



The Global Math Circle

# Fractals, Dots, and Blocks

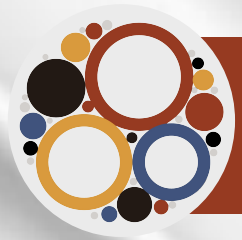
Visual-Spatial Pathways to Sequences & Series

Taylor Yeracaris

Special Session: Math Circles for Makers, Creators, and Artists

Joint Mathematics Meetings (Seattle)

January 8, 2025

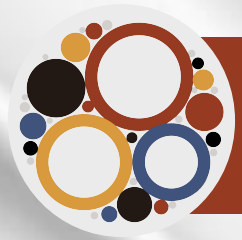


# Bob and Ellen's vision

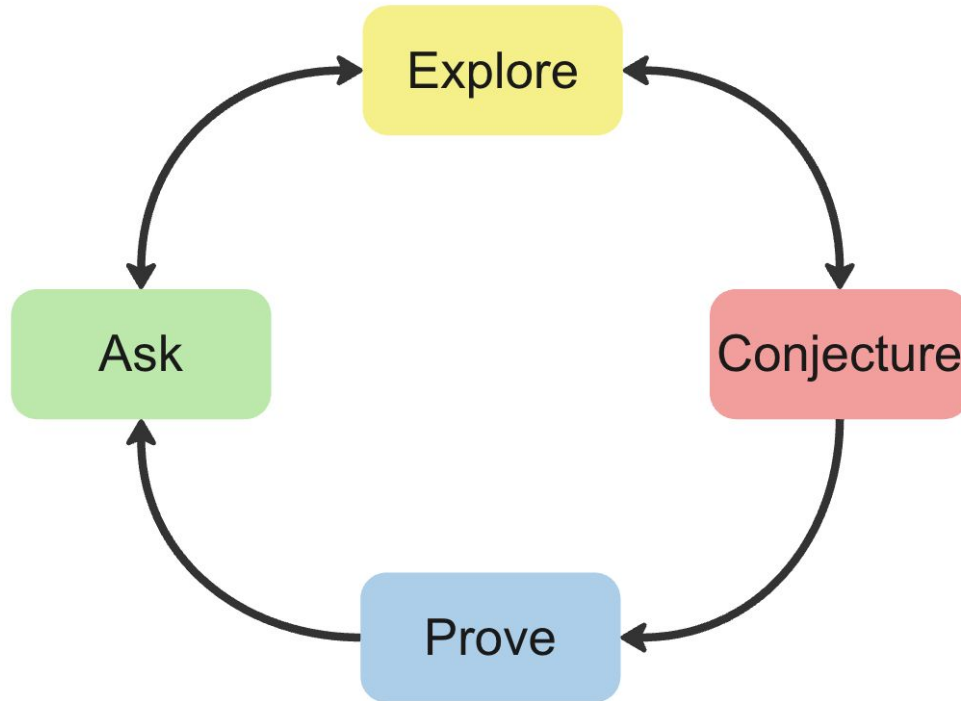


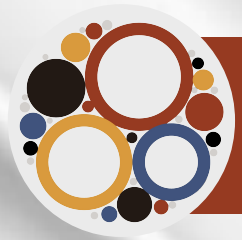
Engage all kids in math circles in order to share the creative, collaborative nature of mathematics, our lost native language.





# A true experience of doing mathematics

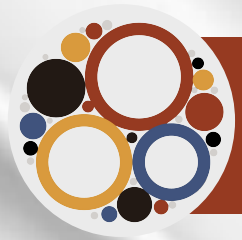




## In this talk

- A framework for preparing circles that give this experience
- Specific artistic and visual-spatial problems for topics in combinatorics

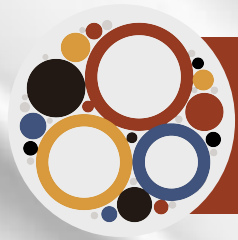




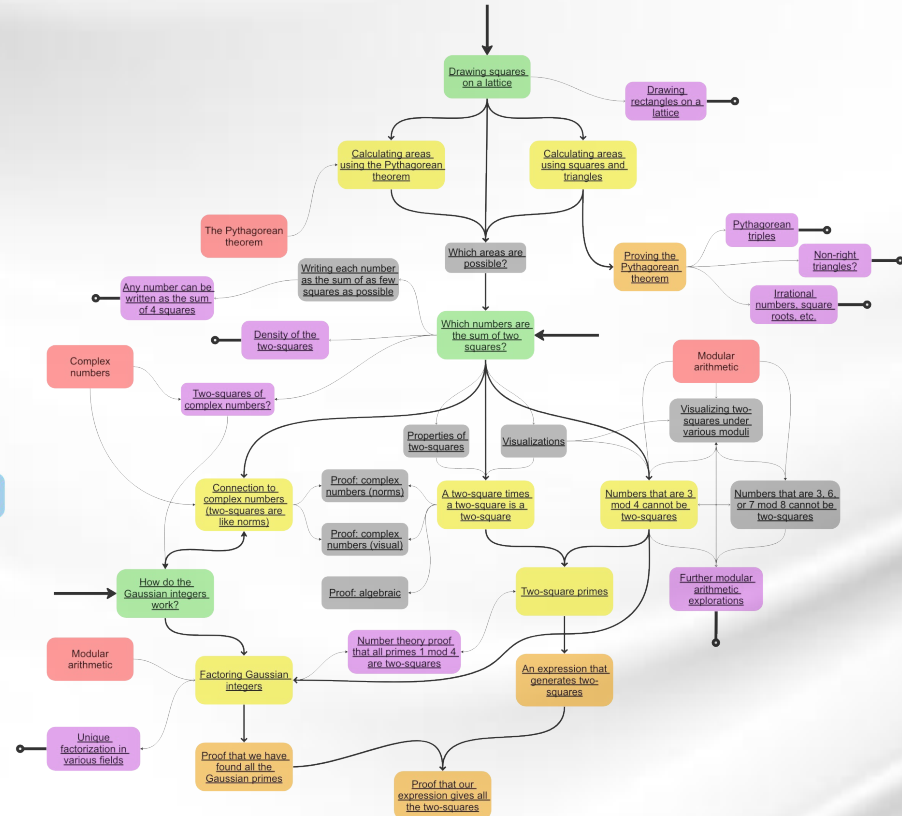
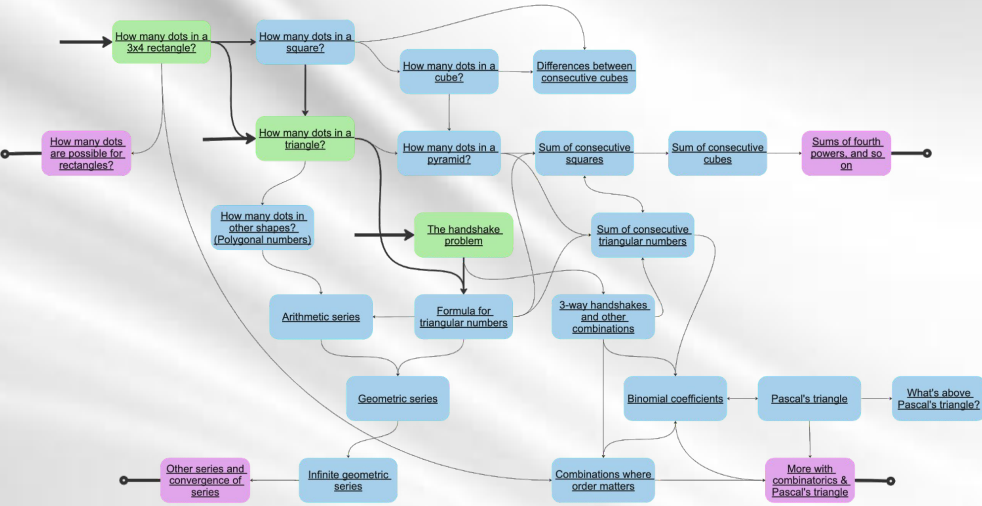
# To give a full experience of doing math...

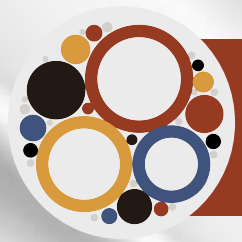
- Accessible mysteries (plot hooks)
- Following learners' questions
- Guide's preparedness



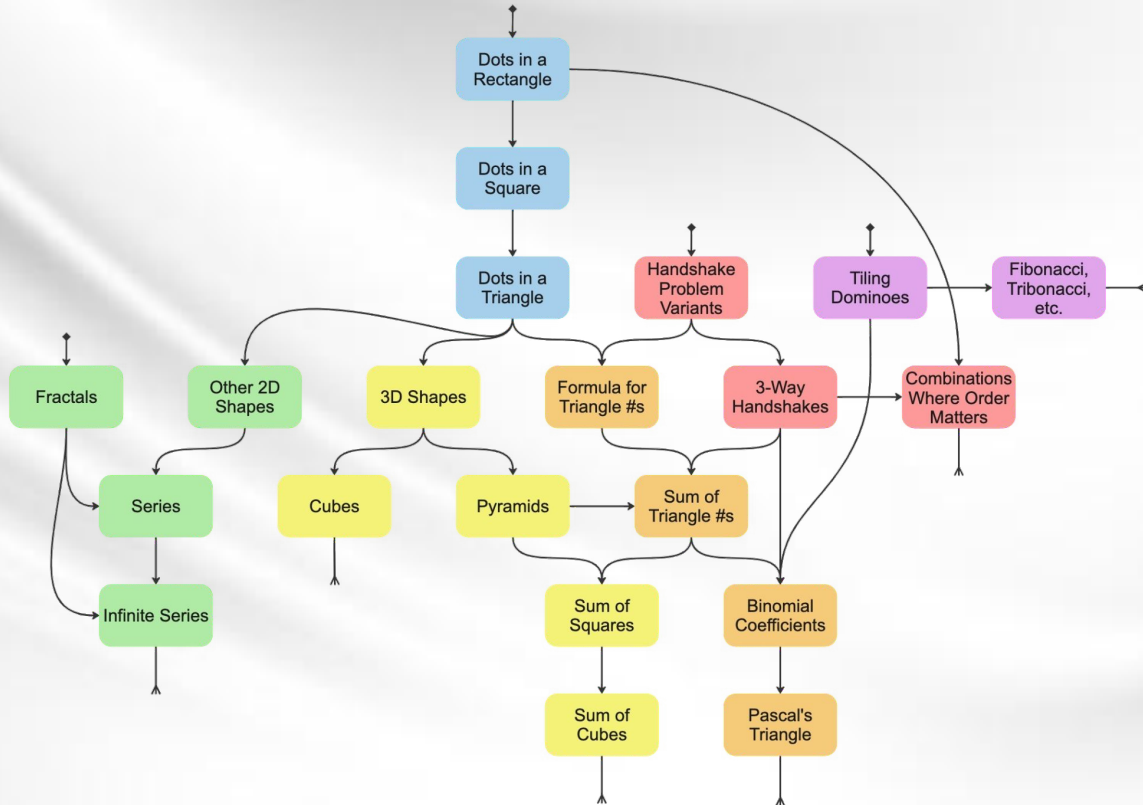


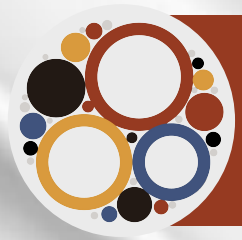
# Flowcharts (the Nexus)



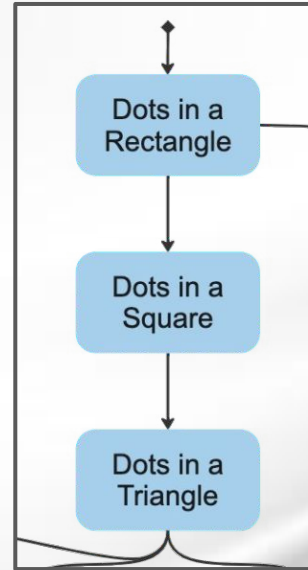


# Artistic/visual paths into combinatorics

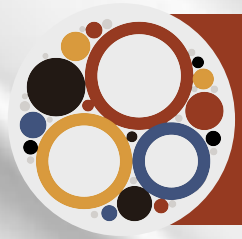




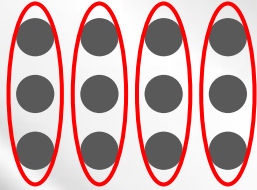
# What are some different ways we can count these dots?



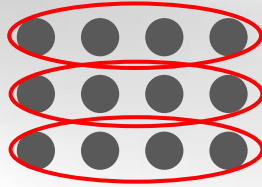




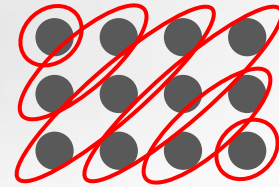
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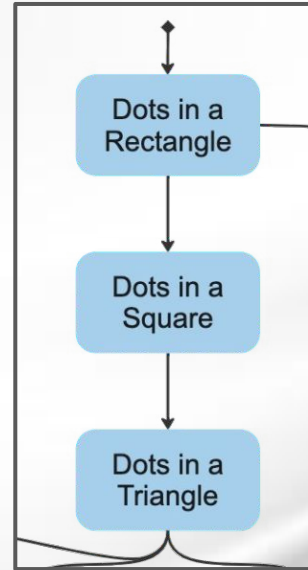
$$3 \times 4 = 12$$

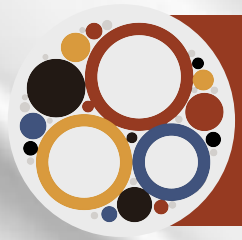


$$4 \times 3 = 12$$

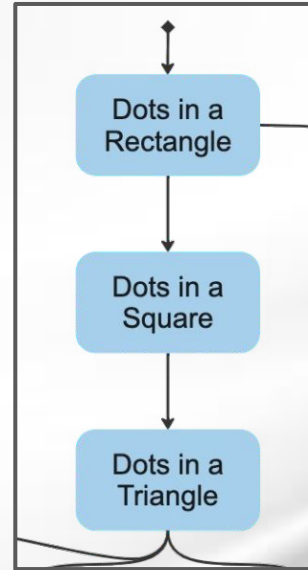
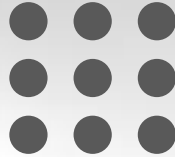


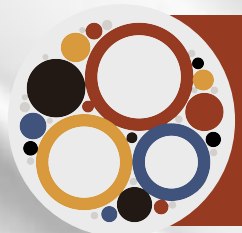
$$1+2+3+3+2+1 = 12$$



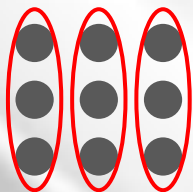


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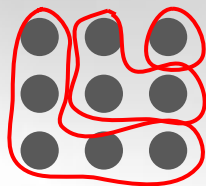




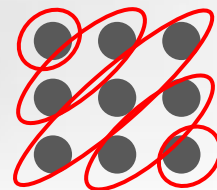
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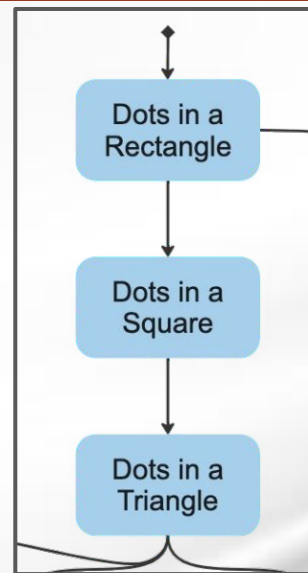
$$3 \times 3 = 9$$

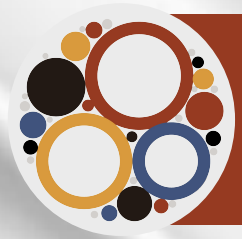


$$5 + 3 + 1 = 9$$

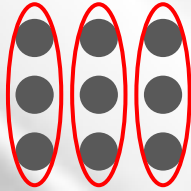


$$1 + 2 + 3 + 2 + 1 = 9$$

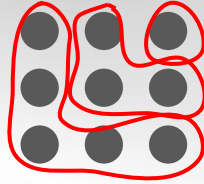




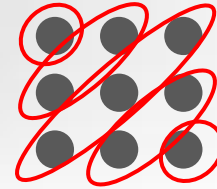
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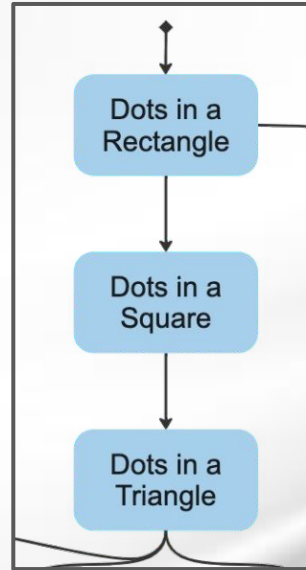
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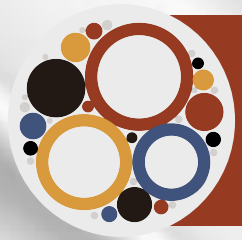


$$5 + 3 + 1 = 9$$

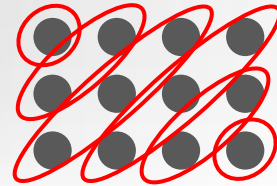


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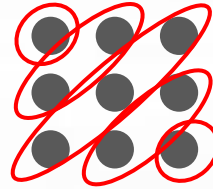




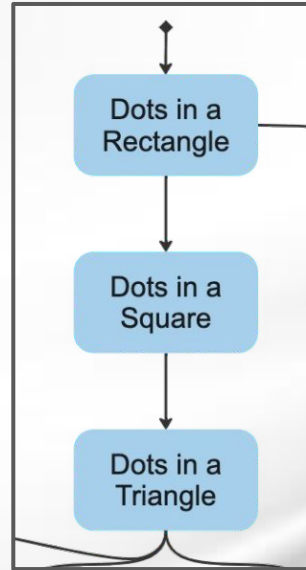
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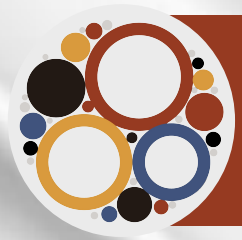


$$1+2+3+3+2+1 = 12$$

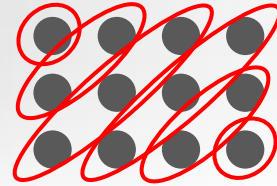
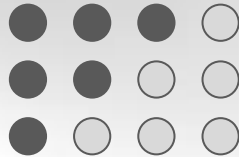


$$1+2+3+2+1 = 9$$



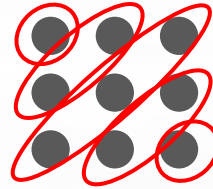
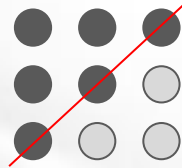


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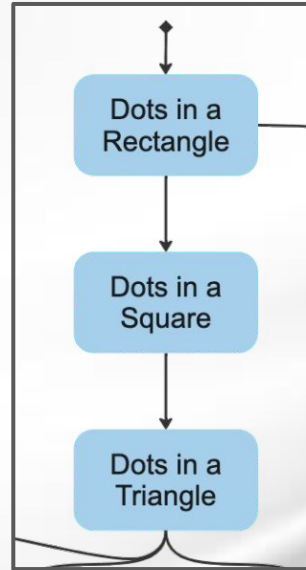
Half of a rectangle  
 $3 \times 4 / 2$

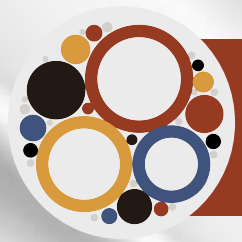
$$1+2+3+3+2+1 = 12$$



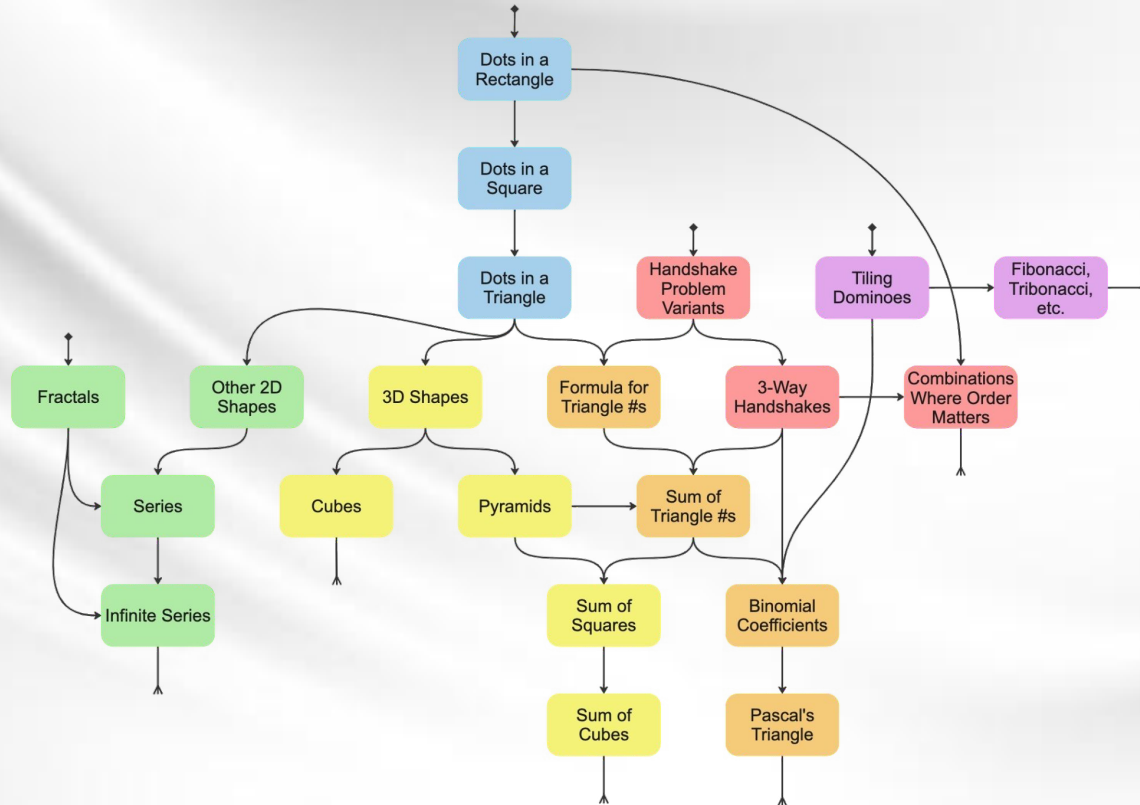
Almost half a square  
 $(3 \times 3 / 2) + (3 / 2)$

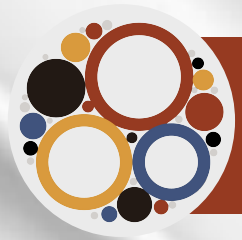
$$1+2+3+2+1 = 9$$





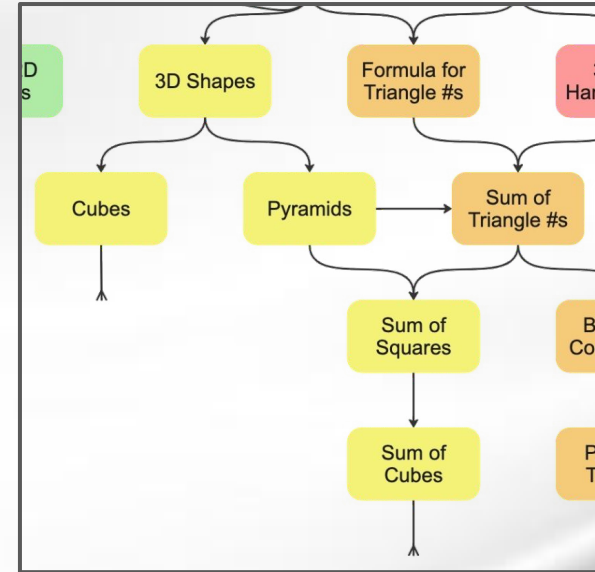
# Artistic/visual paths into combinatorics



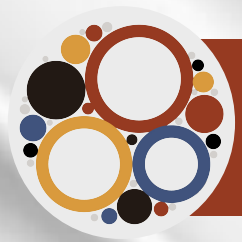


# Three-dimensional counting

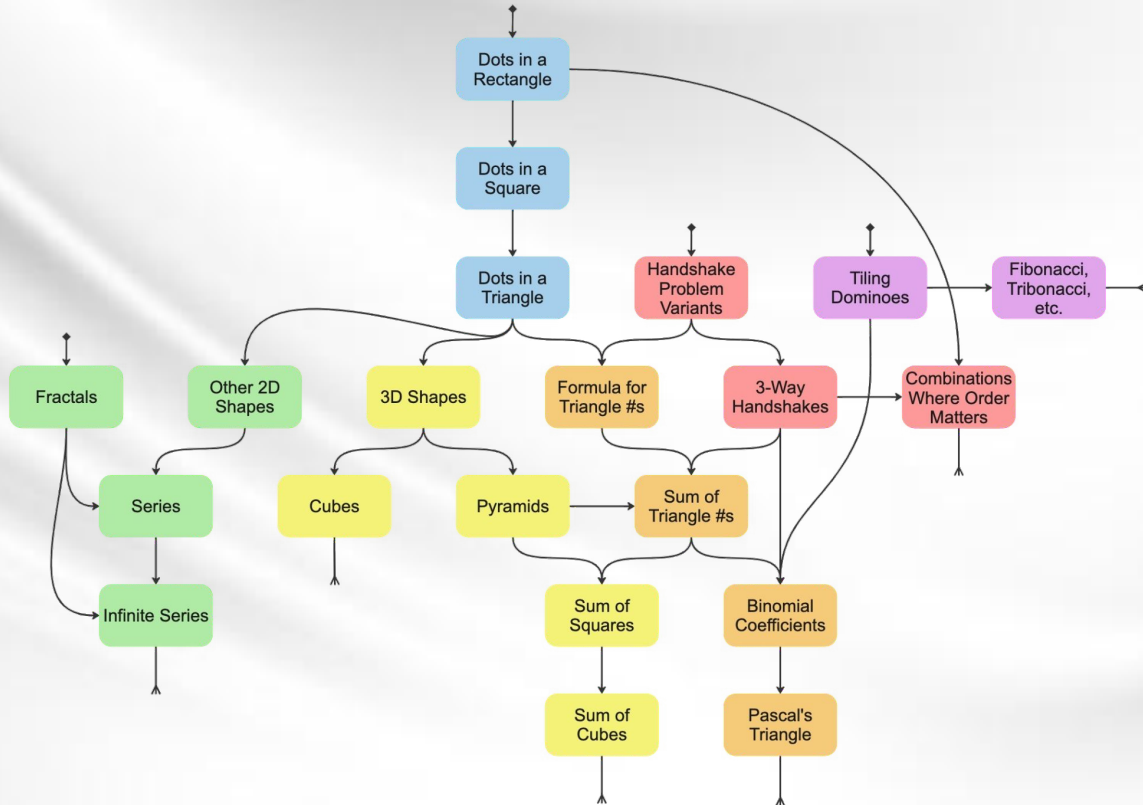
- Dots in a square pyramid?
- Dots in a tetrahedron?
- Diffs. of consecutive cubes

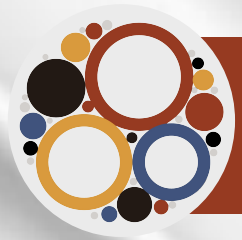






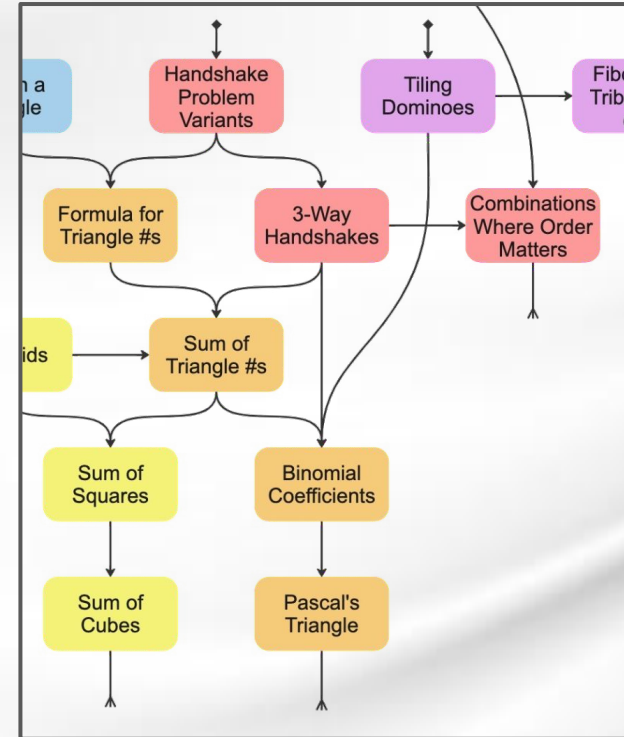
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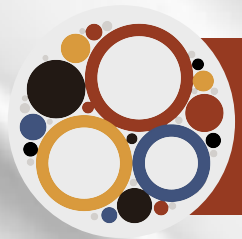




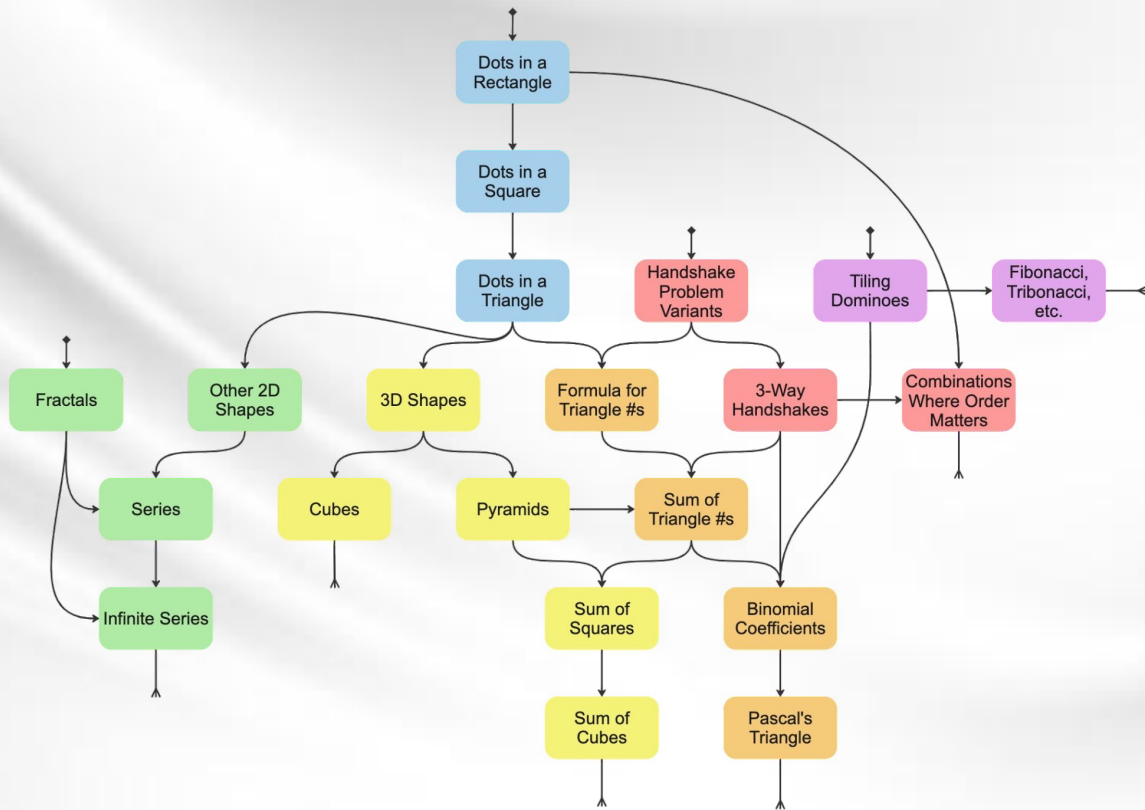
# Handshake problem variants

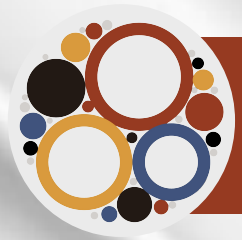
- Color schemes for your room
- Friendship bracelets (combining colors of thread)
- Painting (mixing colors)
- “Soups” (combining random items to make a “menu”)





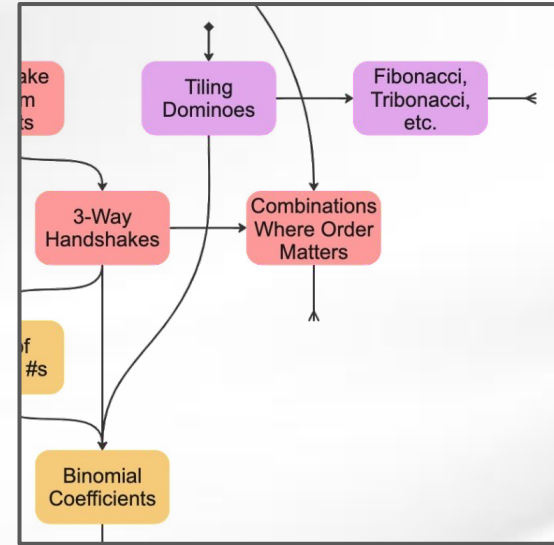
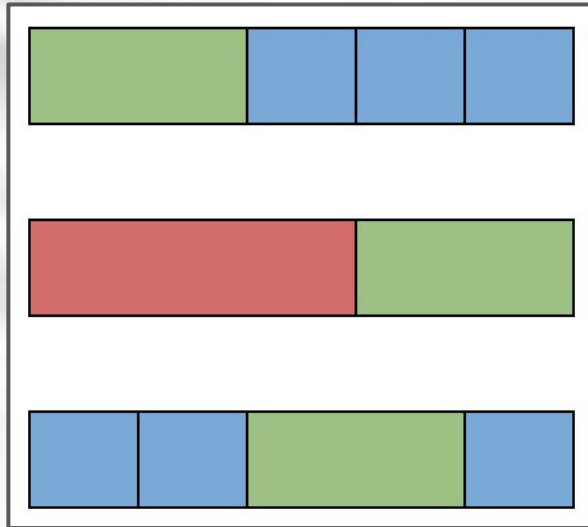
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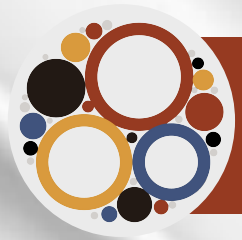




# Tiling with Blocks

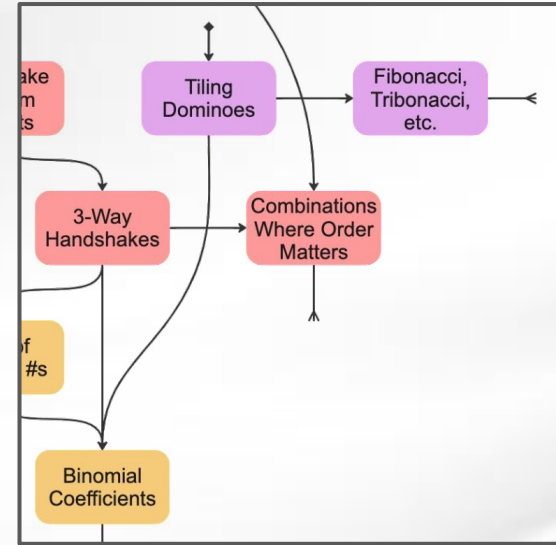
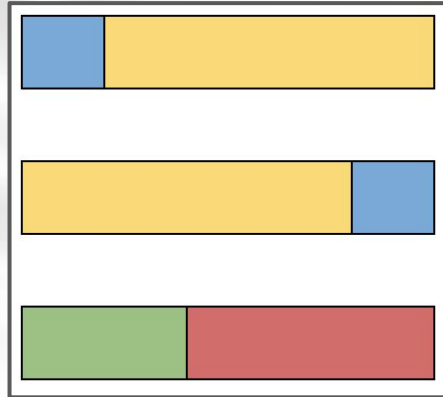
- Cuisenaire rods; ways to tile a  $1 \times 5$  rectangle?

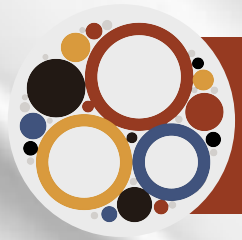




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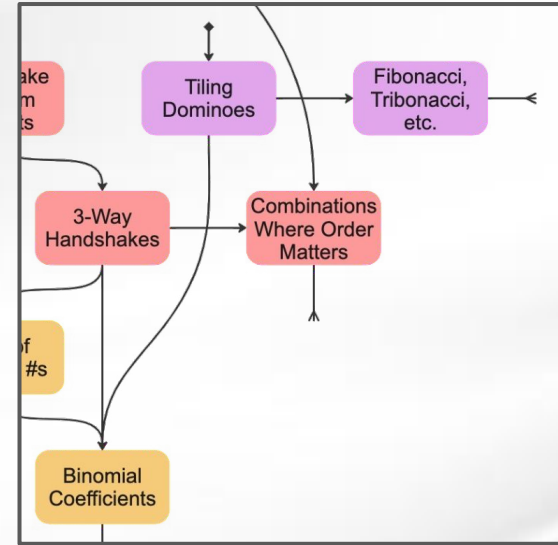
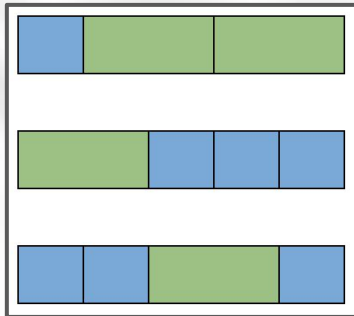
- Cuisenaire rods; ways to tile a  $1 \times 5$  rectangle?
- How many ways with only two blocks?

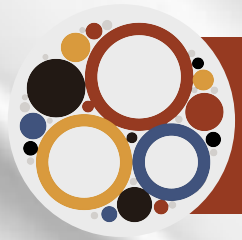




# Tiling with Blocks

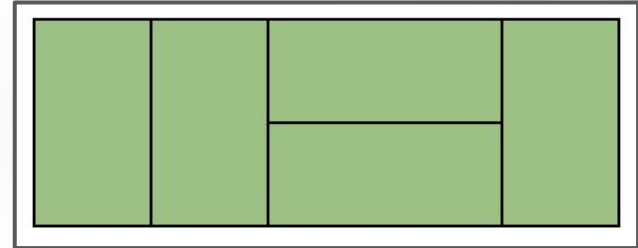
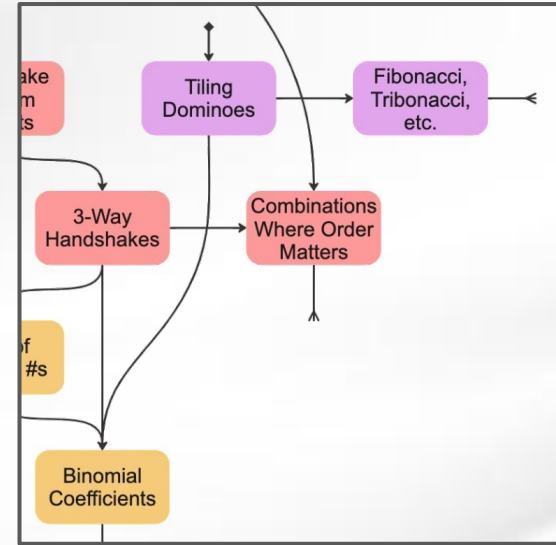
- Cuisenaire rods; ways to tile a  $1 \times 5$  rectangle?
- How many ways with only two blocks?
- What about only  $1 \times 1$  and  $1 \times 2$  blocks?

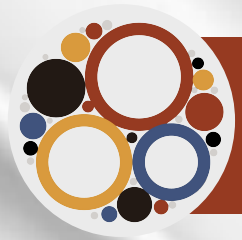




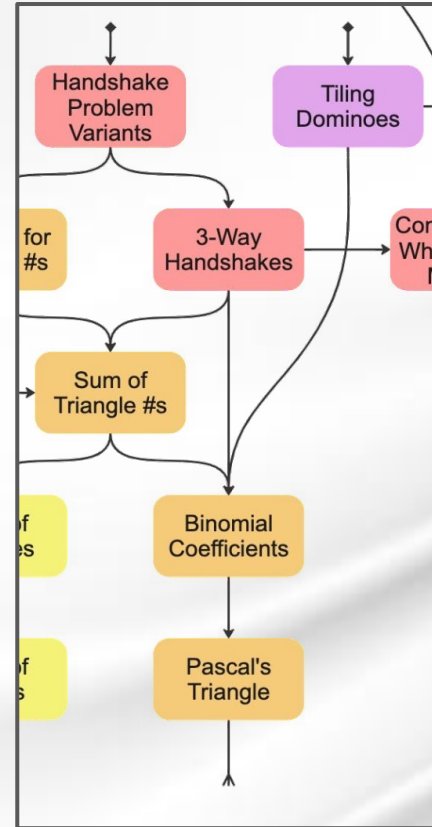
# Tiling with Blocks

- Cuisenaire rods; ways to tile a  $1 \times 5$  rectangle?
- How many ways with only two blocks?
- What about only  $1 \times 1$  and  $1 \times 2$  blocks?
- What about tiling a  $2 \times 5$  rectangle? (Using only  $1 \times 2$ s / dominoes)

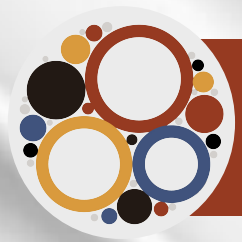




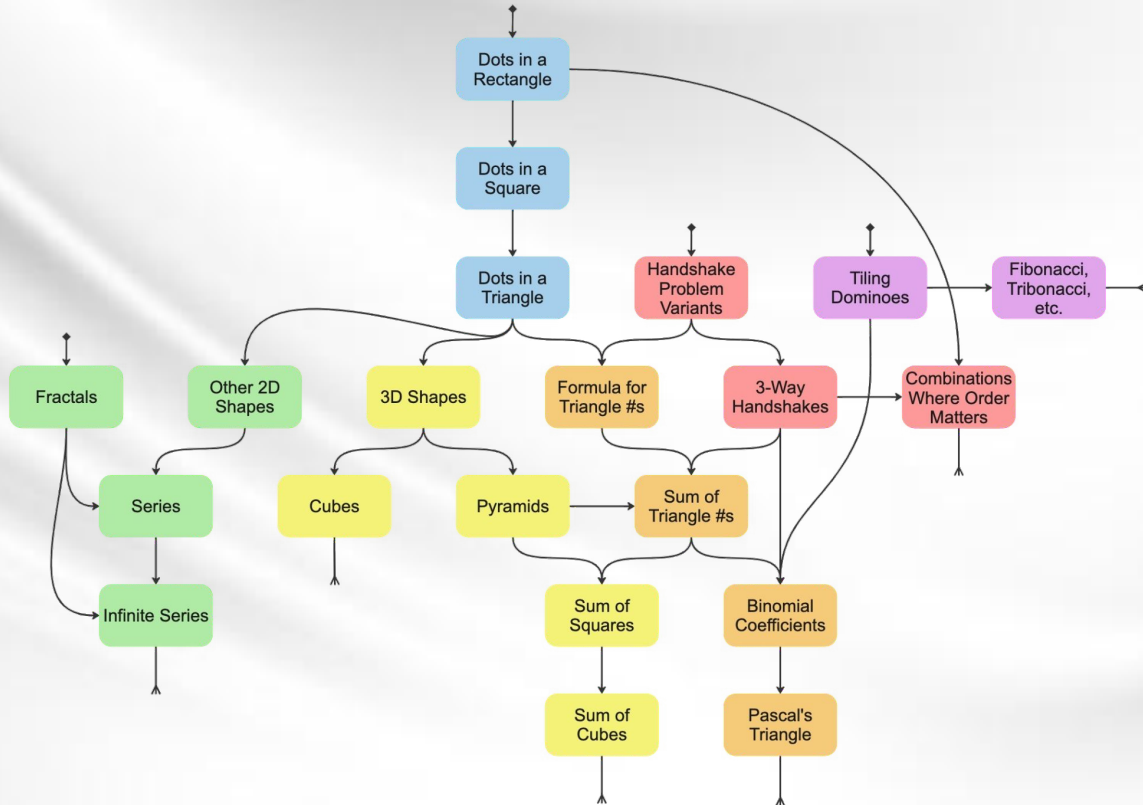
# Pascal's triangle

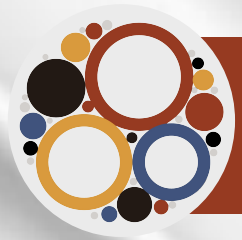






# Artistic/visual paths into combinatorics



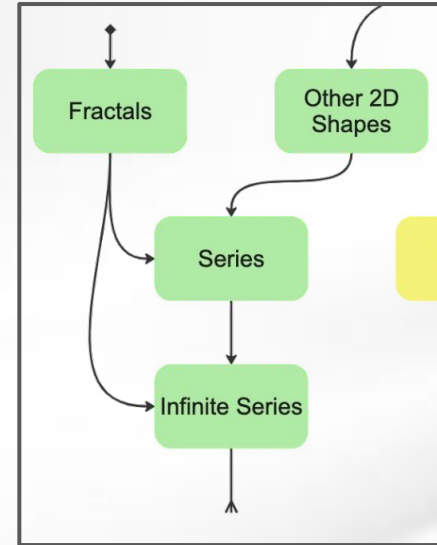
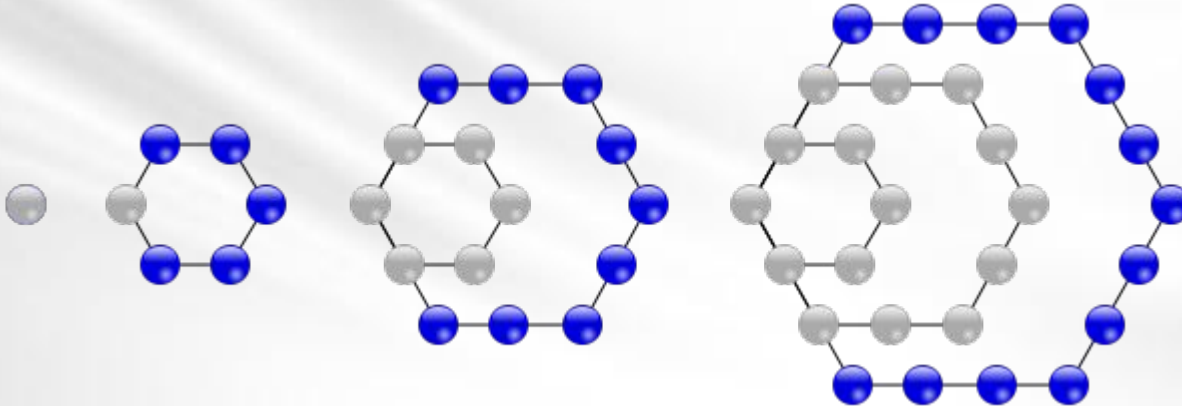


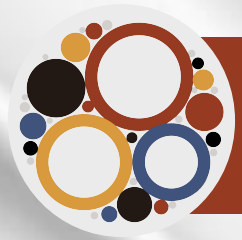
# Other shapes in 2D

Different triangles



Hexagonal numbers (image: Wikipedia)





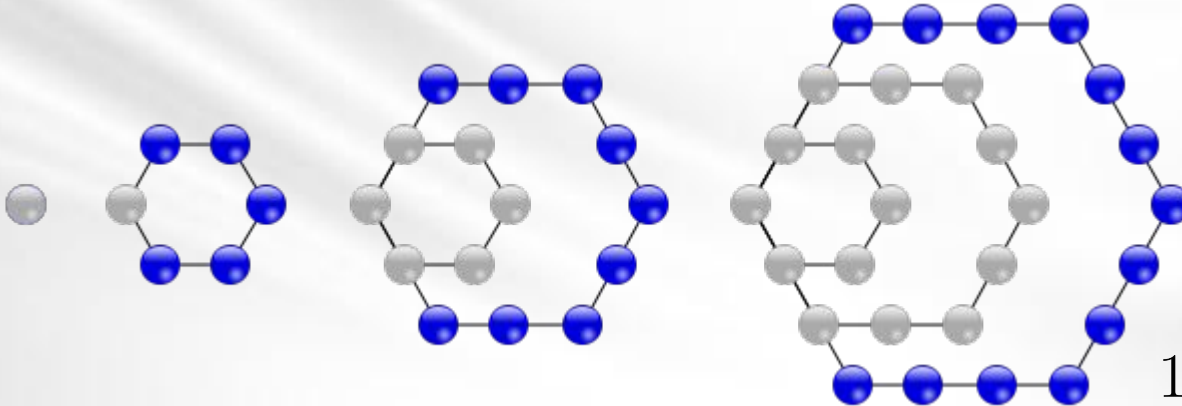
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Different triangles

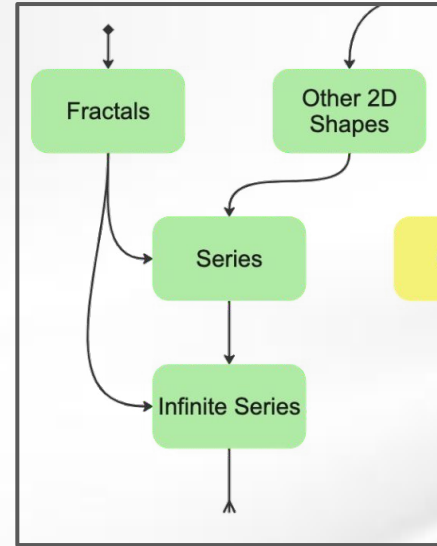


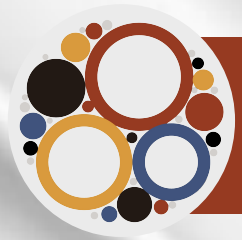
$$1 + 3 + 5 + 7 + \dots$$

Hexagonal numbers (image: Wikipedia)



$$1 + 5 + 9 + 13 + \dots$$





# Series: fertile ground for kids to ask new questions

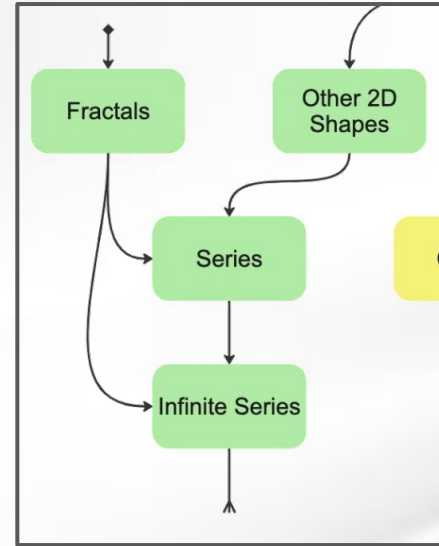
$$1 + 3 + 5 + 7 + \dots$$

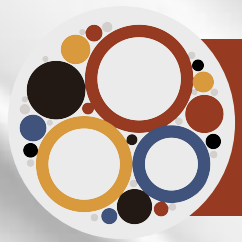
$$1 + 5 + 9 + 13 + \dots$$

$$3 + 2 + 1 + 0 - 1 - 2 - \dots$$

$$2 + 6 + 18 + 54 + 162 + \dots$$

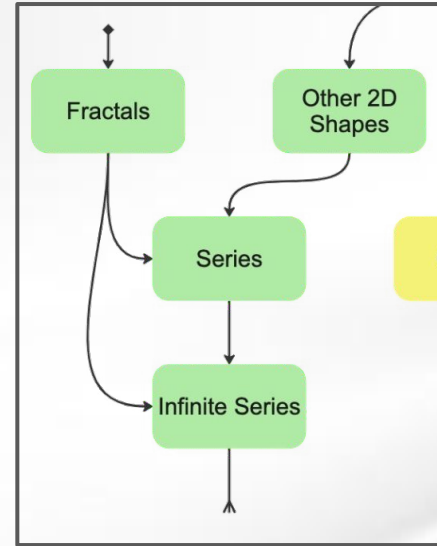
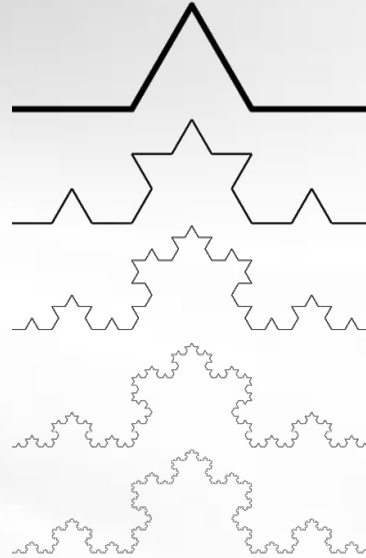
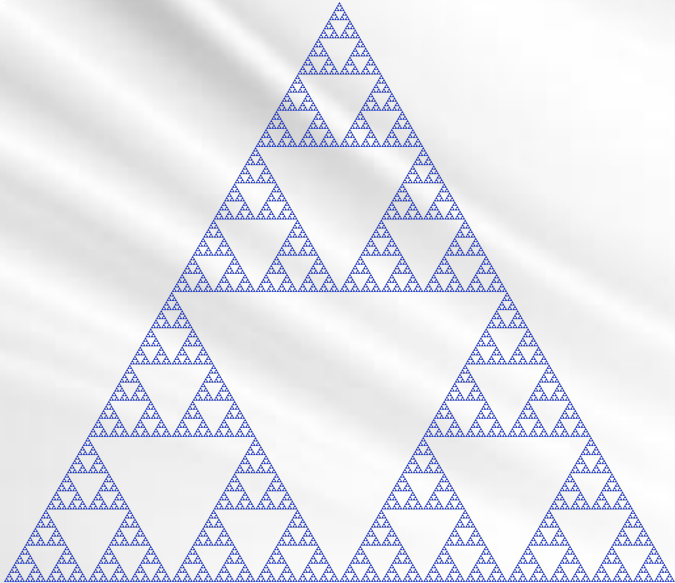
$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$





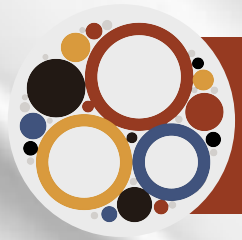
# Fractals as a gateway into series

Sierpinski Triangle (image: Wikipedia)



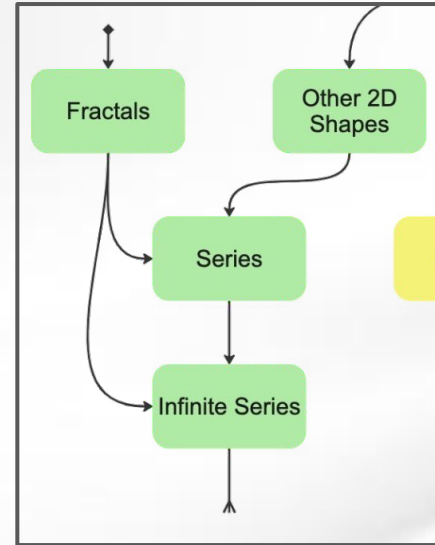
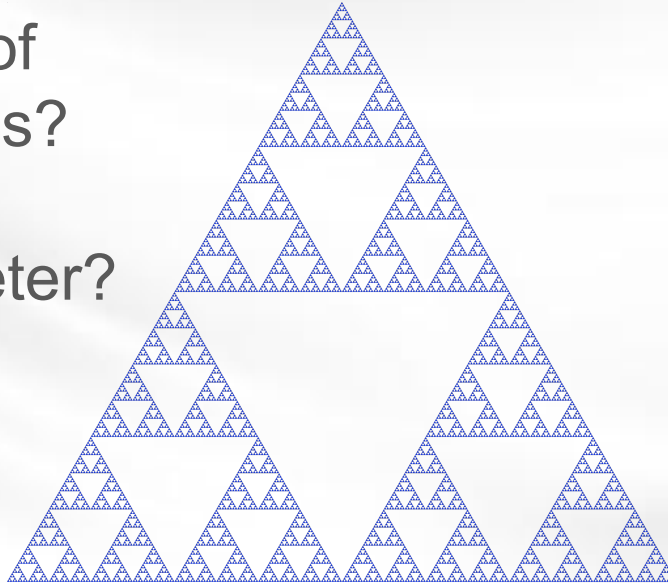
Koch Snowflake (image: Fractal Foundation)

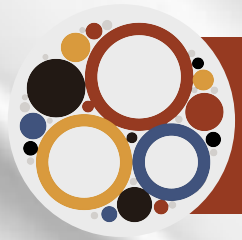




# Sierpinski Triangle

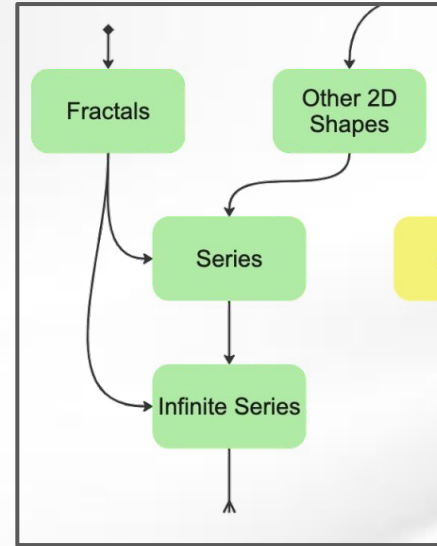
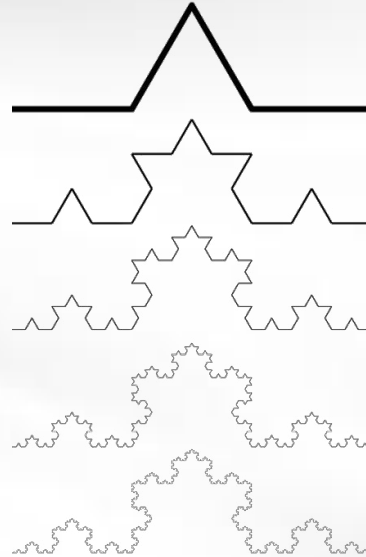
- What's its area?
- What's the area of the white triangles?
- What's its perimeter?

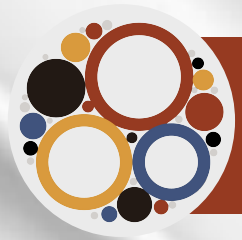




# Koch Snowflake

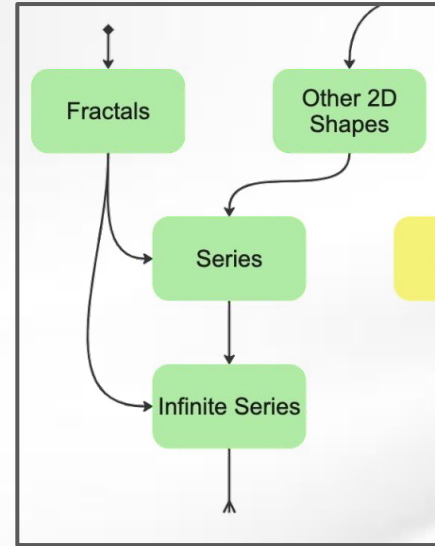
- What's its perimeter?
- What's the area *under* the snowflake?
- How many triangles are there?



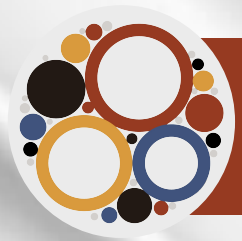


# Create your own fractals!

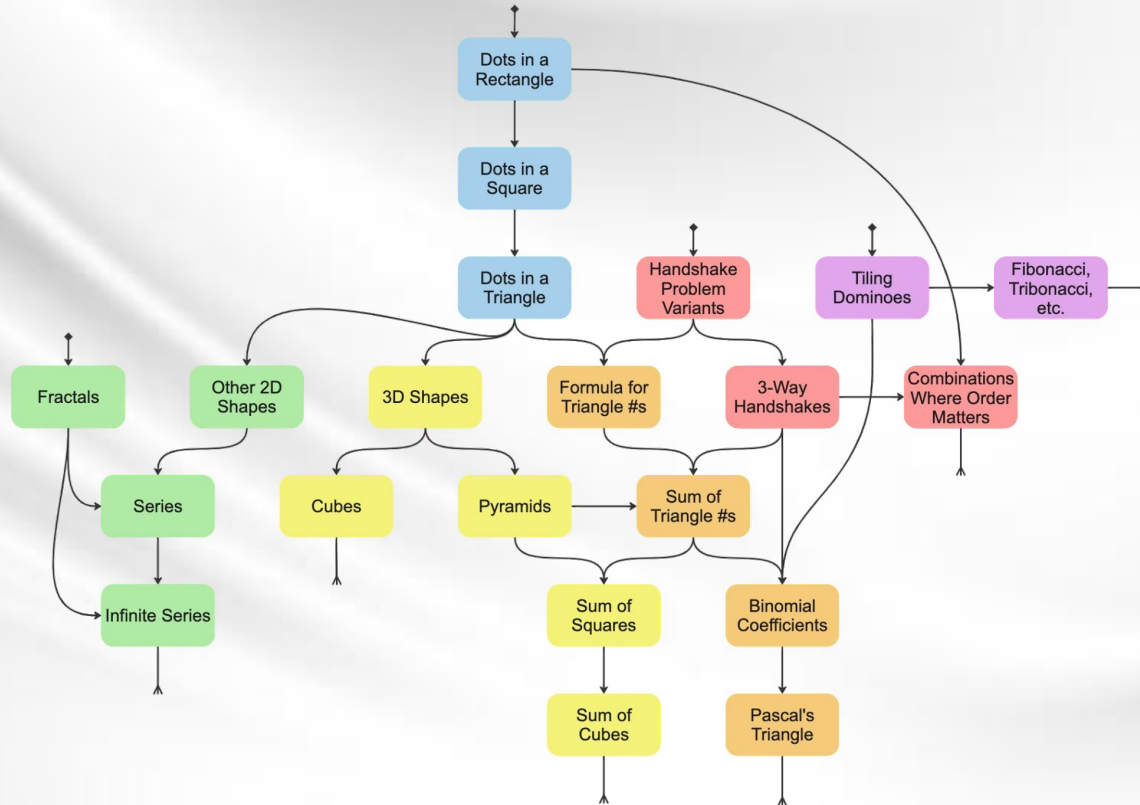
- Perimeter?
- Area?
- How many components?

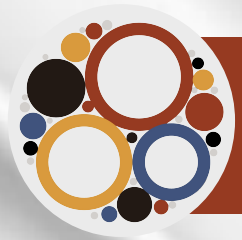






# Artistic/visual paths into combinatorics

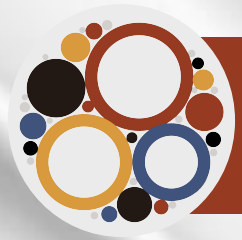




# Key points

- Asking questions is a key part of math
- Preparing with a flowchart map allows you to follow participants' questions
- Flowchart provides multiple entry points, for new angle/plot hook on the same idea





# Please keep in touch!

- Nexus project still in the works
- Math circle guide training institute
- Online math circles for anybody ages 5-13

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**Thank you!**

