The Cell Phone Dropping Problem

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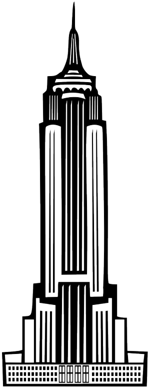


You work for a cell phone company which has just invented a new cell phone protector and wants to advertise that it can be dropped from the N-th floor without breaking.

If you are given 1 or 2 phones and a 100 story building, how do you guarantee you know the highest floor it won't break with the smallest number of trial drops?

With 1 phone: Drop from every floor, may require 100 drops.

With 2 phones, you can try:

- Drop from every 2nd floor. $50 + 1 = 51$ drops
- Drop from every 50th floor. $2 + 49 = 51$ drops
- Drop from every 25th floor. $4 + 24 = 28$ drops
- Drop from every k^{th} floor. $\text{drops}(k) = \lfloor 100/k \rfloor + k - 1$

Try it: drops$(k) = \lfloor 100/k \rfloor + k - 1$

k	drops(k)
7	$\lfloor 100/7 \rfloor + 7 - 1 = 20$
8	$\lfloor 100/8 \rfloor + 8 - 1 = 19$
9	$\lfloor 100/9 \rfloor + 9 - 1 = 19$
10	$\lfloor 100/10 \rfloor + 10 - 1 = 19$
11	$\lfloor 100/11 \rfloor + 11 - 1 = 19$
12	$\lfloor 100/12 \rfloor + 12 - 1 = 19$
13	$\lfloor 100/13 \rfloor + 13 - 1 = 19$
14	$\lfloor 100/14 \rfloor + 14 - 1 = 20$

Or, for those who know some calculus, if $f(k) = 100/k + k - 1$, then $f'(k) = -100/k^2 + 1$. Critical point $f'(k) = 0$ when $k = 10$.

So the minimum seems to be $f(10) = 19$, but this is wrong. What was the incorrect assumption?

SEE-Math

Dr. Yasskin was given this problem by a middle school student at SEE-Math who had learned it at MathPath. Yasskin has used it successfully at the TAMU Math Circle with high school students and at a session of 30 Davidson Young Scholars.

Summer Educational Enrichment in Math
`see-math.math.tamu.edu`

MathPath
`www.mathpath.org`

TAMU Math Circle
`mathcircle.tamu.edu`

Davidson Young Scholars
`www.davidsongifted.org`

SPMPS/BEAM

Used as a Challenge Problem at SPMPS. Solving it earns a movie night at the end of the week for the whole program.

Summer Program in Mathematical Problem Solving (SPMPS)
`www.artofproblemsolving.org/spmps`

12. Marbles

You are standing in front of a building with 100 levels, and you are given two glass marbles. If you drop a glass marble out of the window on level 100, it will break.

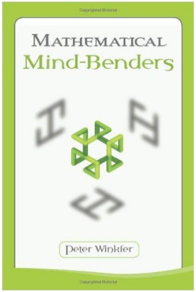
Your job is to determine **the lowest level** at which a glass marble will break if dropped out of the window. The simple-minded method is to drop a marble first from level 1, then from level 2, and so on, testing each level until the marble breaks. This method will require a maximum of 99 tests.

Since you have two marbles, you can use a more efficient method if one marble breaks, you can continue working with the other.

Find the most efficient method for using the two marbles to determine the lowest breaking level. In other words, find the method that has the smallest possible maximum number of tests.

What is this number?

Testing Ostrich Eggs



Mathematical Mind-Benders, Peter Winkler

In preparation for an ad campaign, the Flightless Ostrich Farm needs to test its eggs for durability. The world standard for egg-hardness calls for rating an egg according to the highest floor of the Empire State Building from which the egg can be dropped without breaking.

Flightless's official tester, Oskar, realizes that if he takes only one egg along on his trip to New York, he'll need to drop it from (potentially) *every one* of the building's 101 floors, starting with the first, to determine its rating.

How many drops does he need in the worst cast, if he takes *two* eggs?

Mathematical Mind-Benders by Peter Winkler, 2007. A K Peters/CRC Press

INFORMS Transactions on Education

The Joy of Egg-Dropping in Braunschweig and Hong Kong

Moshe Sniedovich, University Of Melbourne

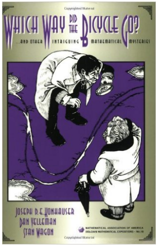
- Operations Research
- Management Sciences
- Dynamic Programming
- Worst Case scenario
- Expected Value scenario

Moshe Sniedovich, (2003) OR/MS Games: 4. The Joy of Egg-Dropping in Braunschweig and Hong Kong. INFORMS Transactions on Education 4(1):48–64.

166. An Egg-Drop Experiment (p.53)

Which Way Did the Bicycle Go?

Suppose that we wish to know which windows in a 36-story building are safe to drop eggs from, and which will cause the eggs to break on landing. We make a few assumptions:



- An egg that survives a fall can be used again.
- A broken egg must be discarded.
- The effect of a fall is the same for all eggs.
- If an egg breaks when dropped, then it would break if dropped from a higher window.
- If an egg survives a fall then it would survive a shorter fall.
- It is not ruled out that the first-floor windows break eggs, nor is it ruled out that the 36th-floor windows do not cause an egg to break.

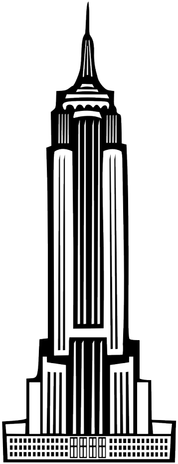
If only one egg is available and we wish to be sure of obtaining the right result, the experiment can be carried out in only one way. Drop the egg from the first-floor window; if it survives, drop it from the second floor window. Continue upward until it breaks. In the worst case, this method may require 36 droppings. Suppose 2 eggs are available. What is the least number of egg-droppings that is guaranteed to work in all cases?

Which Way Did the Bicycle Go? by Joseph Konhauser, Dan Velleman, Stan Wagon. (1996). MAA Dolciani Mathematical Expositions

Macalester College Problem of the Week, 1968–1995. (See `mathforum.org/wagon` for 1995–present).

Stan Wagon's favorite problems/puzzles: `stanwagon.com/wagon/Misc/bestpuzzles.html`

The Solution (part 1): 14 drops is sufficient



105th floor
1
104th floor
2
102nd floor
3
99th floor
⋮
50th floor
11
39th floor
12
27th floor
13
14th floor
14

$$14 + 13 + \cdots + 2 + 1 = 105 > 100$$

The Solution (part 2): 13 drops is not sufficient

$\mathbf{1}$ = phone breaks $\qquad$ $\mathbf{0}$ = phone doesn't break

1	2	3	4	5	6	7	8	9	10	11	12	13
0	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
0	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**1**
0	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**1**	
0	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**1**		

- Each trial yields a string in $\{\mathbf{1}, \mathbf{0}\}$ of length ≤ 13.
- No symbol can appear to the right of a second $\mathbf{1}$.
- No string can be the prefix of another string.
- There must be at least 100 strings.

$$\binom{13}{2} + \binom{13}{1} + \binom{13}{0} = 78 + 13 + 1 = 92 < 100$$

With 14 Drops

$$\binom{14}{2} + \binom{14}{1} + \binom{14}{0} = 91 + 14 + 1 = 106 > 100$$

Why does $\binom{N}{2} + \binom{N}{1} + \binom{N}{0}$ always add up to one more than a triangular number?

What is the solution to the Expected Value scenario?