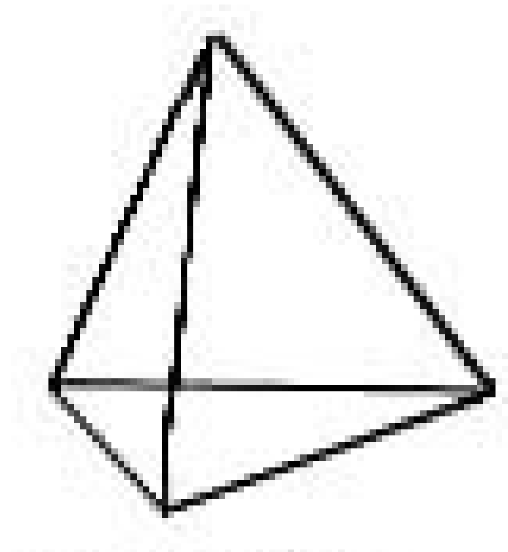
We will be interested in a special truncation of a Platonic solid - called a **uniform truncation**: truncate the Platonic solids until the original faces become regular polygons with double the number of the sides.

[9] http://numb3rs.wolfram.com/406/images/Archimedean.gif

Truncated tetrahedron:

It can be constructed by truncating (cutting off) portions around the four vertices of a regular tetrahedron such that one third of the original edge length is cut of at each end.

Exercise 7: Truncate the tetrahedron below to obtain the truncated tetrahedron.



Question: What is the shape of the boundary of each cut?

Answer: ______________________________

Question: What types of faces will the truncated tetrahedron have?

Answer: ______________________________

Question: How many faces of each type will it have?

Answer: ______________________________

Question: How many faces meet at each vertex? *Answer:* ______________________________

Question: Are they of same type? *Answer:* ______________________________

Question: If not, how many of each type meet at a vertex?

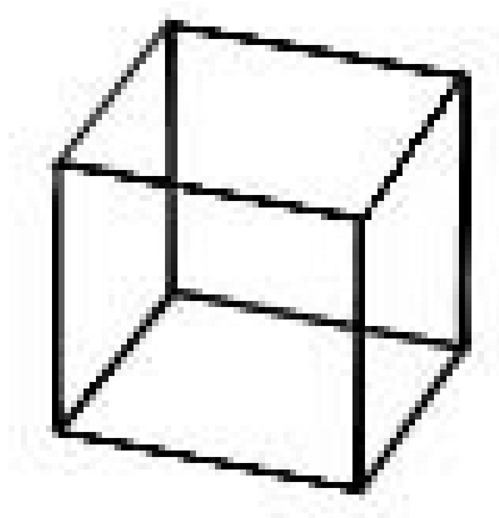
Answer: ____

	F= # of Faces	V= # of Vertices	E= # of Edges	F+V-E
Truncated Tetrahedron				

- Application in architecture

Truncated cube:

Exercise 8: Truncate the cube below to obtain the truncated cube.



Question: What is the shape of the boundary of each cut?

Answer: ______________________________

Question: What types of faces will the truncated cube have?

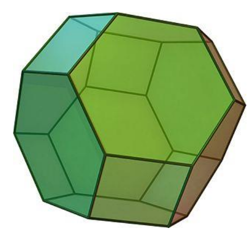
Answer: ______________________________

Question: How many faces of each type will it have?

Answer: ______________________________

	F = # of Faces	V = # of Vertices	E = # of Edges	F+V-E
Truncated Cube		24	36	

Truncated octahedron:



This polyhedron (shown[10] on the right) can be obtained by truncating the octahedron so that:

- the triangular faces become _________________, and
- the vertices are replaced by faces that are _________________.

	F = # of Faces	V = # of Vertices	E = # of Edges	F+V-E
Truncated Octahedron		24	36	

Recall that the cube and the octahedron are duals to each other.

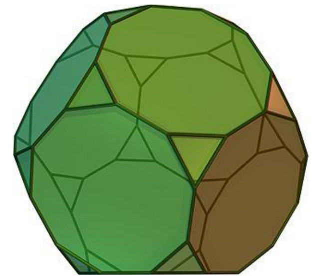
Question: Compare the number of faces, edges and vertices of their associated truncated polyhedra. What can you observe? Why?

Answer: ____

[10] http://en.wikipedia.org/wiki/Truncated_octahedron

Truncated dodecahedron:

This polyhedron (shown[11] on the right) can be obtained from a dodecahedron by truncation so that:

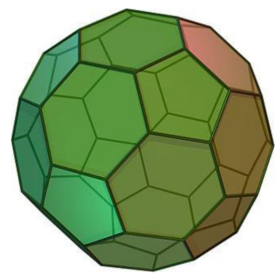


- the pentagon faces become _______________, and
- the vertices are replaced by faces that are ________________.

	F = # of Faces	V = # of Vertices	E = # of Edges	F+V-E
Truncated Dodecahedron	32	60	90	

Truncated icosahedron:

This polyhedron (shown[12] on the right) can be formed from an icosahedron by truncation so that:



- the triangular faces become _______________, and
- the vertices are replaced by faces that are ________________.

To fill the table below, recall that the dodecahedron and the icosahedron were dual to each other.

	F = # of Faces	V = # of Vertices	E = # of Edges	F+V-E
Truncated Icosahedron				

Question: Can you think of any object that looks like the truncated icosahedron?

Answer: ____

Question: Does Euler's polyhedron formula hold for the truncated polyhedra? Why?

Answer: ____

Note: The truncated octahedron is the only Archimedean solid that has an interesting property: identical copies of it can "fill" the space without gaps (something that is called "tessellation" – coming up at Math Girls Rock! next semester!)

[11] http://en.wikipedia.org/wiki/Truncated_dodecahedron
[12] http://en.wikipedia.org/wiki/Truncated_icosahedron

Reference:

1. Edward B. Burger, Michael Starbird, *The Heart of Mathematics: An invitation to effective thinking*, John Wiley & Sons, Inc., 2010.
2. Daud Sutton, *Platonic and Archimedean solids*, Wooden Brooks, 2005.
3. Platonic Solid (Wikipedia, the free encyclopedia) (http://en.wikipedia.org/wiki/Platonic_solid)
4. Archimedean Solid (Wolfram MathWorld) (http://mathworld.wolfram.com/ArchimedeanSolid.html)

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Research Articles

Mentoring Female Undergraduates in Research-Centered Outreach

Violeta Vasilevska & Carolyn Hamilton

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Abstract

Math Girls *R*ock! is a year-long, two-tiered mathematics mentoring program that prepares female undergraduate mentors to facilitate high school girls' engagement in challenging mathematics concepts through a dynamic after-school program. In this article, we describe the distinct educational component of this program in which female faculty members mentor female undergraduate mathematics and mathematics education students in researching and developing the content to be presented at the high schools. In addition, we discuss some of the feedback collected from program participants about various aspects of this program relating to the involvement of the undergraduate students. In conclusion, we share advice for those interested in starting and running similar outreach programs.

Keywords: K–12 math outreach mentoring female role models women in math undergraduate research near-peer

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DISCLOSURE STATEMENT

No potential conflict of interest was reported by the author(s).

Notes

1 *Near-peer* mentoring is used to describe communication or activity between people or organizations that are very similar or nearly equal. A *near-peer* teaching model is one in which a more experienced student acts as the instructor for less experienced students. [5]

2 The stylized R is meant to represent the set of real numbers.

3 In progress at this time.

Additional information

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Notes on contributors

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Violeta Vasilevska is Professor in the Department of Mathematics at Utah Valley University (UVU). She received her Ph.D. from the University of Tennessee and joined Utah Valley University in 2010. Her research interests are diverse, ranging from topics in pure mathematics (topology, algebra, and graph theory) to topics in math education/SoTL. She loves to engage her students in actively building their knowledge in class as well as outside of the classroom setting. Vasilevska co-founded and has been running the UVU outreach program Math Girls *R*ock! for high school women since 2011. In her spare time, she loves doing Origami, reading, hiking, and traveling.

Carolyn Hamilton

Carolyn Hamilton is Associate Professor in the Department of Strategic Management and Operations at Utah Valley University. She received her M.S. in Mathematics from the University of California, Riverside and joined the UVU Mathematics Department in 1993, chairing the department from 2004 to 2009. Hamilton co-lead the Math Girls *R*ock! program from 2011 to 2015. Her awards include the UVU Trustees Award of Excellence and the Deans Award for Excellence in Teaching. She currently coordinates the business calculus program at the Woodbury School of Business.

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