Math Wrangle Problems: Set II

American Mathematics Competitions

April 14, 2012

1. What is the largest positive integer n for which $n^3 + 100$ is divisible by $n + 10$?

2. In a sequence of coin tosses one can keep a record of the number of instances when a tail is immediately followed by a head, a head is immediately followed by a head, etc. We denote these by TH, HH, etc. For example, in the sequence HHTTHHHHTHHTTTT of 15 coin tosses we observe that there are five HH, three HT, two TH and four TT subsequences. How many different sequences of 15 coin tosses will contain exactly two HH, three HT, four TH and five TT subsequences?

3. Let $\triangle ABC$ be a right triangle in the xy-plane with the right angle at C. Given that the length of the hypotenuse AB is 60, and that the medians through A and B lie along the lines $y = x+3$ and $y = 2x+4$, respectively, find the area of $\triangle ABC$.

4. An ordered pair (m, n) of non-negative integers is called "simple" if the addition $m + n$ in base 10 requires no carrying. Find the number of simple ordered pairs of non-negative integers that sum to 1492.

5. Find the area of the region enclosed by the graph of $|x - 60| + |y| = |x/4|$.

6. Find $3x^2y^2$ if x and y are integers such that $y^2 + 3x^2y^2 = 30x^2 + 517$.

7. Let $[r, s]$ denote the least common multiple of positive integers r and s. Find the number of ordered triples (a, b, c) of positive integers for which $[a, b] = 1000$, $[b, c] = 2000$, and $[c, a] = 2000$.

8. Al walks down to the bottom of an escalator that is moving up and he counts 150 steps. His friend, Bob, walks up to the top of the escalator and counts 75 steps. If Al's speed of walking (in steps per unit time) is three times Bob's speed, how many steps are visible on the escalator at any given time? (Assume that this number is constant.)