Math Wrangle Problems: Set I

American Mathematics Competitions

April 14, 2012

1. What is the sum of the solutions of the equation

$$\sqrt[4]{x} = \frac{12}{7 - \sqrt[4]{x}}?$$

2. The pages of a book are numbered 1 though n. When the page numbers of the book were added, one of the page numbers was mistakenly added twice, resulting in the incorrect sum of 1986. What was the number of the page that was added twice?

3. In $\triangle ABC$, $AB = 425$, $BC = 450$ and $CA = 510$. Moreover, P is an interior point chosen so that the segments DE, FG and HI are each of length d, contain P, and are parallel to the sides AB, BC and CA, respectively. Find d.

4. Let the sum of a set of numbers be the sum of its elements. Let S be a set of positive integers, none greater than 15. Suppose no two disjoint subsets of S have the same sum. What is the largest sum a set S with these properties can have?

5. The shortest distances between an interior diagonal of a rectangular parallelepiped (box), P, and the edges it does not meet are $2\sqrt{5}$, $30/\sqrt{13}$ and $15/\sqrt{10}$. Determine the volume of P.

6. By a proper divisor of a natural number we mean a positive integral divisor other than 1 and the number itself. A natural number greater than 1 will be called "nice" if it is equal to the product of its distinct proper divisors. What is the sum of the first ten nice numbers?

7. Find the largest possible value of k for which 3^{11} is expressible as the sum of k consecutive positive integers.

8. Let m be the smallest positive integer whose cube root is of the form $n+r$, where n is a positive integer and r is a positive real number less than $1/1000$. Find n.