CryptoClue

An Unexpected Topic for a Math Circle

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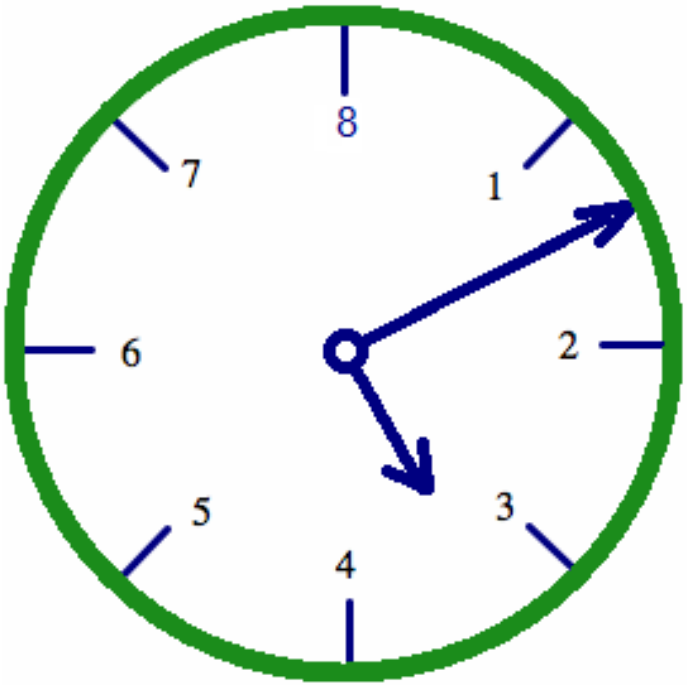
Trinity Christian College

January 6, 2017

An Hour-able Crime!

- Location: Archimedes Junior High



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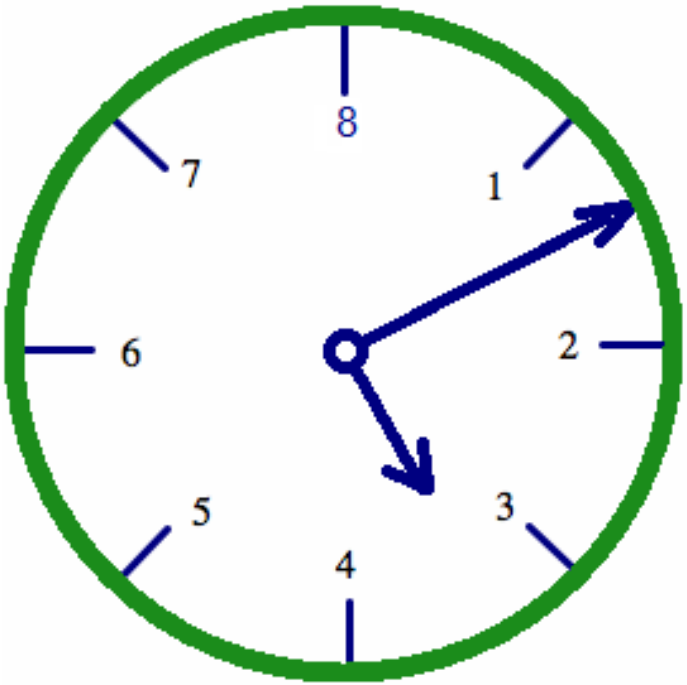

- Principal: **Mrs. Bletchley**
- To solve this **enigmatic** mystery, you will need to solve three clues based on historical cryptosystems.
  - Where?
  - Who?
  - With what?

Narrowing the Options

- Principal Bletchley has identified the following options for the crime:
  - Locations for the criminal act: Outhouse, Walk-in closet, basement, Tree house, Attic, kitchen cabinet
  - Many weapon options, including: a dustbuster, a tennis shoe, a plunger, a squirt gun, shrubbery, bagpipes, etc.
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- Volunteer crytanalysts?

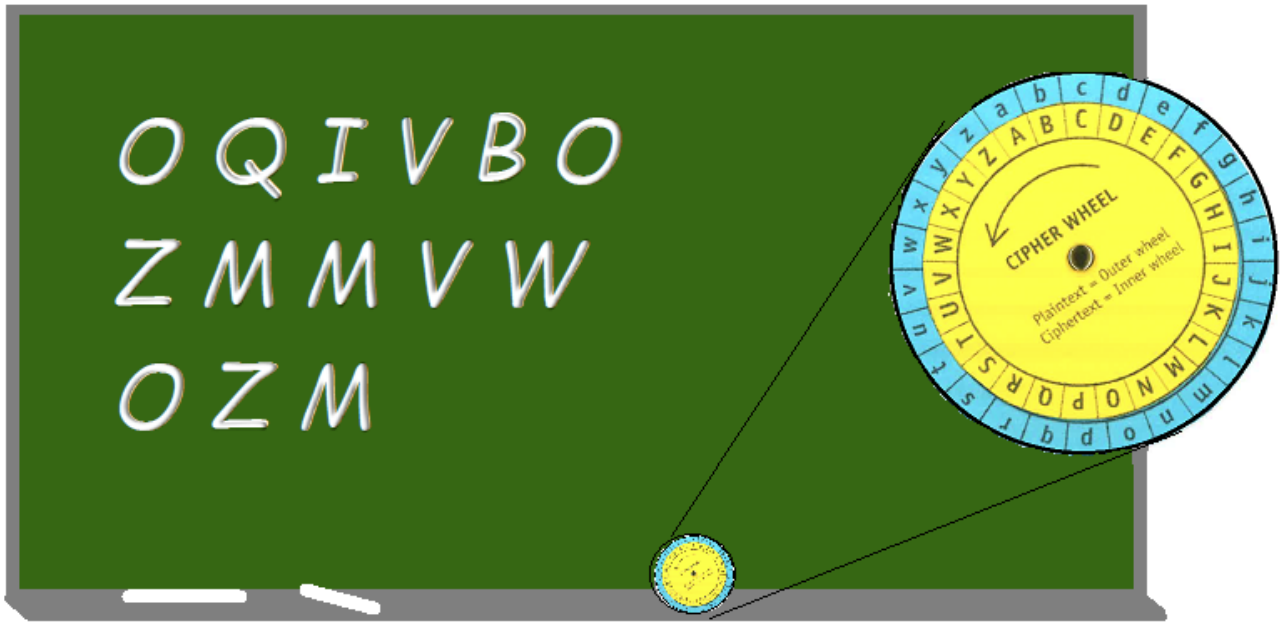
Where? Clue #1: Letters Inscribed on Colored Ribbons

- Curious rods in **Mrs. Oiler's** geometry classroom, ribbons dangling from light fixtures

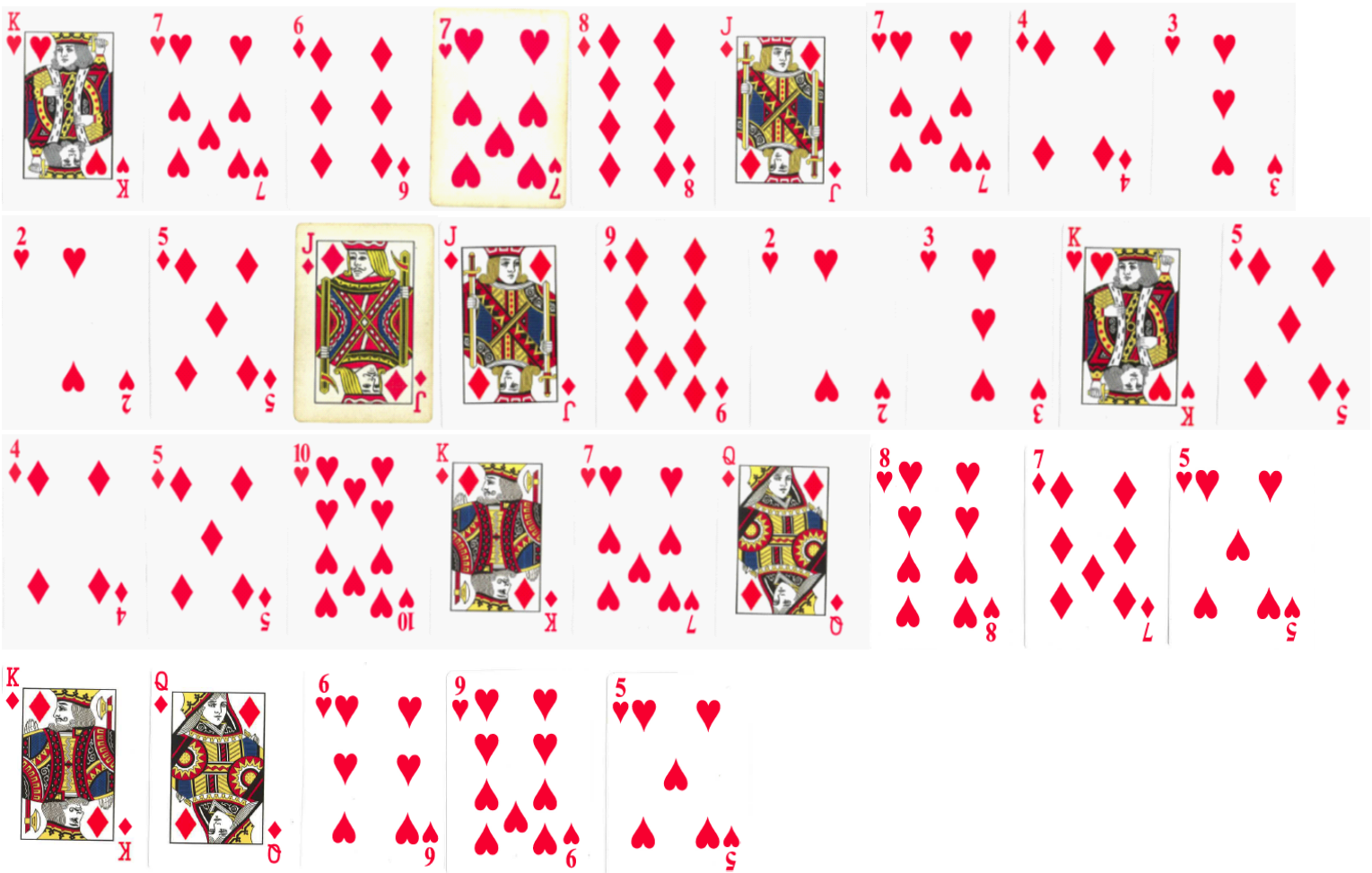
- Hint: To solve the riddle, you must test the ribbons by wrapping them around the rods provided to see if the resequencing makes sense.

Who? Clue #2: A Cipher Wheel and Cipher Text

- In **Mr. Seezer's** classroom:



With What? Clue #3: A sequence of playing card symbols of one color



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- Curious fact: the number of red playing cards is identical to the number of letters in the English alphabet.
- A suspicious challenge by your math tutor, **Ms. R.S. Adleman:** "I'll give you one card-letter clue for each correct problem you answer..."
- Hint: you need not obtain all 26 cards to solve the riddle and determine the weapon of clock destruction.

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- BUT where
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- Cryptosystems are considered secure, if the enemy is unable to decipher the message even if everything about the system is *public knowledge* except for the key. (Kerckhoffs' principle)

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  - The cipher used in this clue is a shift cipher, a variation of the Caesar cipher.
- With What? Clue #3: A sequence of playing card symbols of one color
  - The cipher used in this clue is a substitution cipher, believed to be first used by Arabs prior to the 10^{th} century AD. The Arabs also were the first to do frequency analysis of characters to break these codes!

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- With What: a dustbuster, a tennis shoe, a plunger, a squirt gun, shrubbery, bagpipes, etc..
  - Clue: *I am a plant collection of hardy shrubs.*
- **The clock crime was committed by Shrek with shrubbery in the basement.**

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  - Eight are teaching mathematics,
    - Lauren: I created "a cryptography lesson in Algebra 2 as a tie to inverse functions."
    - Dan: Cryptography "tied together history, humanities, technology, and of course mathematics in a way that made me feel like I was learning 'just for the fun of it.'"

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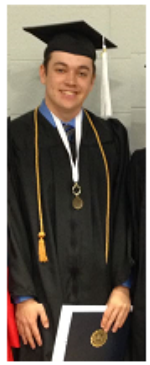
  - Spencer: After the Triathlon, "I ended up buying a book with a bunch of cryptography-related puzzles" to solve the following summer...



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    - Math topics in cryptography include: geometric measurement, function characteristics, modular arithmetic, statistics (frequency analysis), number theory
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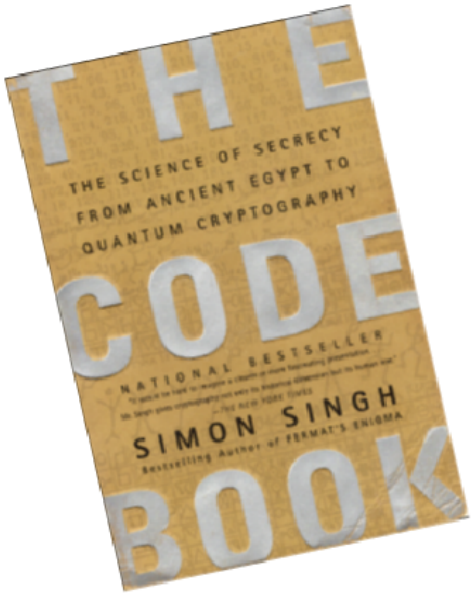
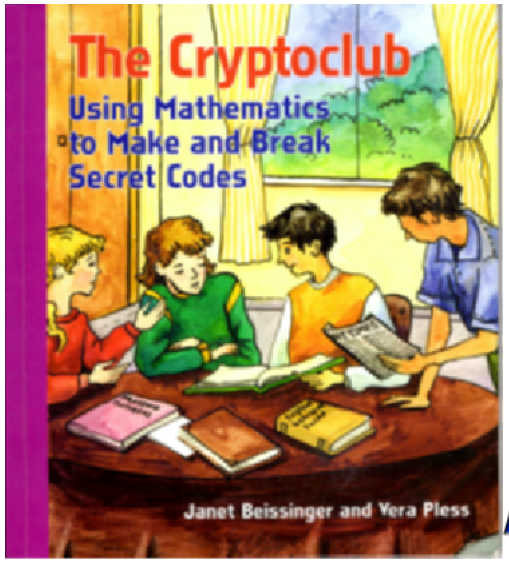
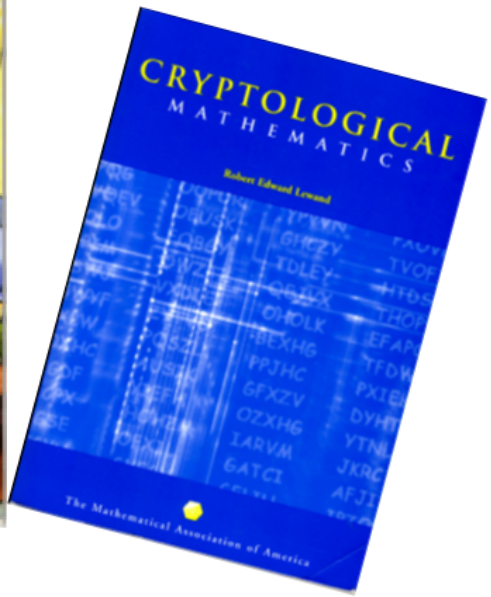
    - Current technology: secure online transactions, passcodes for devices, E-currency
  - It's FUN and can be used to excite students about mathematics

Recommended Beginning Cryptography References







- Online cryptography tool: http://simonsingh.net/The_Black_Chamber/

Fiction and Films that Utilize Cryptography as a Plot Element

- Sir Arthur Conan Doyle, *The Return of Sherlock Homes* "The Adventure of the Dancing Men"
- Eoin Colfer, *Artemis Fowl* series, 8 books (2001-2012)
- Cory Doctorow, *Little Brother* (2008)
- Neal Stephenson, *Cryptonomicon* (some mature content, 1999)
- BBC TV show: "The Bletchley Circle". (Two seasons, 2012-2014)
- Film: "The Imitation Game" (2014)
- Film: "Sneakers" (1992)

Conclusion

Questions?

Contact Information:

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Special thanks to Trinity's maintenance department for cutting the twenty-four sets of scytales!

Extensions: Rearrangement Ciphers

- Scytale uses a geometric pattern to rearrange letters. Extension questions:
  - Encipher a message using the rod of your choice. Then measure the distance between several pairs of consecutive letters. How does this distance compare to the diameter of the rod?
  - If you encipher a single message with rods of differing diameter, one larger and one smaller in diameter, for which rod are the consecutive letters spaced more closely and which farther apart?
  - Give your encoded message to another table group to decipher.
- Another similar cipher to investigate: rail fence cipher

Extensions: Shift Ciphers

- How secure is this method? How long would it take a determined person to break the cipher?
- How many different options for shift cipher exist?
- Are there any shift ciphers that are symmetric, that is, where plaintext and ciphertext are mirrored?
- Shift ciphers can be interpreted in terms of modular arithmetic with mod 26. What number corresponds to the letter shift used to decipher your clue? What number corresponds to the letter shift used to encipher your clue? What is special about these number pairs?
- More complex versions of shift ciphers
  - Vigenere cipher: use a pattern of multiple shifts in a single message, usually based on a keyword.
  - One-time pad: each letter in the message is shifted by a random letter sequence recorded on the pad.
  - Suppose you wished to encipher a message with all identical characters, e.g., "F F F F F F F F F F... F." Is this possible? How?

Extensions: Substitution Ciphers

- How many different options for encoding a 26 character message with a substitution cipher exist?
- Cipher security is not solely based on the number of options: letter frequency analysis for ciphertext of sufficient length
- Common English digraphs (th, er, on, an, re, ...) and trigraphs (the, and, tha, ent, ion, ...) can also be used to complete the cryptanalysis.
- Is it possible to decipher a code to more than one plaintext message? If so, how? If not, why not?
- Other versions of substitution ciphers
  - Retain word spacing vs. uniform letter blocks. Which makes for a more secure cipher? Why?
  - Expand character set beyond the 26 letters. E.g., add punctuation, numbers, spaces, etc. Does this make the system more secure? Why or why not?