Developing the Horizon Content Knowledge of Teachers through a Math Teachers' Circle

Creating Coherence in K–12 Mathematics, II

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Outline

A few leading questions:

- What is Horizon Content Knowledge?
- Why is it important?
- Can teachers develop theirs through participating in a math teachers' circle?

Some Background

- Shulman - Pedagogical Content Knowledge.
- Ball, Hill and colleagues - Math Knowledge for Teaching.
- Can it be measured? Does it affect student outcomes? Which parts are most important?

Horizon Content Knowledge

From Ball, Thames, Phelps (2008):

> *Horizon knowledge is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum.*

This is refined by Ball and Bass(2009).

> *We define horizon knowledge as an awareness - more as an experienced and appreciative tourist than as a tour guide - of the large mathematical landscape in which the present experience and instruction is situated. It engages those aspects of the mathematics that, while perhaps not contained in the curriculum, are nonetheless useful to pupils' present learning, that illuminate and confer a comprehensible sense of the larger significance of what may be only partially revealed in the mathematics of the moment.*

Some Loose Data

I sampled via a questionnaire the 11 teachers at my MTC.

My teaching improves when I learn new pedagogical techniques.

Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
4	7	0	0	0

The mathematics I learned in college connects to the mathematics I now teach.

Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
2	3	3	3	0

Good teaching is informed by a deep understanding of mathematics.

Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
8	1	0	1	0

Data Continued

I teach mathematics because I enjoy doing mathematics.

Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
8	1	1	0	0

I have an answer when students ask "when will I ever use this?"

Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	7	1	1	0

Example 1: Algebra

Question

What horizon knowledge should teachers have in order to teach the quadratic formula well?

- (Its derivation via completing the square.)
- Applications of finding roots.
- Some history of solving algebraic equations e.g., Cardano's formula.
- Fundamental Theorem of Algebra.
- Abel-Ruffini Theorem

This is an example of starting with the content of school mathematics and looking ahead to its applications, consequences, and theoretical underpinnings.

Example 2: Function Composition

Question

Take a strip of paper, make a mark in it and then alternatively fold the left and right edges to the most recent fold. What happens if you continue this process?

- Modeling with mathematics.
- Thinking about limiting processes.
- Dynamical systems.
- Contraction mapping principle.

This is an example of starting with straightforward question and looking ahead to its applications, consequences, and theoretical underpinnings.

Example 3: Decimals

Question

What horizon knowledge should teachers have to teach decimal numbers? (Why is $0.\overline{9} = 1$*?)*

- Place value.
- Infinite series.
- Natural Numbers $\longrightarrow$ Integers $\longrightarrow$ Rationals $\longrightarrow$ Reals $\longrightarrow$ Complex Numbers.

Concluding Thoughts

- Is this a good way to develop HCK?
- Can it be measured? Does that matter?
- MTCs should be beneficial to teachers regardless of the choice of content, but one can organize a circle around this concept and perhaps over a period of time give teachers opportunities to grow in their knowledge of the mathematical horizon and see more coherence in the curriculum.

Thanks for your time.

References

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