CryptoClue
An Unexpected Topic for a Math Circle

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January 6, 2017
An Hour-able Crime!

- Location: Archimedes Junior High

Diagram of a clock with numbers 1 through 12 and arrows pointing to the numbers 2, 6, and 10.
An Hour-able Crime!

Location: Archimedes Junior High

Principal: Mrs. Bletchley

To solve this enigmatic mystery, you will need to solve three clues based on historical cryptosystems.

Where?
Who?
With what?
Narrowing the Options

- Principal Bletchley has identified the following options for the crime:
  - Locations for the criminal act: Outhouse, Walk-in closet, basement, Tree house, Attic, kitchen cabinet
  - Many weapon options, including: a dustbuster, a tennis shoe, a plunger, a squirt gun, shrubbery, bagpipes, etc.
  - Her list of usual suspects:
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  - Volunteer cryptanalysts?
Where? Clue #1: Letters Inscribed on Colored Ribbons

- Curious rods in **Mrs. Oiler’s** geometry classroom, ribbons dangling from light fixtures

- Hint: To solve the riddle, you must test the ribbons by wrapping them around the rods provided to see if the resequencing makes sense.
Who? Clue #2: A Cipher Wheel and Cipher Text

In Mr. Seezer's classroom:

OQIVBO
ZMMVW
OZM
With What? Clue #3: A sequence of playing card symbols of one color
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- Curious fact: the number of red playing cards is identical to the number of letters in the English alphabet.

- A suspicious challenge by your math tutor, Ms. R.S. Adleman: “I’ll give you one card-letter clue for each correct problem you answer…”

- Hint: you need not obtain all 26 cards to solve the riddle and determine the weapon of clock destruction.
What is Cryptography?

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  - allies can communicate information accurately
  - allies are able to be assured of information validity
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- Cryptosystems are considered secure, if the enemy is unable to decipher the message even if everything about the system is *public knowledge* except for the key. (Kerckhoffs’ principle)
Historical Notes

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  - The ribbons and tubes are a modern version of an ancient Spartan cipher tool called a **scytale** from the fifth century BC.
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- With What? Clue #3: A sequence of playing card symbols of one color
  - The cipher used in this clue is a substitution cipher, believed to be first used by Arabs prior to the 10th century AD. The Arabs also were the first to do frequency analysis of characters to break these codes!
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Crime Resolved!

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  - Clue: *Sometimes this possibly scary and dark place is called the cellar or the lower level but most people just call it this eight letters.*
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- **Who:**
  - Clue: *Giant Green Ogre.*
- **With What:** a dustbuster, a tennis shoe, a plunger, a squirt gun, shrubbery, bagpipes, etc..
  - Clue: *I am a plant collection of hardy shrubs.*
- **The clock crime was committed by Shrek with shrubbery in the basement.**
Overview of CryptoClue Development

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  - Four are working in computing fields
    - Brandon: “...cryptography showed the mysterious side of mathematics, and that sometimes math can be used for fun instead of ‘solving problems’…”
    - Maria: “…the familiarity with cryptography and the history and purpose behind it has definitely boosted my ability to contribute to [a] work project” implementing a data obscuring tool
Crypto Clue

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  - Eight are teaching mathematics,
    - Lauren: I created “a cryptography lesson in Algebra 2 as a tie to inverse functions.”
    - Dan: Cryptography “tied together history, humanities, technology, and of course mathematics in a way that made me feel like I was learning ‘just for the fun of it.’”
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Trinity Math Triathlon Half-time

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- Student response
  - The shift cipher was very challenging for the students, even with cipher wheel provided.
  - The students very quickly solved the scytale puzzle.
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- The students very quickly solved the scytale puzzle.
- Spencer: After the Triathlon, “I ended up buying a book with a bunch of cryptography-related puzzles” to solve the following summer...
Math Circle Event

- Challenge teachers to solve the puzzles with guided instruction in each cryptosystem
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  - Many applications to middle school and high school mathematics curriculum!
    - Math topics in cryptography include: geometric measurement, function characteristics, modular arithmetic, statistics (frequency analysis), number theory
    - More ideas (and puzzles to try!) on handout
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  ➤ Great way to link to other subjects!
    ➤ Historical context of cryptography: story characters, use of cryptography in intrigue and war
    ➤ Literature involving cryptography as plot element
    ➤ Current technology: secure online transactions, passcodes for devices, E-currency
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- It’s FUN and can be used to excite students about mathematics
Recommended Beginning Cryptography References

Online cryptography tool: http://simonsingh.net/The_Black_Chamber/
Fiction and Films that Utilize Cryptography as a Plot Element

- Eoin Colfer, *Artemis Fowl* series, 8 books (2001-2012)
- Neal Stephenson, *Cryptonomicon* (some mature content, 1999)
- BBC TV show: “The Bletchley Circle”. (Two seasons, 2012-2014)
- Film: “The Imitation Game” (2014)
- Film: “Sneakers” (1992)
Conclusion

Questions?

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*Special thanks to Trinity’s maintenance department for cutting the twenty-four sets of scytales!*
Extensions: Rearrangement Ciphers

- Scytale uses a geometric pattern to rearrange letters. Extension questions:
  - Encipher a message using the rod of your choice. Then measure the distance between several pairs of consecutive letters. How does this distance compare to the diameter of the rod?
  - If you encipher a single message with rods of differing diameter, one larger and one smaller in diameter, for which rod are the consecutive letters spaced more closely and which farther apart?
  - Give your encoded message to another table group to decipher.
- Another similar cipher to investigate: rail fence cipher
Extensions: Shift Ciphers

- How secure is this method? How long would it take a determined person to break the cipher?
- How many different options for shift cipher exist?
- Are there any shift ciphers that are symmetric, that is, where plaintext and ciphertext are mirrored?
- Shift ciphers can be interpreted in terms of modular arithmetic with mod 26. What number corresponds to the letter shift used to decipher your clue? What number corresponds to the letter shift used to encipher your clue? What is special about these number pairs?
- More complex versions of shift ciphers
  - Vigenere cipher: use a pattern of multiple shifts in a single message, usually based on a keyword.
  - One-time pad: each letter in the message is shifted by a random letter sequence recorded on the pad.
  - Suppose you wished to encipher a message with all identical characters, e.g., “F F F F F F F F F…” F.” Is this possible? How?
Extensions: Substitution Ciphers

- How many different options for encoding a 26 character message with a substitution cipher exist?
- Cipher security is not solely based on the number of options: letter frequency analysis for ciphertext of sufficient length
- Common English digraphs (th, er, on, an, re, ...) and trigraphs (the, and, tha, ent, ion, ...) can also be used to complete the cryptanalysis.
- Is it possible to decipher a code to more than one plaintext message? If so, how? If not, why not?
- Other versions of substitution ciphers
  - Retain word spacing vs. uniform letter blocks. Which makes for a more secure cipher? Why?
  - Expand character set beyond the 26 letters. E.g., add punctuation, numbers, spaces, etc. Does this make the system more secure? Why or why not?