A Sample of Mathematical Puzzles

Compiled by
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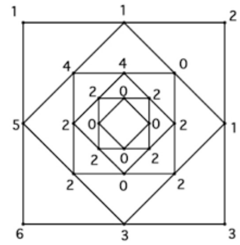


jrmf.org

Difference Engine

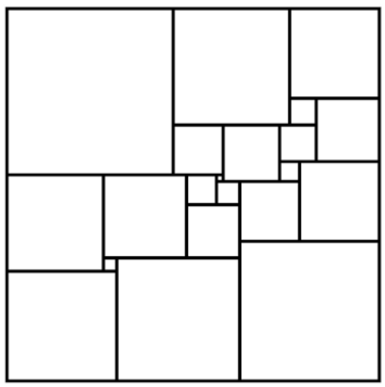
Julia Robinson Mathematics Festival

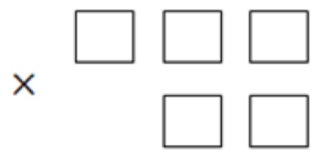
thesmartkitchenblog.com

Squarable Numbers

drmichaelshow.com

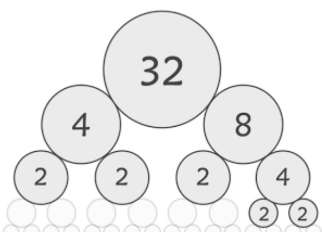
The Muffin Puzzle





Greatest Product

INTEGRAL



www.MathPickle.com

FISSION

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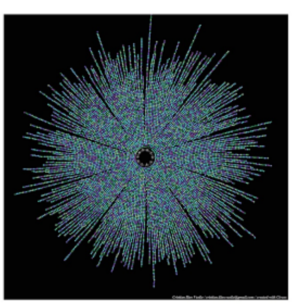
Squaring Puzzles

Deluxe Edition Cover of Adele 21

21 Equals



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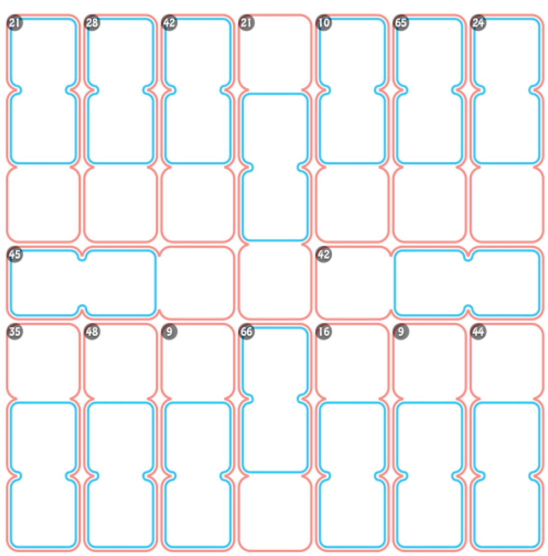
Digit Sums



www.MathPickle.com

Cartouche Puzzles

For math festivals for students and additional puzzles, visit the Julia Robinson Mathematics Festival website jrmf.org.

Intersted in organizing a festival, email info@jrmf.org.


Carrot apple muffins from drmichaelshow.com

The Muffin Puzzle

invented by recreational mathematician
Alan Frank
and described by
Jeremy Copeland
in the New York Times Numberplay Online Blog
wordplay.blogs.nytimes.com/2013/08/19/cake

You have 3 muffins and 5 students. You want to divide the muffins evenly, but no student wants a tiny sliver. What division of the muffins maximizes the smallest piece?

Here are some other questions to consider:

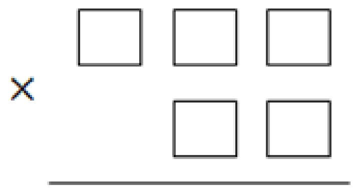
- How would you divide 5 muffins between 3 students?
- How would you divide 6 muffins between 10 students?

What muffin puzzles do you suggest I include in the next version of this booklet?

Multiplication: Finding the Greatest Product

By FAWN | *Published:* OCTOBER 26, 2014 - fawnnguyen.com/multiplication-finding-the-greatest-product

From a set of 1 through 9 playing cards, I draw five cards and get cards showing 8, 4, 2, 7, and 5. I ask my 6th graders to make a 3-digit number and a 2-digit number that would yield the greatest product. I add, **"But do *not* complete the multiplication — meaning do not figure out the answer. I just want you to think about place value and multiplication."**



I ask for volunteers who feel confident about their two numbers to share. This question brings out more than a few confident thinkers — each was so confident that he or she had the greatest product. (I'm noting here that I wasn't entirely sure what what the largest product would be. After this lesson, I asked some math teachers this question, and I appreciate the three teachers who shared. None of them gave the correct answer.)

1) 8 7 5 × 4 2

2) 8 7 2 × 5 4

3) 8 4 2 × 7 5

4) 8 7 4 × 5 2

5) 8 4 5 × 7 2

6) 5 2 4 × 8 7

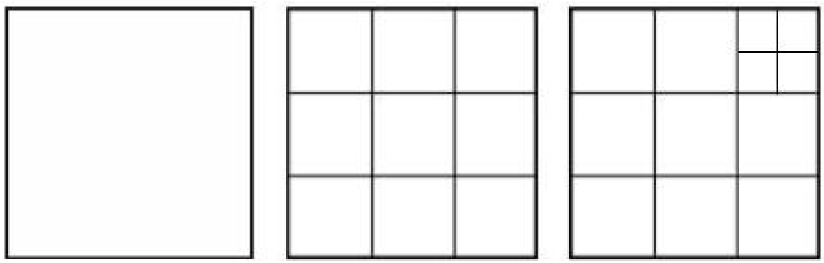
7) 5 4 2 × 8 7

I say, "Well, this is quite lovely, but y'all can't be right." Look at the seven "confident" submissions and see if you can reason that one yields a greater product than another. Which one or ones are do you suspect are not the largest? Which ones are greater or less than one or more other ones? Take 30 seconds to quietly examine the products and put a star next to the one that you believe yields the greatest product. Now cast your vote.

Squarable Numbers

by Daniel Finkel and Katherine Cook, Math for Love

The number n is "squarable" if we can build a square out of *n* smaller squares (of any size) with no leftover space. The squares need not be the same size. For example, 1, 9, and 12 are all squarable, since those numbers of squares can fit together to form another square.



Is there a simple way to tell if a number is squarable or not?

Which numbers from 1 to 30 are squarable? Experiment. Every time you come up with a way to break a square into some number of squares, circle that number.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Is there a pattern? Can you predict squarability in general?

Here's why Dr. Finkel proposed this problem to Gary Antonick, who published it in the New York Time Numberplay online blog, wordplay.blogs.nytimes.com/2013/04/08/squareable.

> *I think this puzzle is amazing because it's compelling right away, and you can work on it without worrying too much about wrong answers. If you're trying to show 19 is squarable and can't, maybe you'll accidentally show 10 is squarable on the way. (Of course, neither of those numbers is necessarily squarable. No spoilers here.) It's great to be able to experiment with a puzzle in an environment where virtually everything you do gives you some positive gains.*
>
> *I also like it because the willy-nilly approach most people start with eventually leads to a more strategic approach, and it takes a combination of deeper strategies to solve the problem. I also like it because just about anyone can get started on it, and make some serious headway —you don't need a sophisticated math background.*

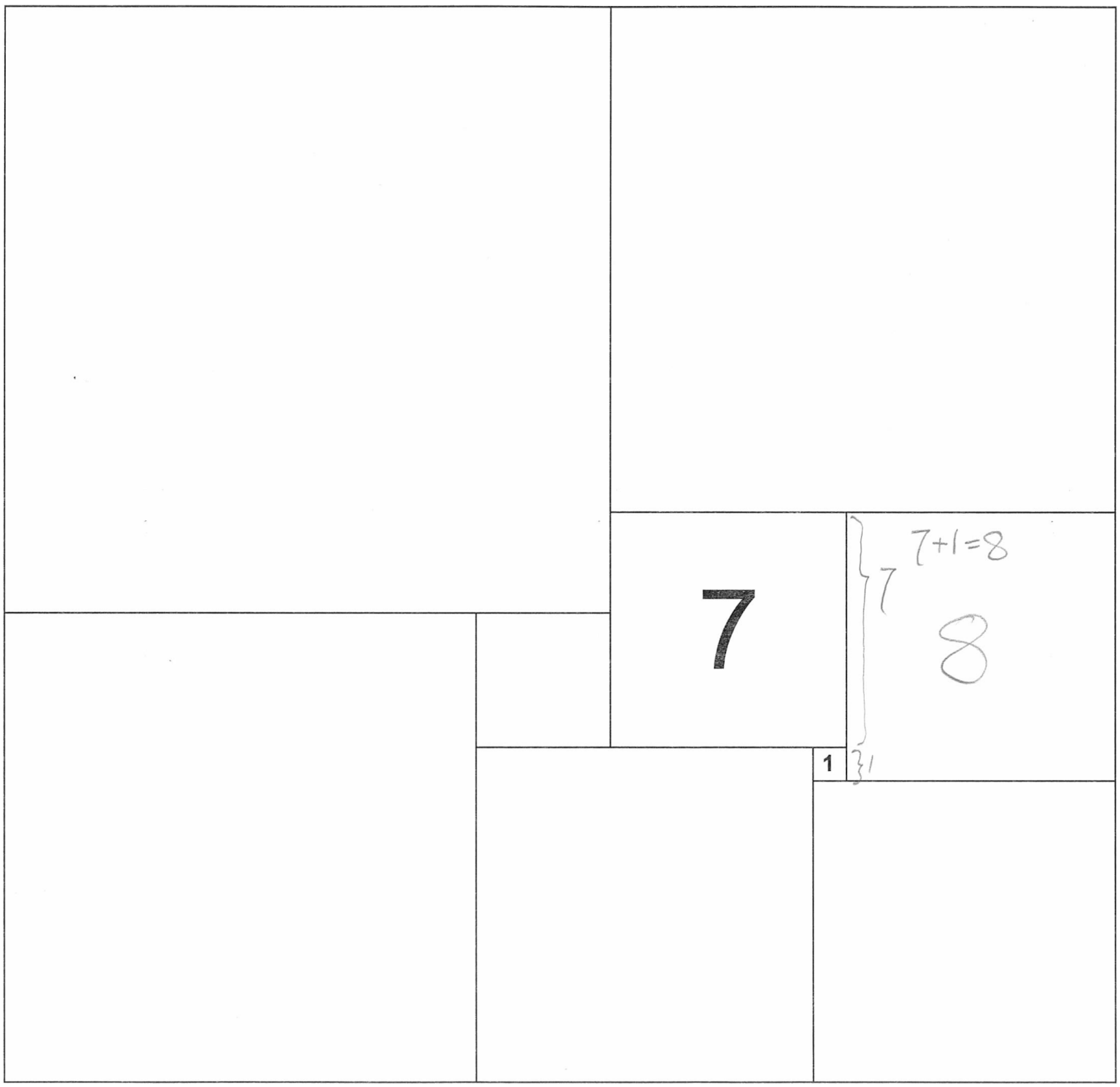
Find this and other Math for Love puzzles online at mathforlove.com/lessons.

Squaring Puzzles

by Gord Hamilton, Math Pickle

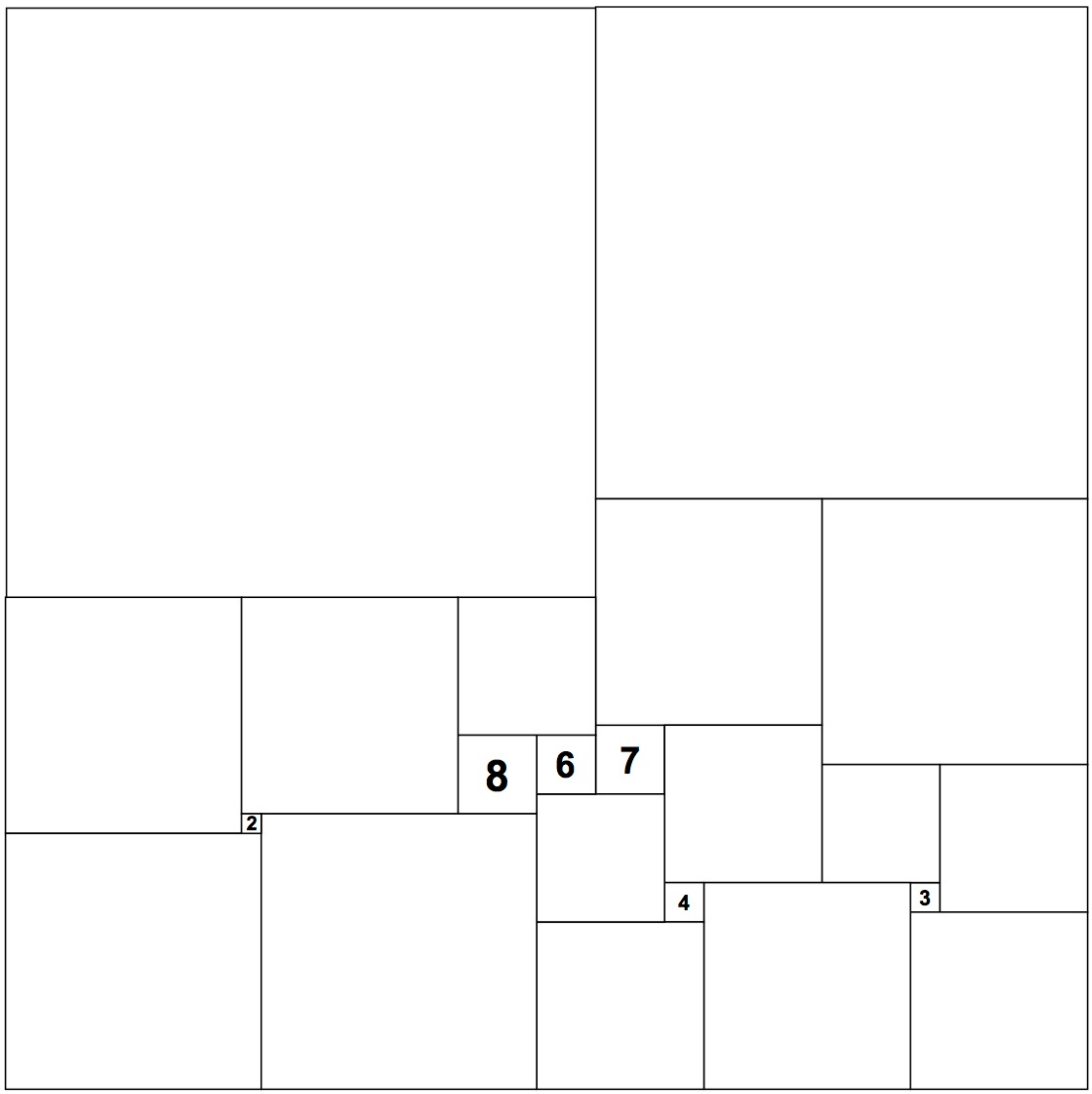
These abstract squaring puzzles give students addition and subtraction practice with numbers usually below 100. They also link these numerical activities to geometry. What a beautiful way to practice subtraction! *—Gord Hamilton, Founder of Math Pickle.*

The number in each square represents the length of the sides of that square. Determine the dimensions of all the squares in this rectangle and the lengths of the sides of the rectangle.

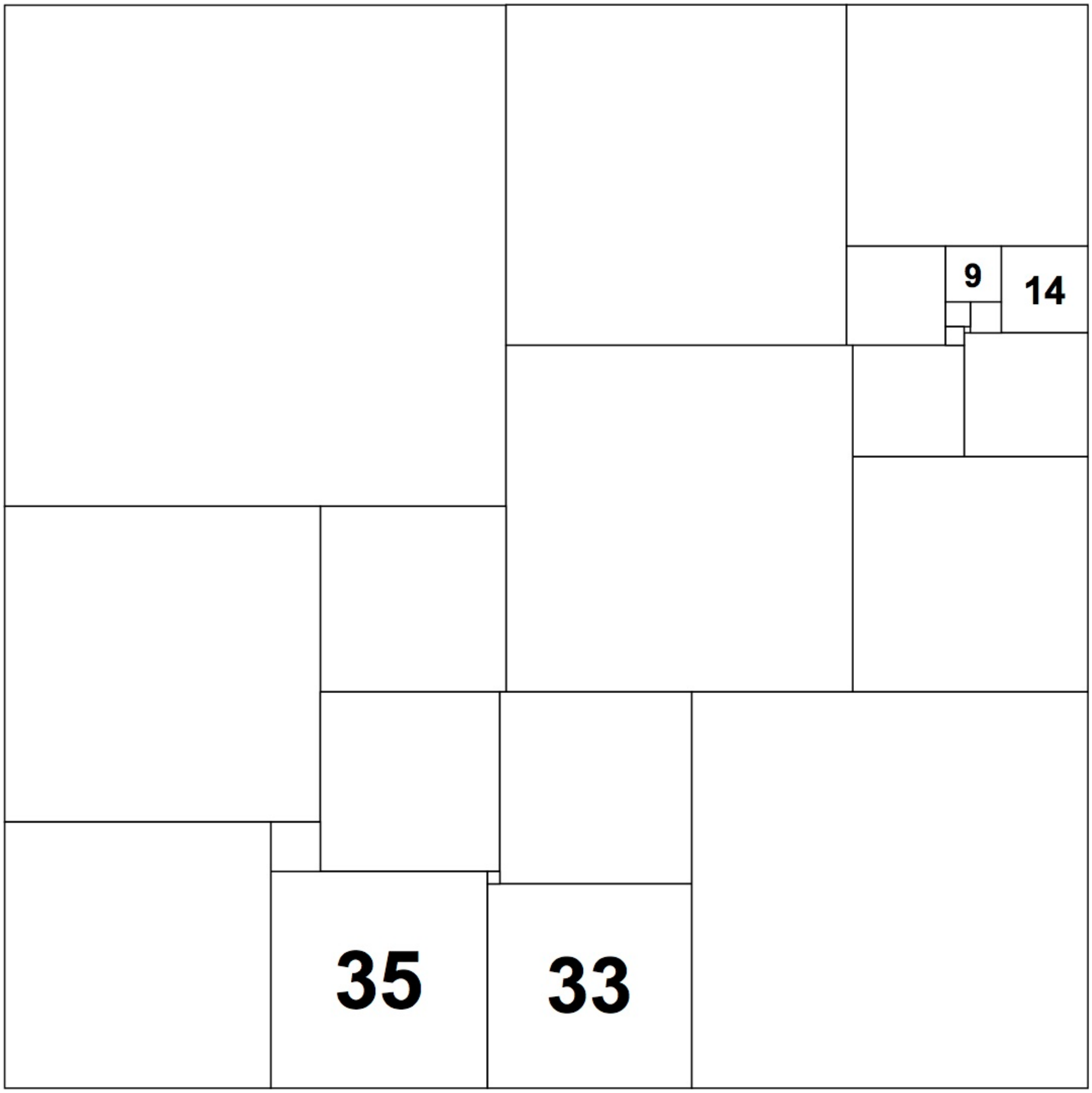


Find more square and subtracting puzzles online at http://goo.gl/eQN5fU.

Here's a more challenging puzzle. As in the previous puzzle, the number in each square represents the length of the sides of that square. Determine the dimensions of all the squares in this rectangle and the lengths of the sides of the rectangle.



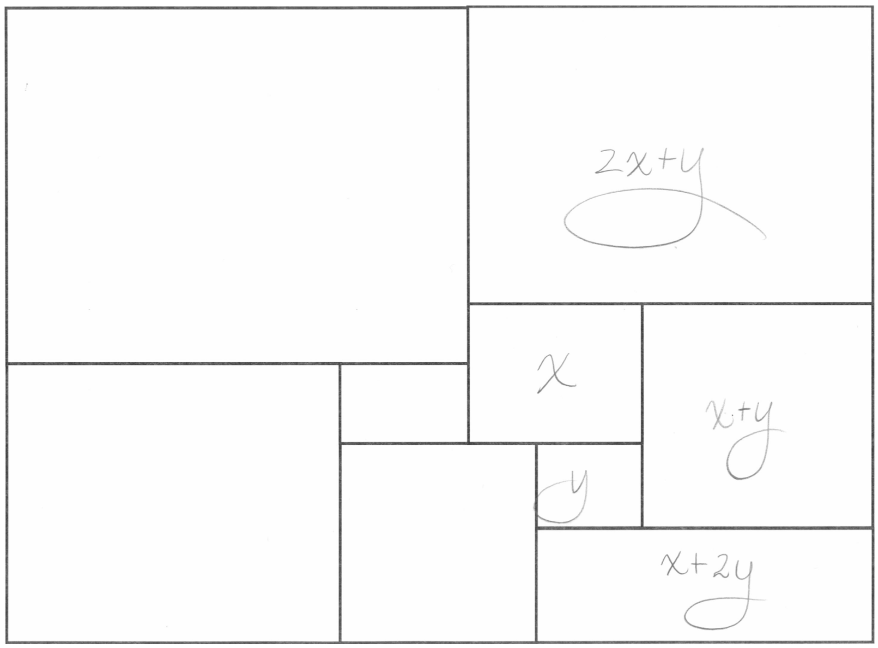
If you want even more of a challenge, try the following puzzle.



Algebra on Squares

by Gord Hamilton, Math Pickle

Imagine all the interior rectangles are squares, even though many look more like rectangles than squares. Determine the size of each square.

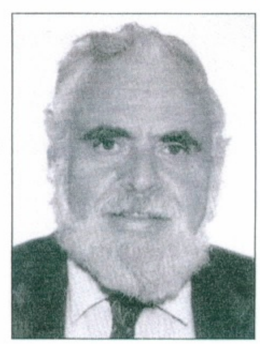


Find more Math Pickle algebra rectangle puzzles online at http://goo.gl/hXV19S.

If you want even more of a challenge, try the following puzzle.

Golomb's Puzzle Column™ Number 35: Rectangles With Consecutive-Integer Sides

Solomon W. Golomb



The sides (lengths and widths) of five rectangles measure each of the values 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 (units), in an unspecified order. As one of many possibilities, the five rectangles could be 1×6, 2×3, 4×10, 5×8, and 7×9, in which case their total area would be $6 + 6 + 40 + 40 + 63 = 155$ (square units).

1. How many different sets of five rectangles are possible? (The sequential order of the five rectangles does not matter, and we do not distinguish between an $a \times b$ and a $b \times a$ rectangle.)

2. What are the maximum and minimum values (A_{max} and A_{min}) for the total areas of the five rectangles?

3. Between A_{min} and A_{max}, which integer values of total areas are possible, and which are impossible?

4. There are a few sets of five rectangles (of the type we are considering) which can be assembled (without gaps or overlaps) to form a square.
   a.) Can you show, by a simple argument, that the total number of such sets of rectangles must be *even*?
   b.) Can you show that the side of any square so formed must have an *odd* length?

5. Can you exhibit any or all of the sets of rectangles, and the squares they form, as described in Problem 4?

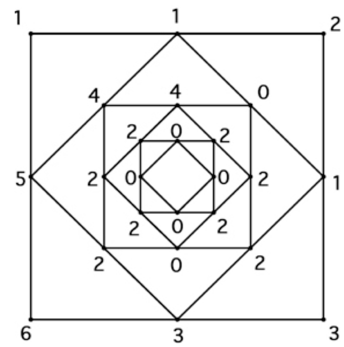
Reproduced with permission of puzzle originator Solomon W. Golomb.

September 1996 *IEEE Information Theory Society Newsletter*

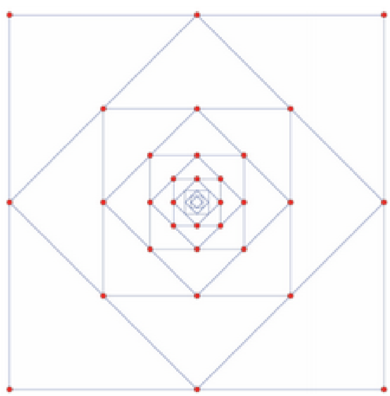
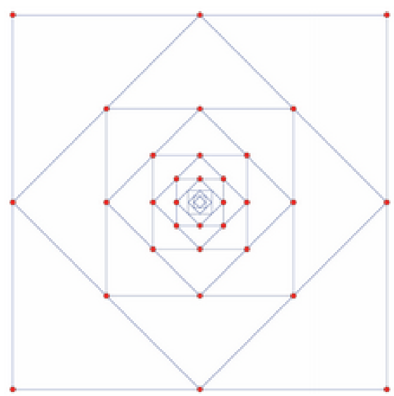
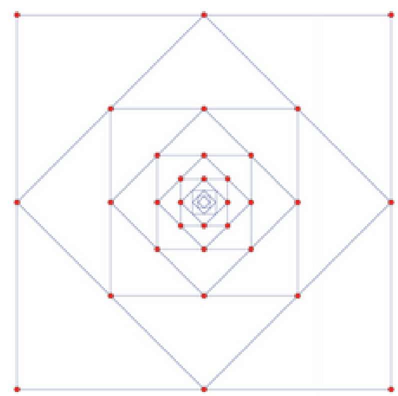
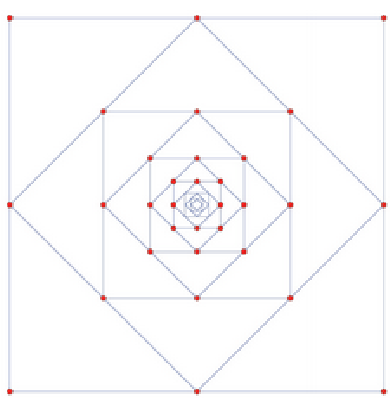
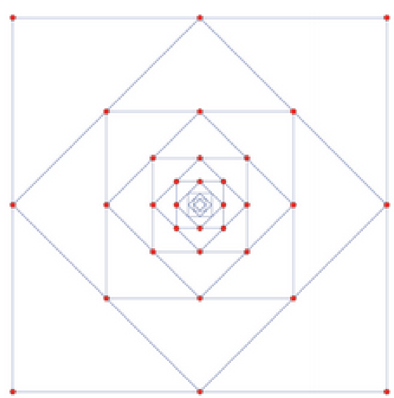
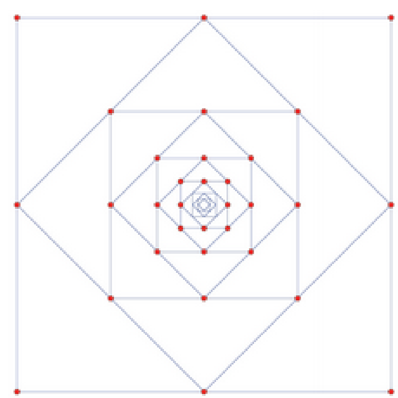
Difference Engine

Josh Zucker, Director of Mathematics
at the Julia Robinson Mathematics Festival
jrmf.org/problems/DifferenceEngine.pdf

Take any four starting numbers at the corners of the square, like 1, 2, 3, 6 in the example shown. At each midpoint, put the positive difference of the numbers at the endpoints. Repeat.



Experiment with lots of different starting lists of four numbers. What sets of numbers last a long time before arriving at all zeros? Do you always end up with all zeros?

Twenty One Equals (Page 1 of 2)

Compute 21 using the 4 numbers in the list on the left.

page 1 of 2

{3,5,8,9}	5	×	9	−	8	×	3	F {−, ×, ×}
{2,3,17,24}								
{4,8,8,9}								
{1,1,1,24}								
{2,2,2,2}								
{1,2,4,8}								
{1,4,4,4}								
{2,2,7,7}								
{4,4,7,8}								
{6,13,18,18}								
{3,3,9,9}								
{2,5,15,15}								
{1,2,4,6}								
{2,3,5,10}								
{5,10,13,17}								
{11,11,11,11}								
{2,2,16,17}								
{2,5,6,7}								
{2,6,8,9}								
{4,5,5,5}								
{5,9,9,12}								

Twenty One Equals

{3,5,8,9} can make 21 as follows 5×9-8×3=45-24=21. Arrange each other lists of four positive integers in the four squares and insert +, −, ×, or ÷ into the space between the squares so that the result is equal to 21. Follow the usual order of operations without using parentheses. Note: No solution is among the possibilities.

A	no solution
B	{ ×, ×, × }
C	{ ÷, ÷, ÷ }
D	{ −, −, − }
E	{ +, +, + }
F	{ −, ×, × }
G	{ +, ×, × }
H	{ ×, ×, ÷ }
I	{ +, ÷, ÷ }
J	{ −, ÷, ÷ }
K	{ ÷, ÷, × }
L	{ +, +, × }
M	{ +, +, ÷ }
N	{ +, +, − }
O	{ −, −, × }
P	{ −, −, ÷ }
Q	{ +, −, − }
R	{ +, ×, − }
S	{ +, ×, ÷ }
T	{ +, −, ÷ }
U	{ −, ×, ÷ }

Decode each set of operators to a letter using the table listed above, for example, {-, ×, ×} → F.

Don't use parentheses. Remember order of operations (for example, 9−8÷2÷2 = 7). Things in curly braces are not necessarily in the order in which they will appear. Anagram the final set of letters into a single "symbol" as your answer.

Twenty One Equals (Page 2 of 2)

Compute 21 using the 4 numbers in the list on the left.

List								Letter
{3,4,11,12}	12	×	3	−	11	−	4	O {−, −, ×}
{2,4,7,9}								
{4,12,13,14}								
{3,7,7,10}								
{1,2,7,11}								
{6,6,6,9}								
{3,4,10,10}								
{2,5,6,8}								
{3,5,5,8}								
{3,3,18,19}								
{3,3,3,4}								
{3,8,14,16}								
{7,15,15,15}								
{2,14,15,16}								
{2,6,7,9}								
{3,4,19,24}								
{1,5,5,5}								
{3,9,12,16}								
{1,4,7,9}								
{3,4,6,8}								
{1,2,7,13}								

Twenty One Equals

{3,4,11,12} can make 21 as follows 12×3-11-4=36-15=21. Arrange each other lists of four positive integers in the four squares and insert +, −, ×, or ÷ into the space between the squares so that the result is equal to 21. Follow the usual order of operations without using parentheses. Note: No solution is among the possibilities.

A	no solution
B	{ ×, ×, × }
C	{ ÷, ÷, ÷ }
D	{ −, −, − }
E	{ +, +, + }
F	{ −, ×, × }
G	{ +, ×, × }
H	{ ×, ×, ÷ }
I	{ +, ÷, ÷ }
J	{ −, ÷, ÷ }
K	{ ÷, ÷, × }
L	{ +, +, × }
M	{ +, +, ÷ }
N	{ +, +, − }
O	{ −, −, × }
P	{ −, −, ÷ }
Q	{ +, −, − }
R	{ +, ×, − }
S	{ +, ×, ÷ }
T	{ +, −, ÷ }
U	{ −, ×, ÷ }

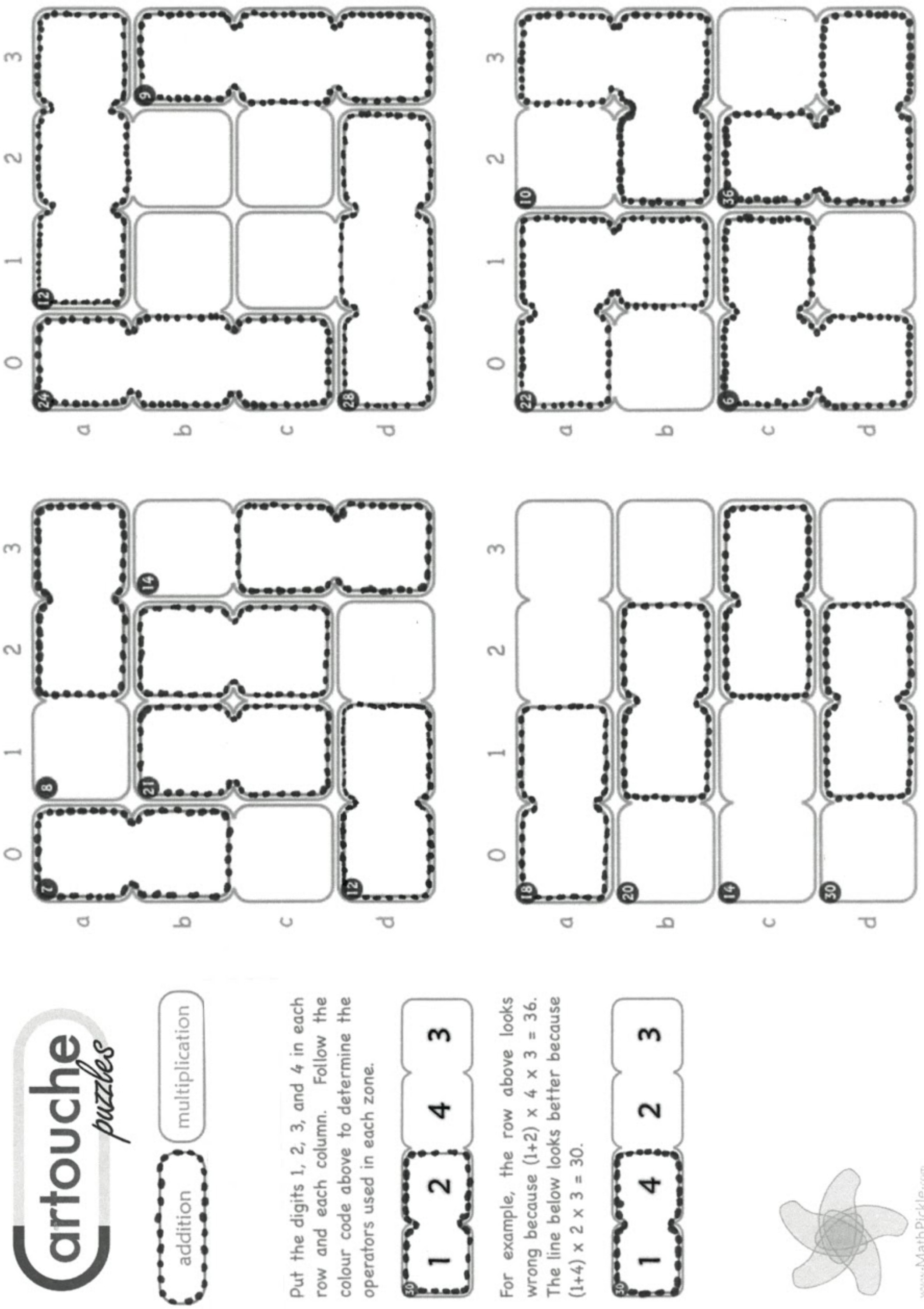
Decode each set of operators to a letter using the table listed above, for example, {-, -, ×} → O.

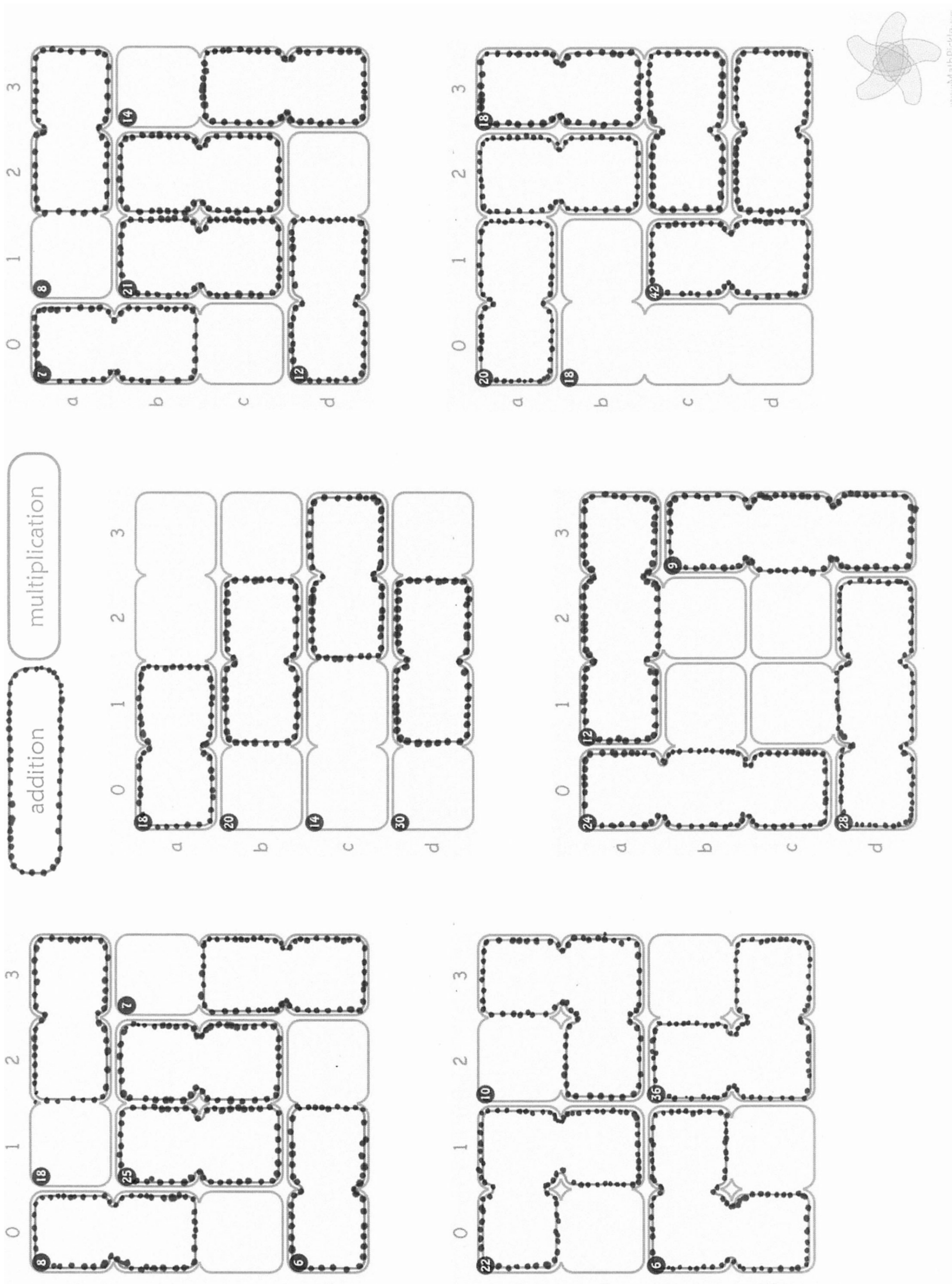
Don't use parentheses. Remember order of operations (for example, 9−8÷2÷2 = 7). Things in curly braces are not necessarily in the order in which they will appear. Anagram the final set of letters into a single "symbol" as your answer.

Cartouche Puzzles

by Gord Hamilton, Math Pickle

Cartouche is referred to as “KenKen on steroids.”





Find more Math Pickle Cartouche puzzles online at http://goo.gl/QzZNov.

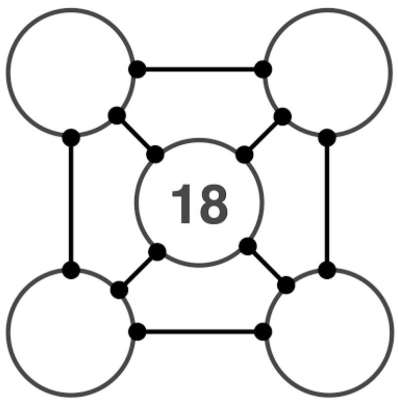
Julia Robinson Mathematics Festival

Directed By Joshua Zucker

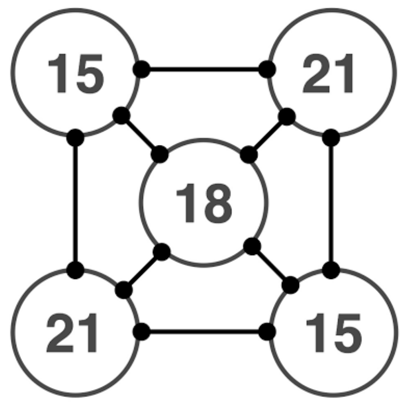
Digit Sums and Graphs

In each diagram, fill in the circle with positive whole numbers in such a way that each circle's number is the sum of the digits of all the numbers connected to it. Thanks to Erich Friedman for this idea!

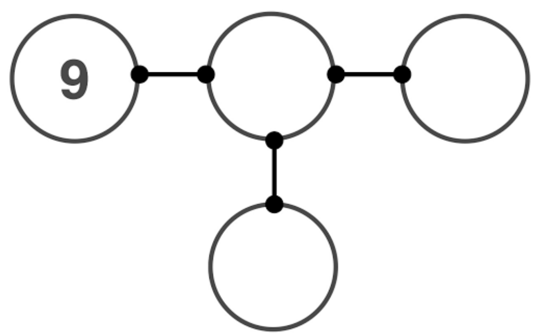
EXAMPLE

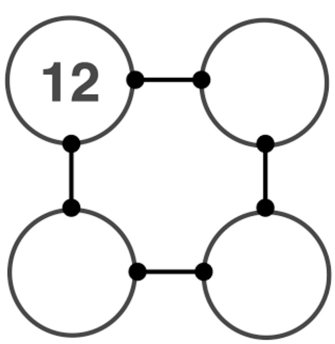


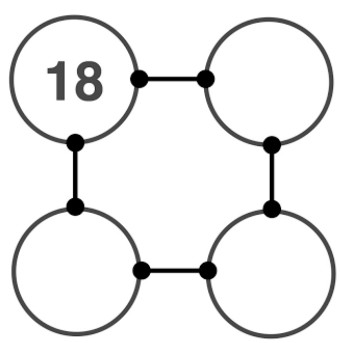
SOLUTION



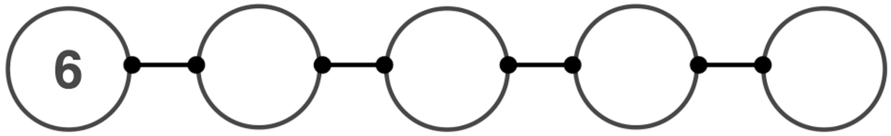
The solution works because
15 = (2+1) + (1+8) + (2+1) for the two corners
21 = (1+5) + (1+8) + (1+5) for the other two corners
18 = (1+5) + (2+1) + (1+5) + (2+1) in the center

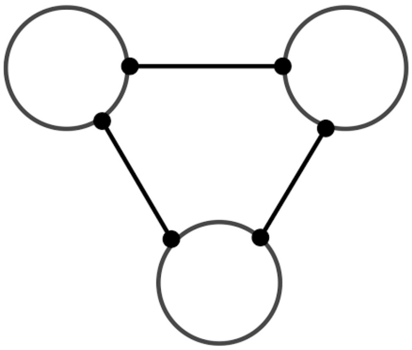


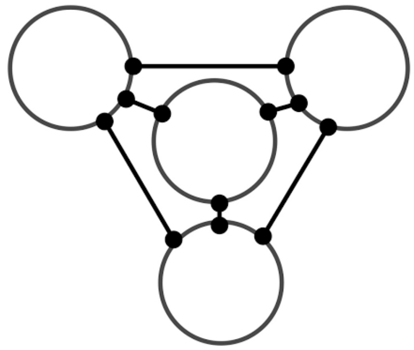


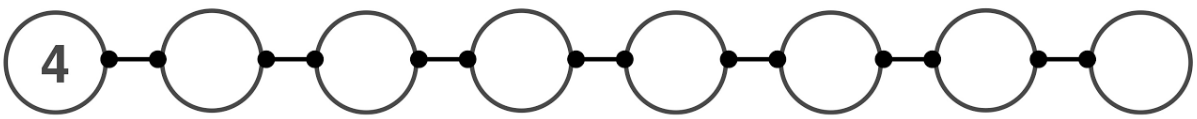


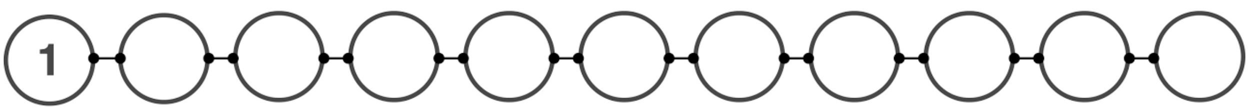
Some of these may have more than one solution

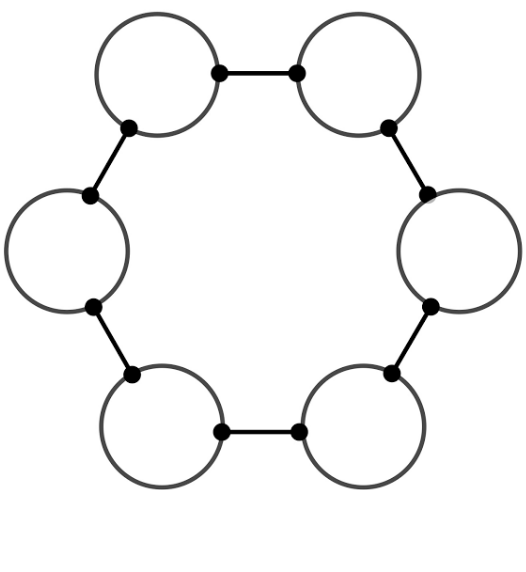


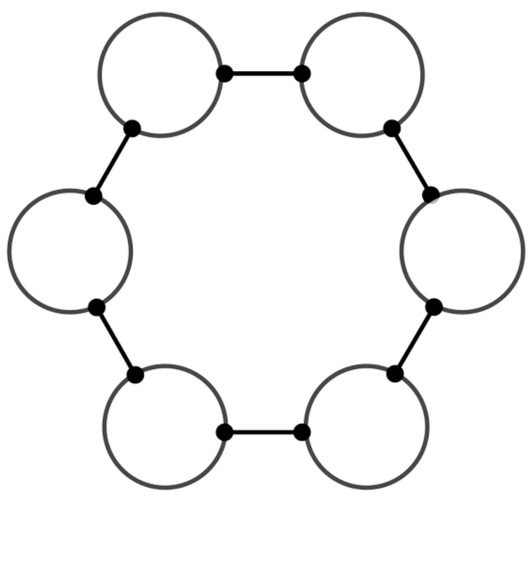






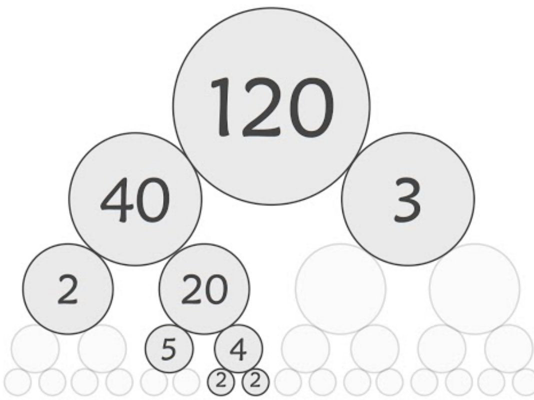


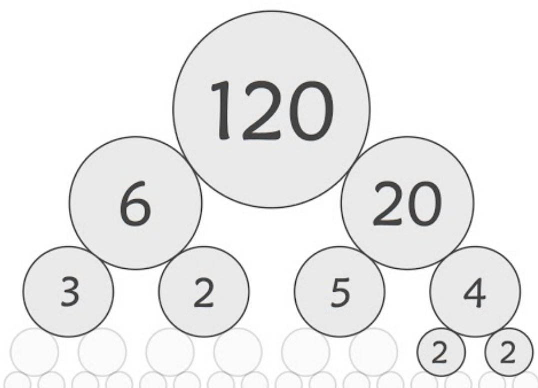




INTEGRAL FISSION

In the 20th century, integers peacefully dreamed of their prime factorization trees. A prime factorization tree is created by splitting an integer into factors ≥ 2 and repeating the splitting on these fractions etc.. Most integers have many prime factorization trees. Here are some for the number 120:

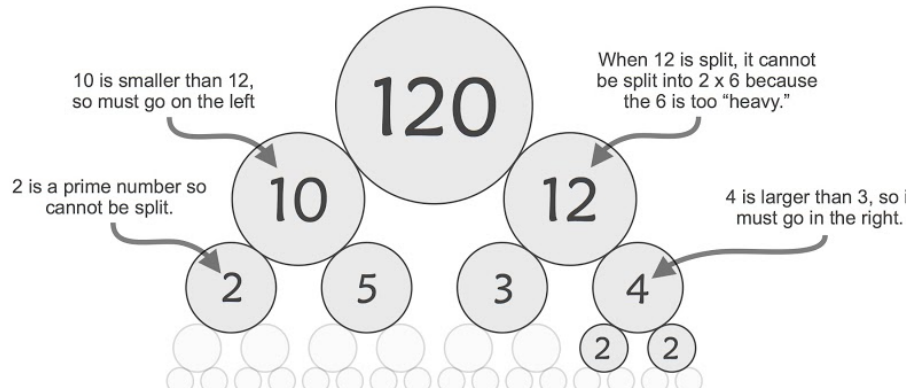




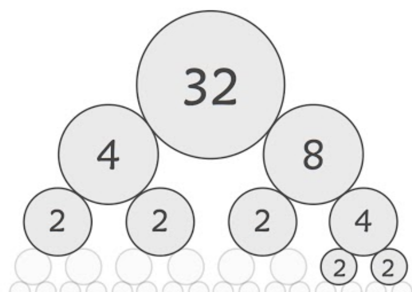
In the 21st century, numbers now dream of exploding in integral fission. There are two additional rules for integral fission:

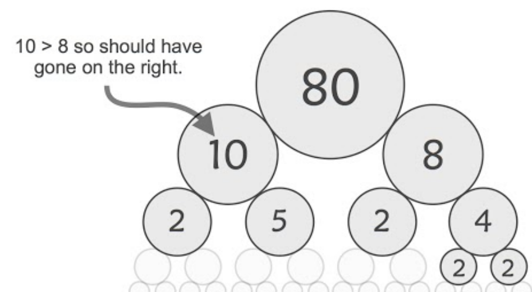
1) Splits must be as equal as possible.

2) If one of the splits is bigger than the other, it must go on the right.



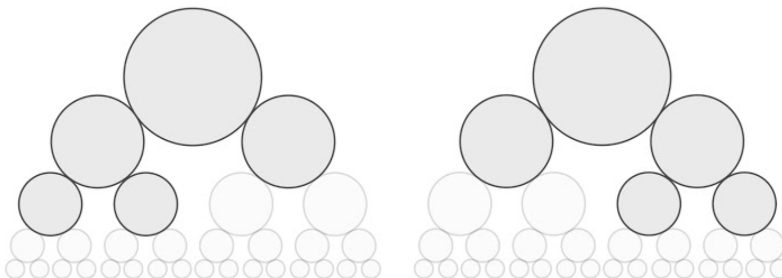
Unlike prime factorization trees, integral fission is unique for each positive integer. The "shape" of an integer is the pattern remaining when the numbers are removed. 32 is the smallest integer which shares the same shape as 120. Does 80 have the same shape? No.





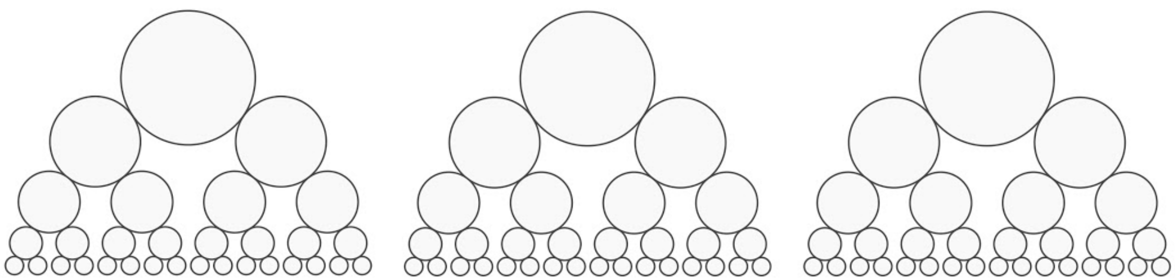
Some grade 5 / 6 students in River Valley School challenge you to find:

1) When the shape of the left first appears?
(The shape on the right first appears at 8.)

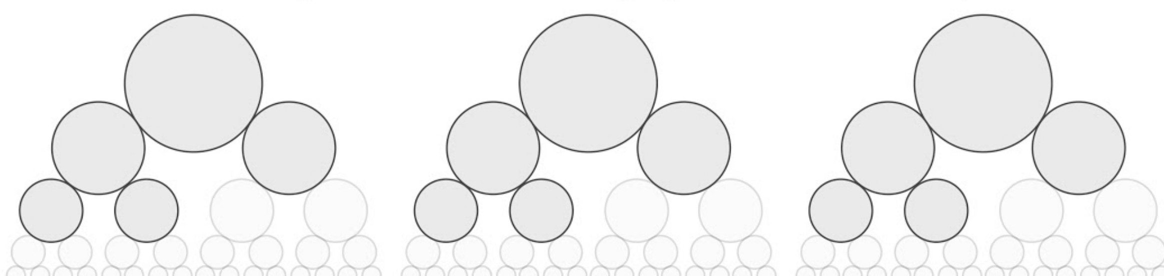


Bonus: Max wants you to find the first time an integer k has the shape on the left and k+1 has the shape on the right.

2) Three consecutive integers between 2 and 50 that have the same shape.
(They're not telling you what the shape is)



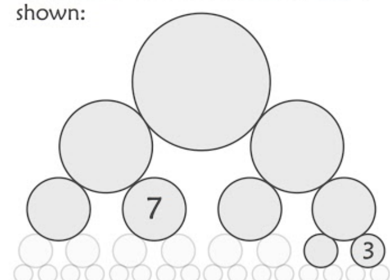
3) The first time a certain shape arises as a pair of consecutive integers is 116 and 117. Find the first time this shape arises in three consecutive integers (hint: it's between 160 and 180):



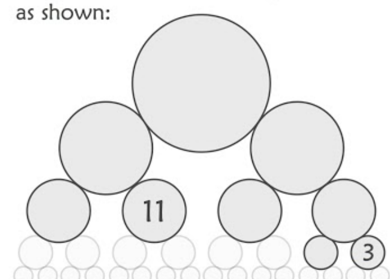
Mr. Pickle asks you to find:

1) The first 25 shapes to appear.

2) The five integers which fission in this pattern with the integers 7 & 3 as shown:



3) The five integers which fission in this pattern with the integers 11 & 3 as shown:

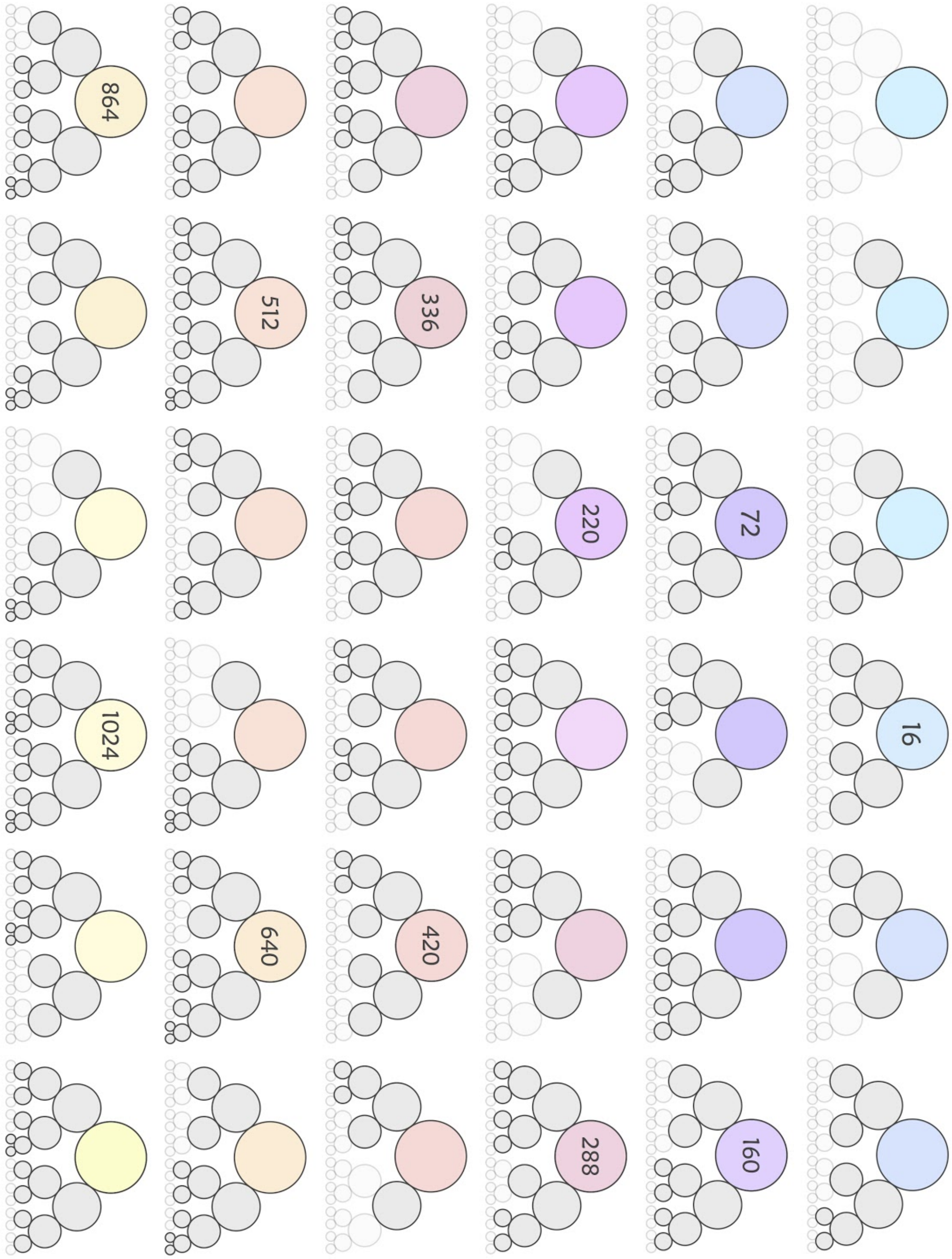


4) What shape do the integers 1431 and 1432 share?

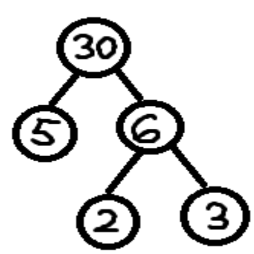
5) What shape do the integers 1885, 1886 and 1887 share?

6) Is there a shape that appears for the first time associated with an odd integer? (I don't know - $100 reward)

7) Is there an integer >1 which is a cube and has a shape that has mirror symmetry? (I don't know - $100 reward)



Each fission must be as equal as possible. If a large circle contains the number 30, you may not fission into 2*15 or 3*10. You must use 5*6.



If the two parts of a split are not equal, then the larger part must be on the right. Therefore, thirty fissions as shown to the right →.

A Couple of Zandra Vinegar's Favorite Math Circle Puzzles

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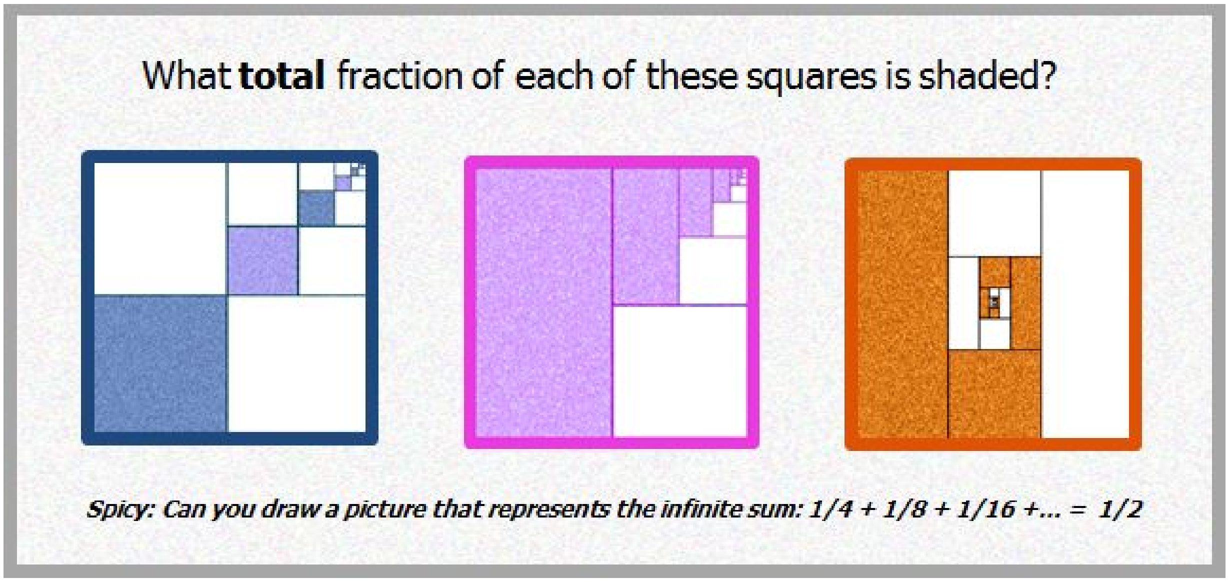

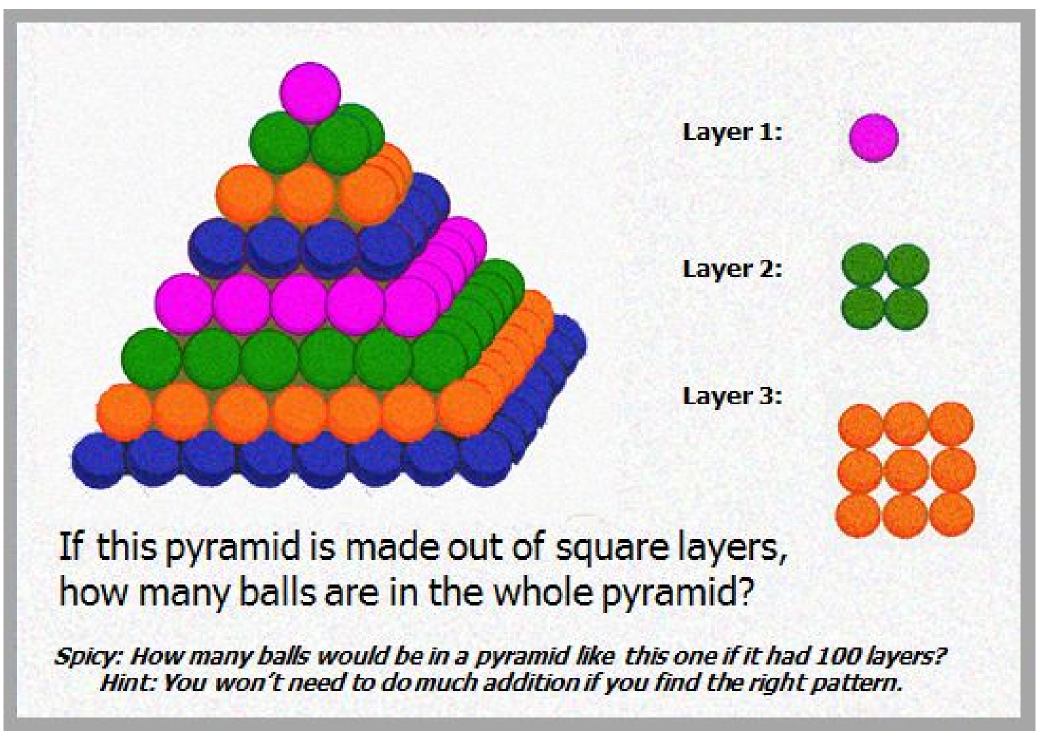

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