Complex Behavior from Simple Rules

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Cellular Automata for Math Circles

Cellular Automata: A grid of cells, each of which is in one of a finite set of states, and which evolve following a set of specified rules.

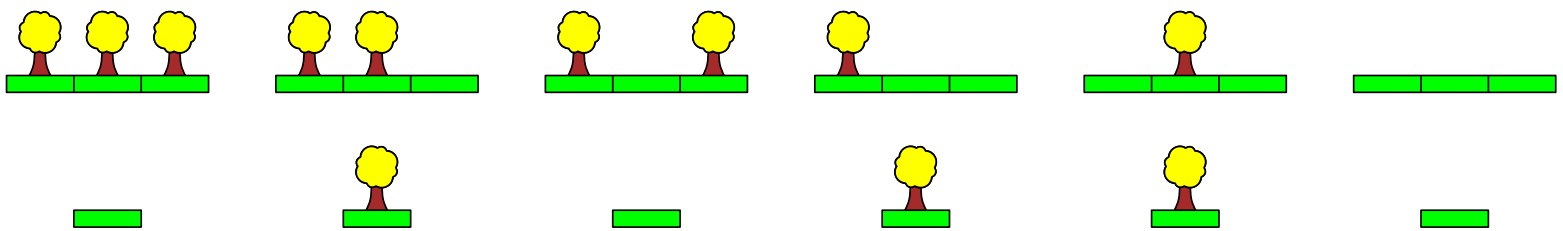
Audience: Elementary-aged students.

Goals: Help students see first-hand how complex behavior can emerge from very simple rules, and can try to predict the patterns that will develop. They also see how real world systems can be modeled using simple mathematical rules.

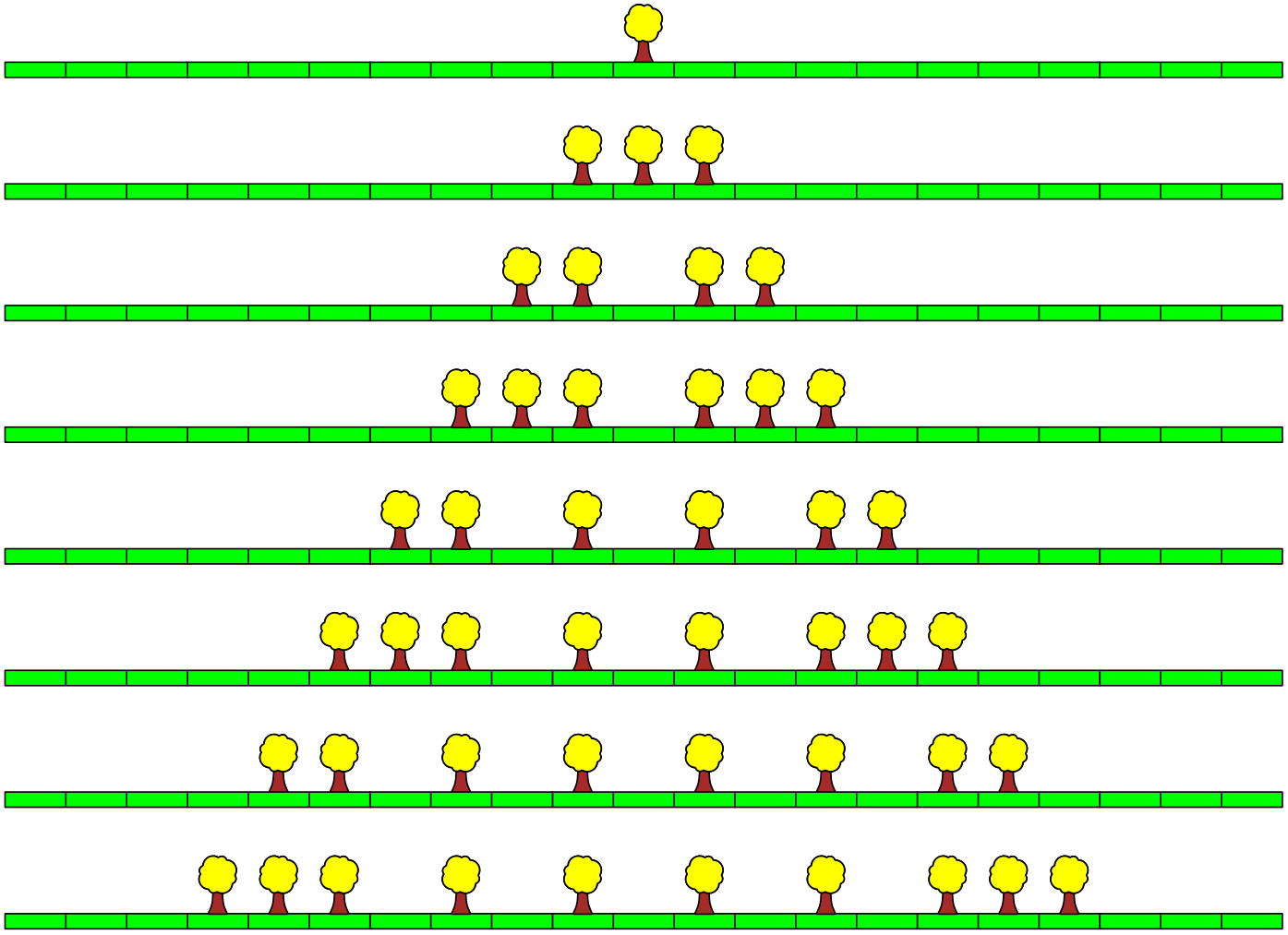
Great Yellow Skrinkleberry Tree

Grows on the side of a single volcano in Iceland. Each year it drops two seeds, one on either side.

They need lots of direct water and sunlight, and can only survive if there is not another tree on both its left and right sides.

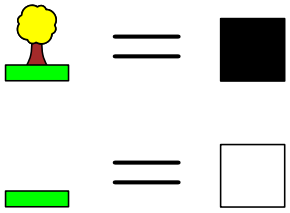


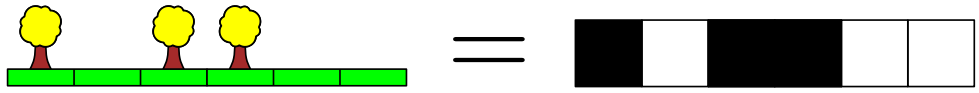
Great Yellow Skrinkleberry Tree

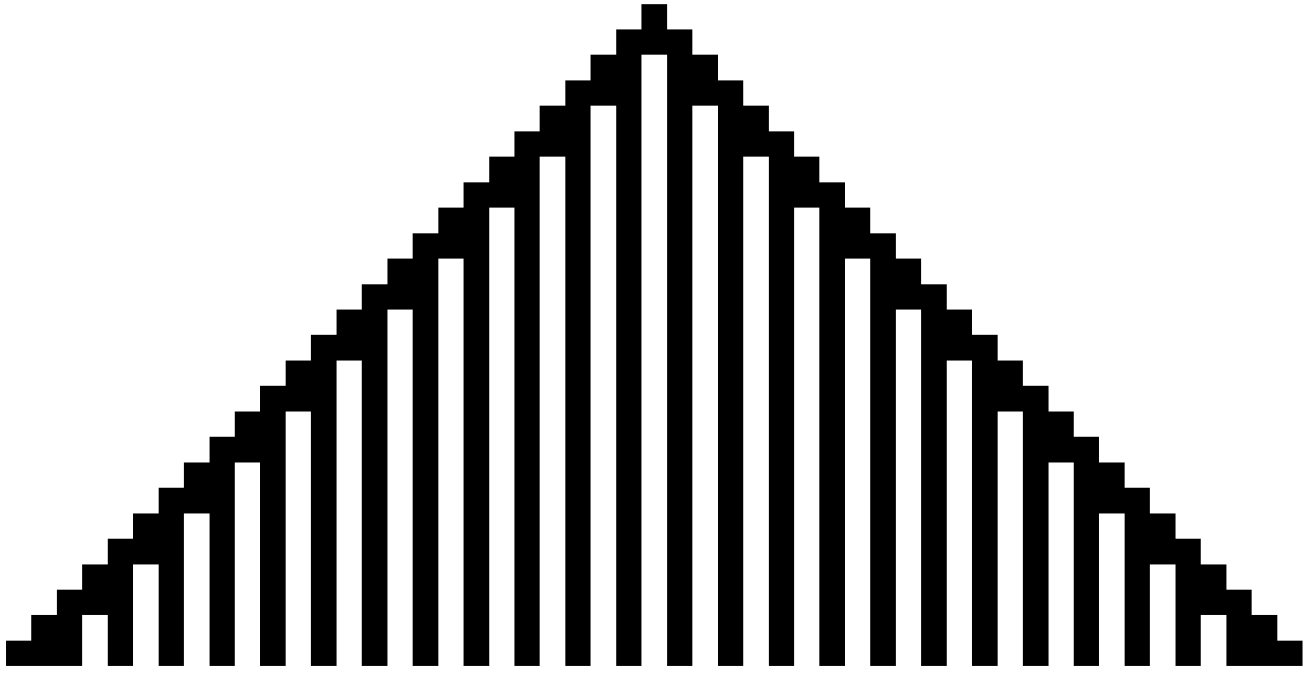


Great Yellow Skrinkleberry Tree

Represent each space on our mountainside as a cell, which can either contain a tree or not:



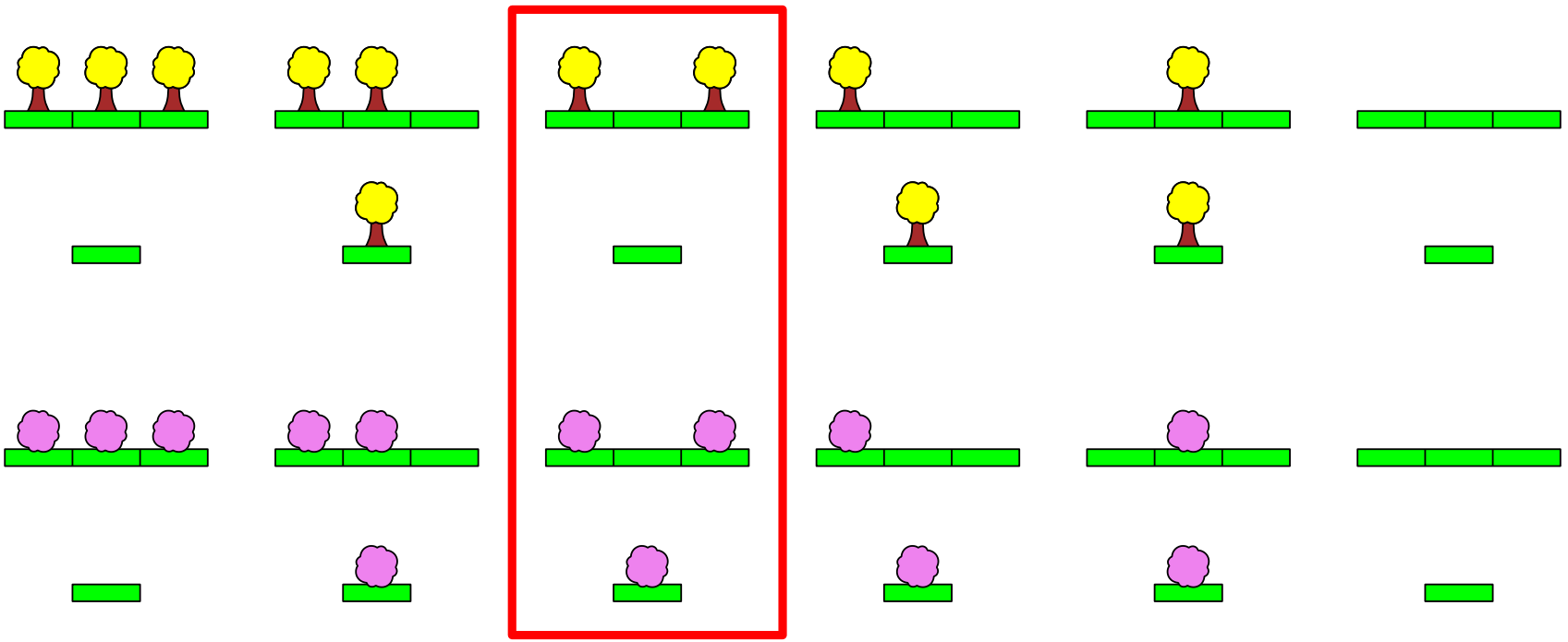




Pink Toed Snorkleberry Bush

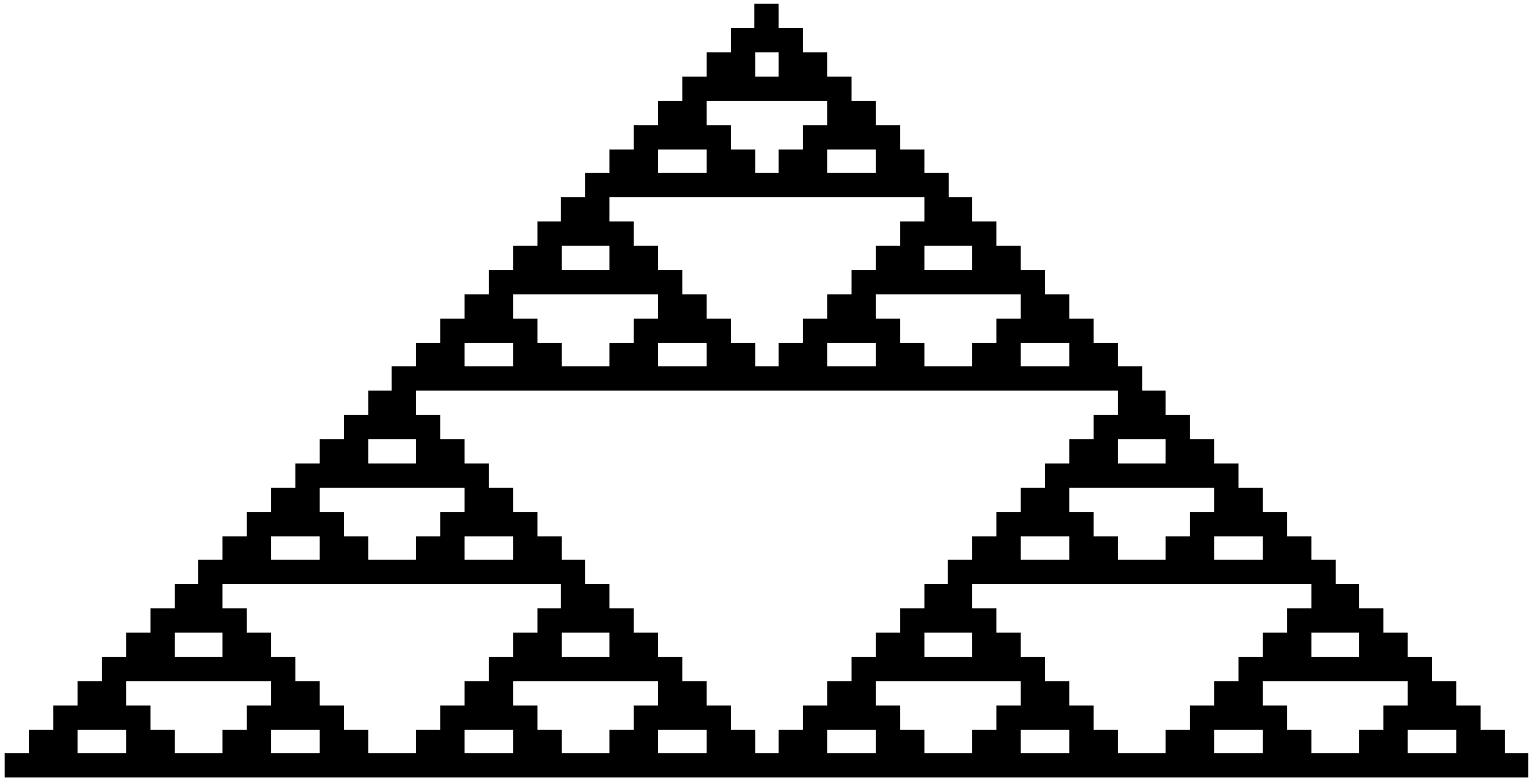
Cousin of Great Yellow Skrinkleberry Tree.

The only difference is that when two bushes are separated by a single space, a bush will still grow in between them for a single year.

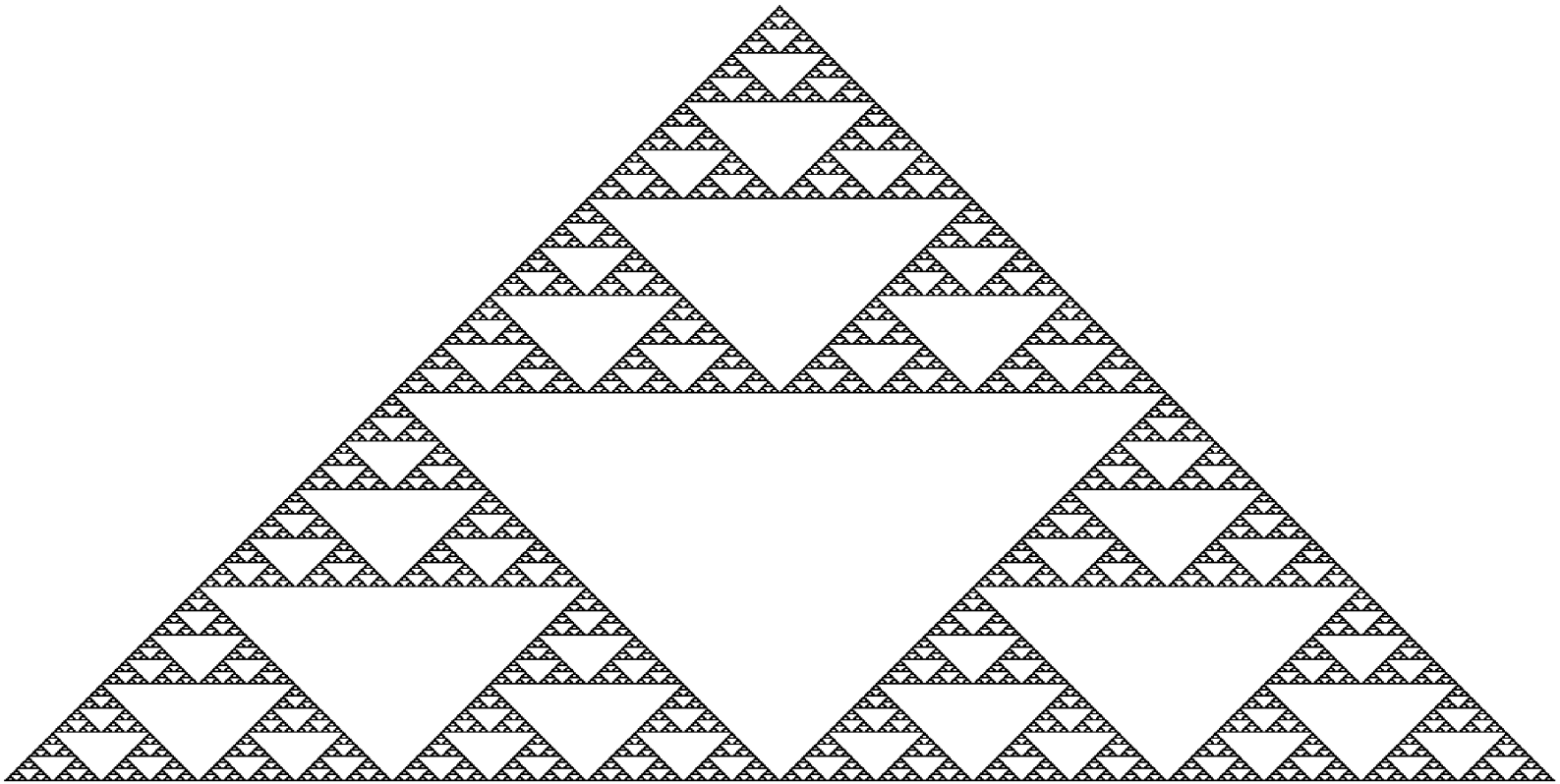


How does changing a single rule change the pattern of growth?

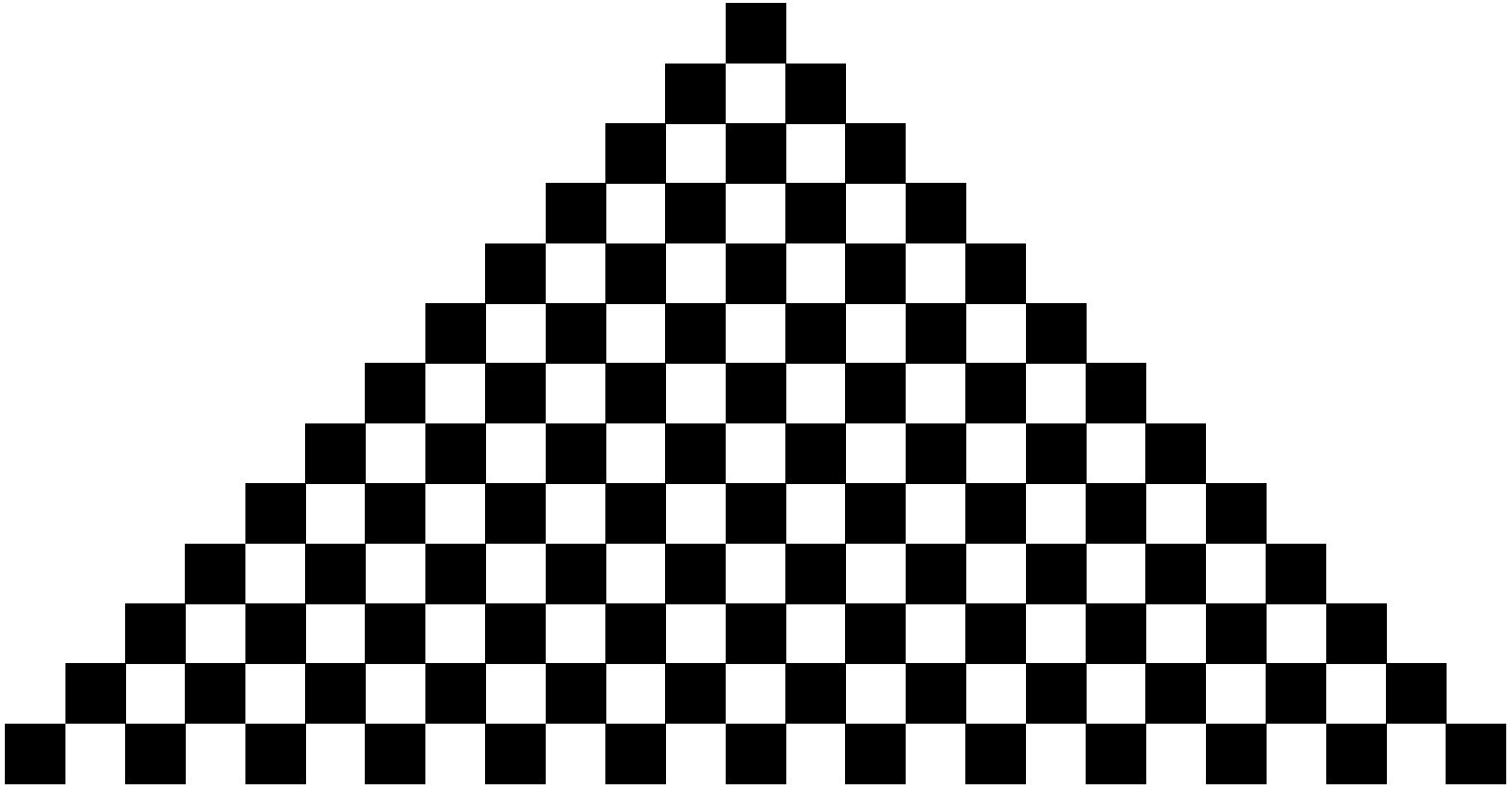
Pink Toed Snorkleberry Bush



Pink Toed Snorkleberry Bush



Purple Wartnose Petunia

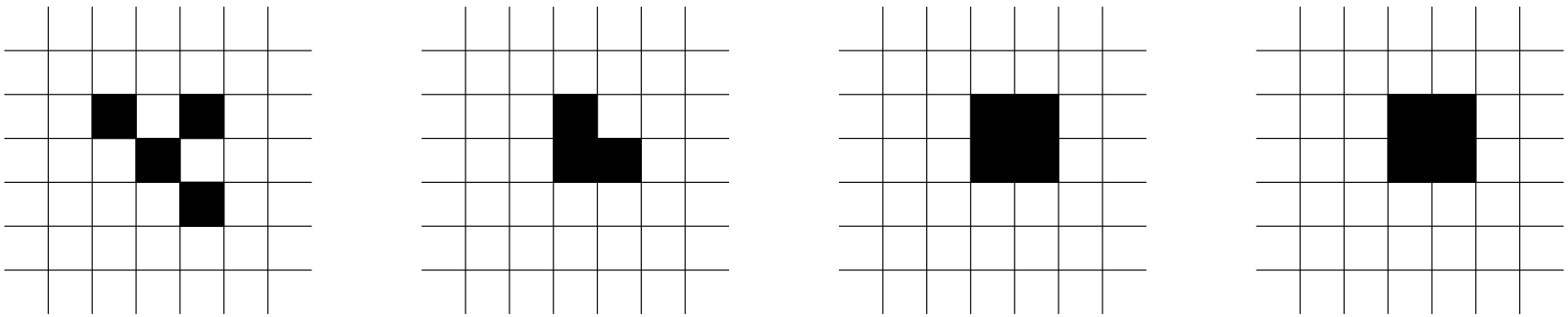


Conway's Game of Life

Example of a 2-dimensional cellular automata.

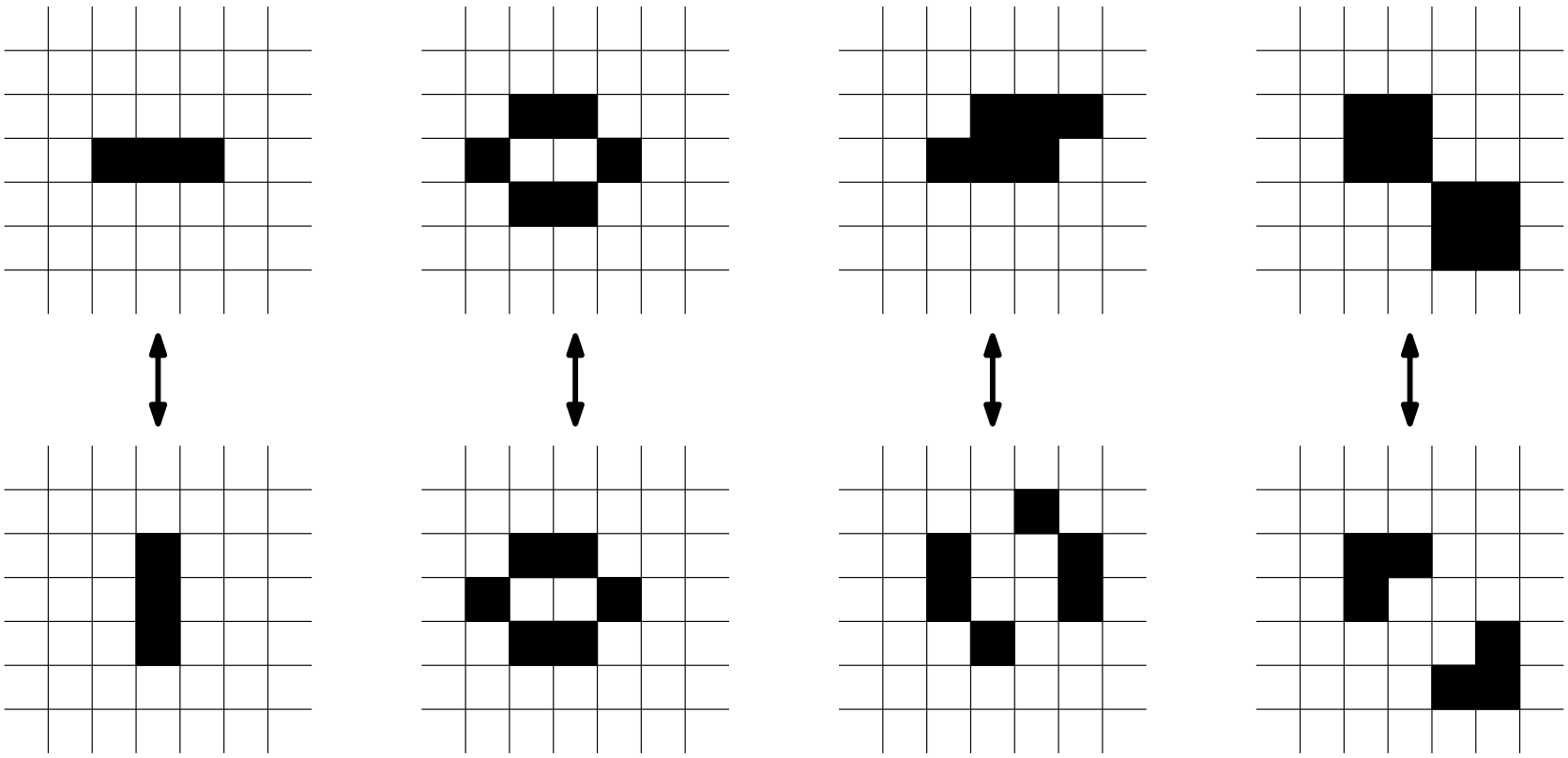
Rules:

1. Cells only survive if they have 2 or 3 living neighbors.
2. Dead cells with precisely 3 neighbors come to life.



Conway's Game of Life

Students used paper and pencil to explore evolution of simple patterns:



Students used tablets (and freely available apps) to explore more complicated patterns.

Observations

Students were given the opportunity to:

- see examples of how problems in biology can be modeled by mathematics,
- practice following specifies rules to observe how the systems evolved,
- interpret their observations in the context of their original problem,
- observe how simple rules can give rise to complicated patterns, and how these patterns change unpredictably as the underlying rules do, and
- use technology in exploring an interesting mathematical topic.

Thank You!