The Whats and Whys of the AMS MSRI Mathematical Circles Library

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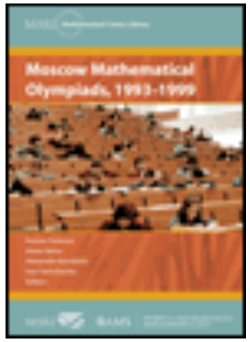
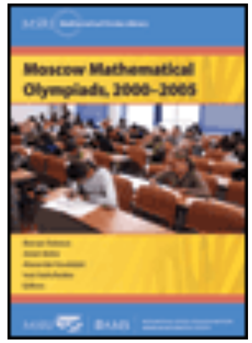
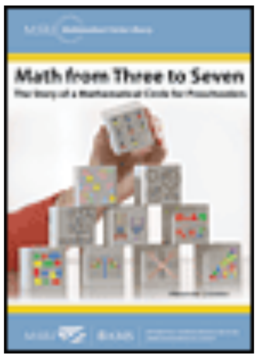
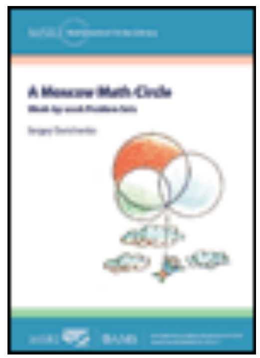
Circle in a Box

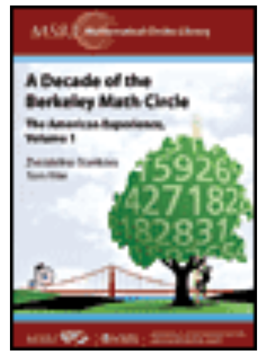

Moscow Mathematical Olympiads, 1993–1999

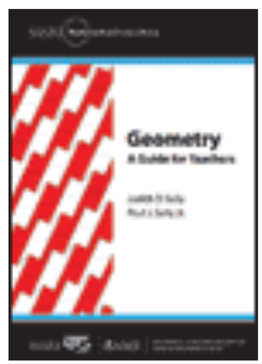

Moscow Mathematical Olympiads, 2000–2005

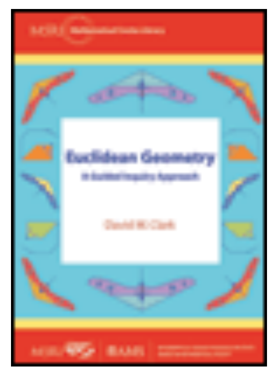

Introduction to Functional Equations

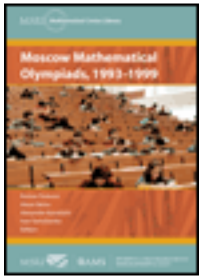

Math from Three to Seven


A Moscow Math Circle


A Decade of the Berkeley Math Circle
The American Experience, Volume 1


Geometry
A Guide for Teachers


Euclidean Geometry



Problems

Hints

Answers

Solutions

Example: 1996 Olympiad Level D

- *Problem 3.* A point is chosen on each of eight evenly spaced parallel planes. Can these points be the vertices of a cube?

- *Hint:* Write the equations of parallel planes drawn through the vertices of the unit cube in such a way that the distances between any two neighboring planes are the same.
- *Answer:* Yes, they can.

Solution:

Any set of evenly spaced parallel planes is equivalent to any other under a similarity, and any cube is also similar to any other.

So the question can be rephrased thus: Can the vertices of the unit cube lie on evenly spaced parallel lines?

Consider planes $ax + by + cz = k$ for fixed a, b, and c but varying k. This is a family of parallel planes. The distance between two planes is proportional to the differences between the corresponding values of k: in particular, the distance between the planes corresponding to to k_1 and k_2 is the same as the distance between the planes corresponding to k_3 and k_4 if and only if $k_2 - k_1 = k_4 - k_3$.

We wish to find a, b, c so that the values of the function on the vertices of the unit cube are evenly spaced. These values are $0, a, b, c, a+b, b+c, a+c$, and $a+b+c$. A moment's thought will show that our condition will be satisfied if, for example, $a = 1$, $b = 2$, and $c = 4$. In other words, the planes defined by $x + 2y + 4z = k$, for $k = 0, 1, \ldots, 7$, contain each one vertex of the unit cube.

Problem 5.13 (Putnam, 1947)

Find the continuous functions $f: \mathbb{R} \to \mathbb{R}$ satisfying the following equation:

$$f(\sqrt{(x^2+y^2)}) = f(x)f(y)$$

for all $x, y \in \mathbb{R}$.

Problem 19.16 (Greece 1996)

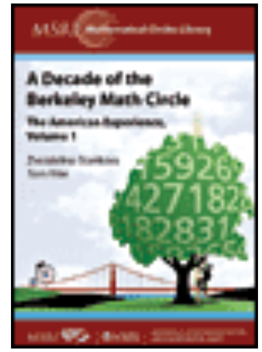
Let $f : (0, \infty) \to \mathbb{R}$ be a function such that

a) f is strictly increasing;

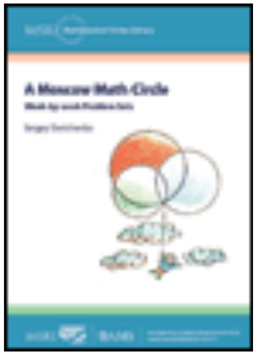
b) $f(x) > -1/x$ for all $x > 0$;

c) $f(x)f(f(x) + 1/x) = 1$ for all $x > 0$.

Find $f(1)$.

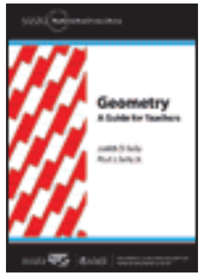


1. Inversions in the Plane. Part I
2. Combinatorics. Part I
3. Rubik’s Cube. Part I
4. Number Theory. Part I
5. A Few Words About Proofs. Part I
6. Mathematical Induction
7. Mass Point Geometry
8. More Proofs. Part II
9. Complex Numbers. Part I
10. Stomp. Games with Invariants
11. Favorite Problems of BMC. Part I
12. Monovariants. Part I



Problem 8.10: A Xork lands on a planet that contains 100 Yorks. Every day after that, a battle takes place where each Xork destroys one York; right after the battle, every Xork and every York split into two. Prove that , sooner or later, all Yorks will be destroyed. How long will it take for that to happen?

Problem 28.12: In three-dimensional space, we are given a point light source emitting light in all directions. Is it possible to choose (a) 100 opaque balls, (b) 4 opaque balls, and place them in space in such a way that they don't intersect each other, don't touch the light source, and completely block its light?



"Geometry has always been a central part of the school curriculum for two reasons: it provides an understanding of the physical space we live in and it offers a workshop in logical reasoning that is required in all walks of life.

The elementary/middle school curriculum is more directed toward the former while the high school/college version emphasizes the latter.

The two work together to build both skills over a period of years. Appropriately, Sallys' book takes a description analytical approach, with multiple activities to familiarize children with geometric properties and help them learn to articulate them correctly.

My book gives an axiomatic development, fostering skill in logical reasoning as the students develop a hierarchy of successively more advanced ideas, one building on the other. " – *David Clark*

Titles to appear:

1. A Year of a Mathematical Circle. Workbook for Grades 5 to 7, by Anna Burago
2. Invitation to a Math Festival, by Ivan Yashchenko
3. Number Theory for Teachers, by Judith and Paul Sally
4. A Decade of the BMC, v.2, by Tom Rike and Zvezda Stankova
5. Collected Works for High School Students, by Vladimir Arnold
6. The SMART Circle – 32 years in Edmonton, by Andy Liu
7. The World Youth Mathematics Intercity Competition, by Andy Liu and Hua-Wei Zhu
8. Math Circle for Elementary School Students, by Natalia Rozhkovskaya
9. Math Teachers' Circle Book, by Paul Zeitz and Tatiana Shubin