



Computed Tomography (CT) in High School Mathematics

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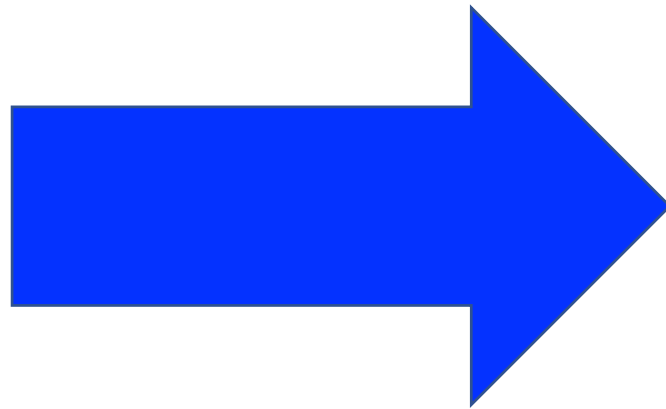
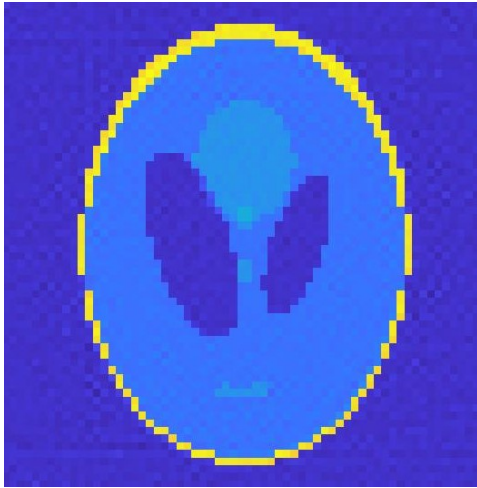


Acknowledgements

- Dr. Lauren Rose, JMM 2022 Special Session Organizer.
- Dr. James Nagy, Emory University Summer 2021 RET experience.



Introduction



Question to Consider:

- How can these concepts be introduced in High School mathematics to motivate students exploration of mathematics?

Mathematical Concepts in High School Algebra

Gaussian Elimination

- Systems of Equations
- Matrix Form: $Ax = b$
- Augmented Matrix

• Possible Matrix Forms for a System of Three Linear Equation in Three Variables

Form 1: Exactly one solution

$$\left[\begin{array}{ccc|c} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{a} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{b} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{c} \end{array} \right]$$

a, b, c, d are real numbers; d ≠ 0

Form 2: Infinitely many solutions

$$\left[\begin{array}{ccc|c} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{a} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{b} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array} \right]$$

Form 3: No solution

$$\left[\begin{array}{ccc|c} \mathbf{1} & \mathbf{0} & \mathbf{a} & \mathbf{b} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{c} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{d} \end{array} \right]$$



Instructional Model

Lesson Objectives

- Determine the difference between infinitely number of solutions and a unique solutions as it applies to x-ray imaging.
- Solve a system of equations using Gaussian Elimination.

Lesson Structure: *The 5E Model*



Ref:<https://nasaclips.arc.nasa.gov/teachertoolbox/the5e>



The 5E Model

Lesson Ideas

- 1) Engage: Can Mathematics be used to save people's lives?
- 2) Explore: My Square!
- 3) Explain: An Infinitely Number of Solutions!
- 4) Elaborate: One Unique Solution!
- 5) Evaluate: Determine the difference between both solutions as it applies to x-ray imaging.



The 5E-Model Engage

ENGAGE

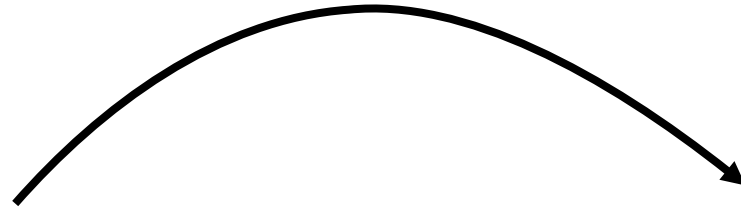
Math
can
Help Save
LIVES 



Students read an article about x-ray imaging.

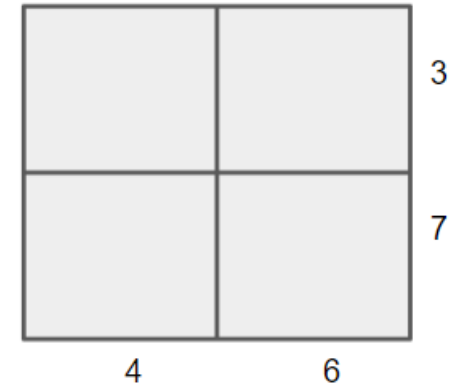


The 5E Model- Explore



EXPLORE

What goes in my Square?



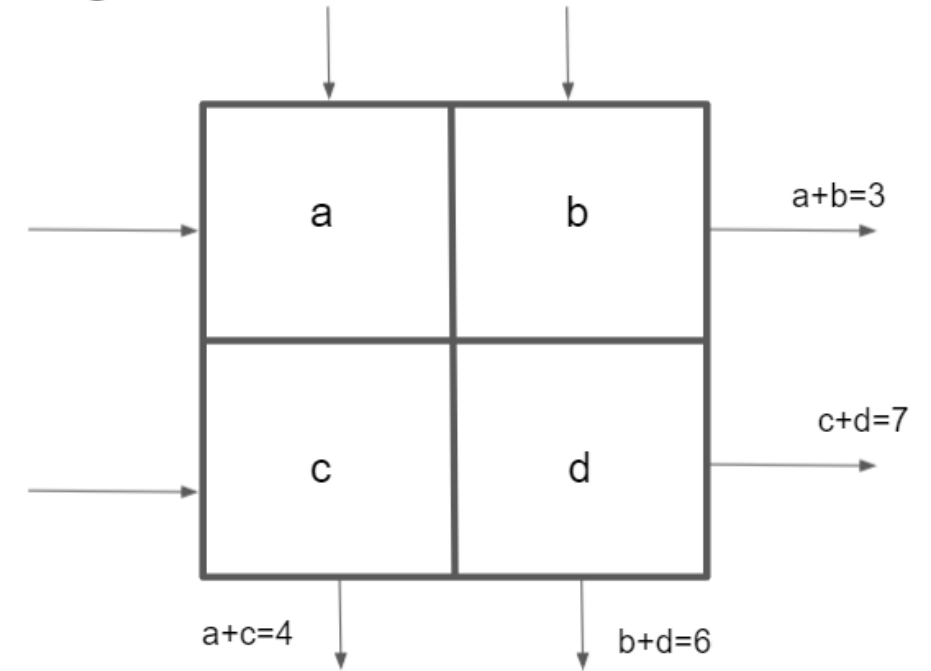
Students explore what numbers go in the squares



The 5E-Model Explain

Finding an infinite number of solutions.

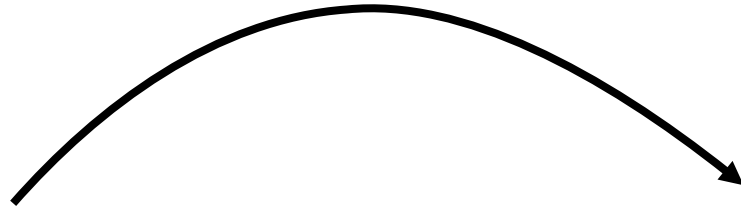
EXPLAIN



Teacher models the Gaussian Elimination method to students.



The 5E-Model Explain



EXPLAIN

```
3  
4 - clear all  
5 - close all  
6 - clc  
7  
8 - A = [1 1 0 0; 0 0 1 1; 1 0 1 0; 0 1 0 1];  
9 - b= [3 7 4 6]';  
10  
11 - Aug=[A b];  
12  
13 - C=rref(Aug)  
14  
15
```

Command Window

```
C =  
  
1 0 0 -1 -3  
0 1 0 1 6  
0 0 1 1 7  
0 0 0 0 0
```

fx >>

Teacher models how to check their solution using MATLAB programming language.



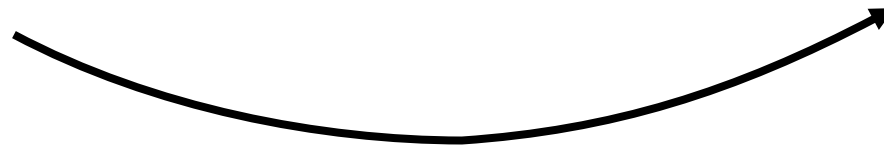
The 5E-Model Explain

EXPLAIN

Definition of a Well-Posed Problem

For a problem to be well-posed:

1. A solution exists.
2. The solution is unique.
3. The solution's behavior changes continuously with the inputs.

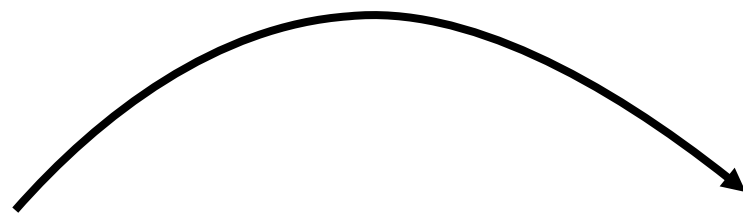


Teacher provides definition of a well-posed problem.

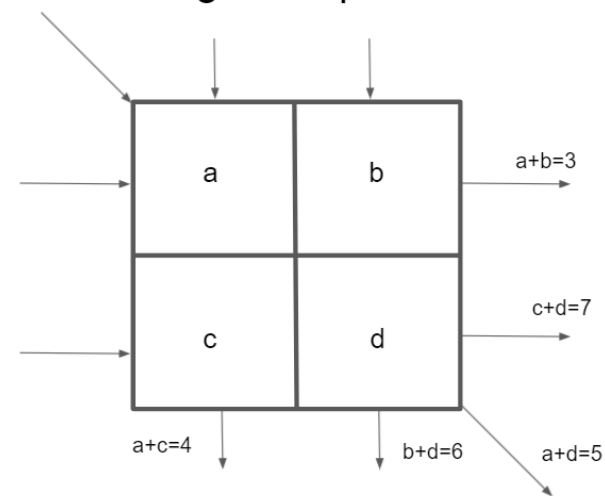


The 5E-Model Elaborate

E**L****A****B****O****R****A****T****E**



Finding a unique solution.



Students practice independently from the teacher to find a unique solution.

The 5E-Model Elaborate

ELABORATE

```
1 - clear all
2 - close all
3 - clc
4
5 - %A = [1 1 0 0; 0 0 1 1; 1 0 1 0; 0 1 0 1];
6 - %b= [3 7 4 6]';
7
8 - A = [1 1 0 0; 0 0 1 1; 1 0 1 0; 0 1 0 1; 1 0 0 1];
9 - b= [3 7 4 6 5]';
10
11 - Aug=[A b];
12 - %C=rref(Aug)
13
14 - p=rref(Aug)
15 - x=A\b
16
17
```

Command Window

D =

1	0	0	0	1
0	1	0	0	2
0	0	1	0	3
0	0	0	1	4
0	0	0	0	0

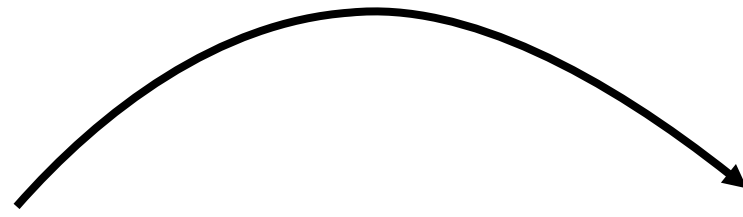
x =

1.0000
2.0000
3.0000
4.0000

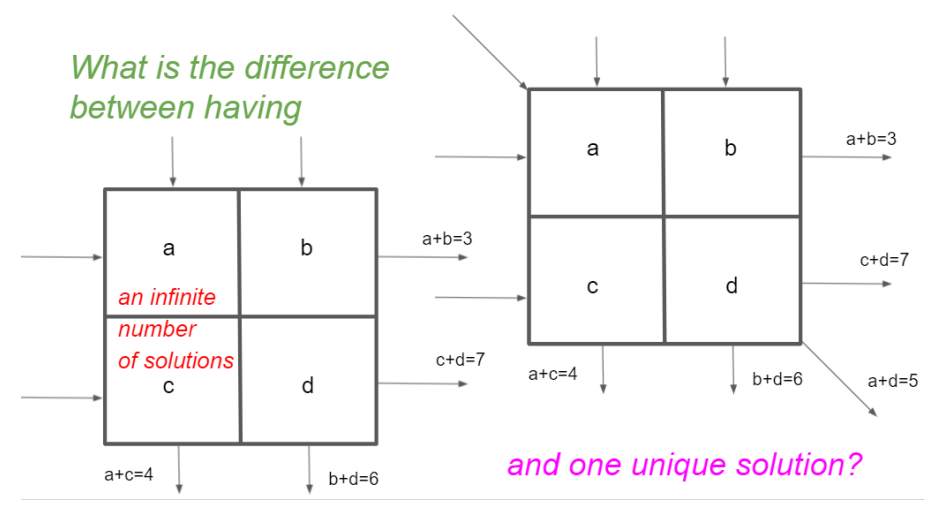
Students check their solution independently using MATLAB programming language.



The 5E-Model Elaborate



EVALUATE



Teacher assess student knowledge.



The 5E Model- Making Learning Meaningful

I learned

- 1) What is the difference between having one solution and an infinite number of solutions.
- 2) How to check my solution with MATLAB.
- 3) Why imaging problems tend to be ill-posed.



Computed Tomography (CT) and X-Ray Imaging

- The "CT" Scanner.



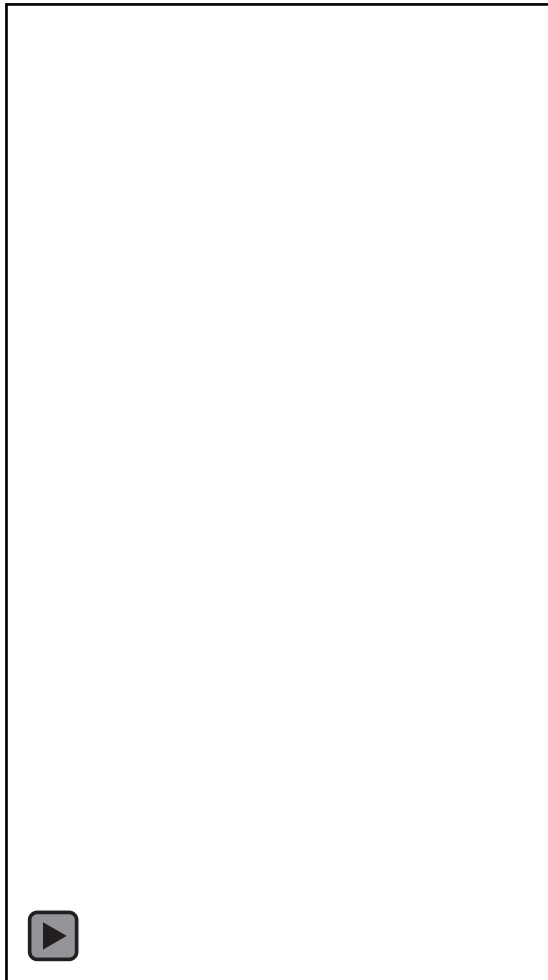
"Flash CT Scanner" by [Scott & White Healthcare](#) is marked with [CC BY-NC-ND 2.0](#).

→ when teaching this to students, we need to explore this in more detail.

Q: How do CT scanners work?



Images



```
clear all
close all
clc

RGB = imread('RotatedPoodle.jpg');
imshow(RGB);

I=rgb2gray(RGB);
figure
imshow(I)

level=graythresh(I);
BW=im2bw(I, level);

figure
imshow(BW);
imtool(BW)
```




Future Directions

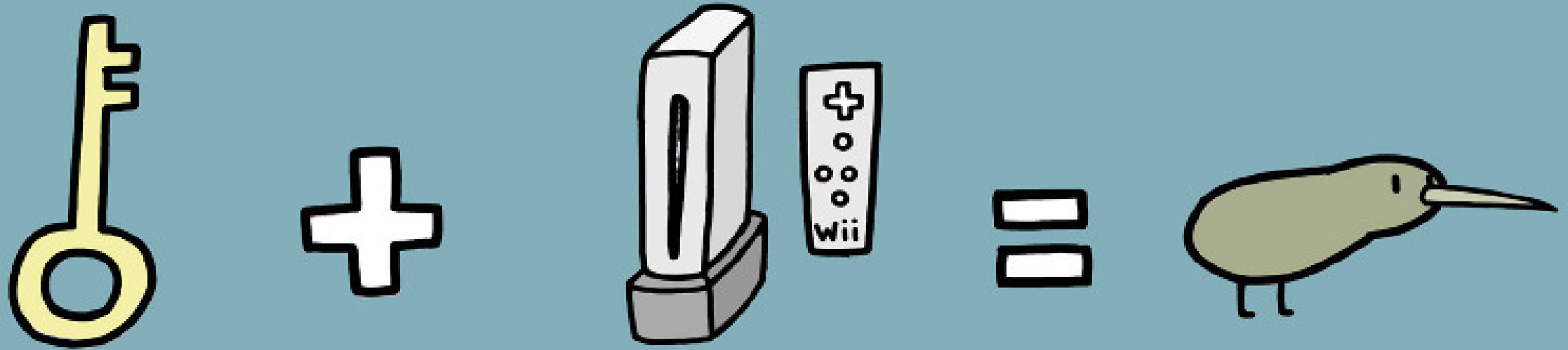
- Continue reading to expand my knowledge of image processing.
- Continue reading to expand my knowledge of mathematical algorithms that allow for better quality images.
- Continue using MATLAB and create some of my own code to explain these examples to students.



References

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THANK YOU!



Math. It explains everything.