MAA Session: Unexpected Topics for a Math Circle

Dancing in Math Circles

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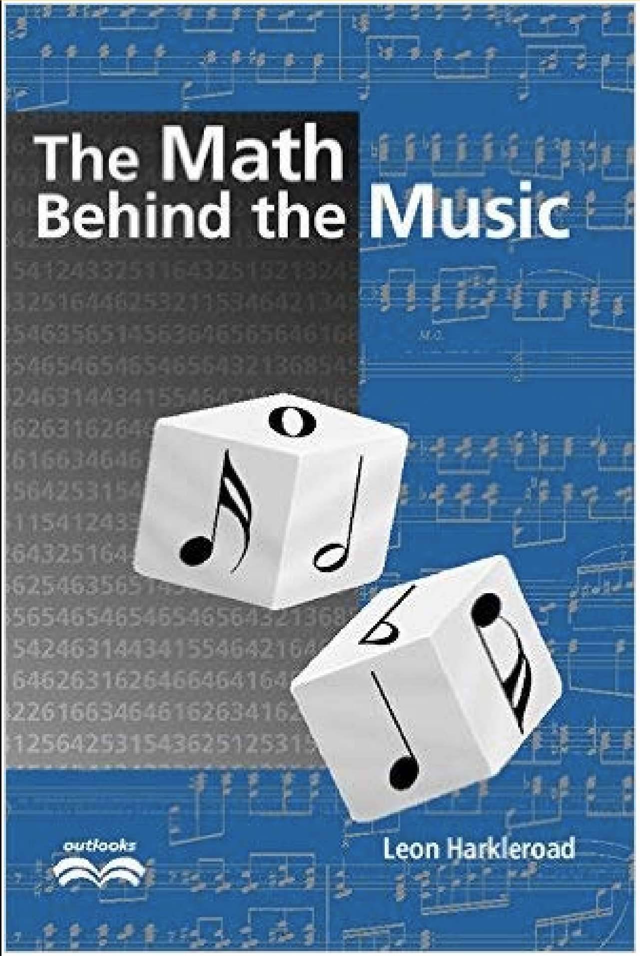
Inspiration



Leon Harkleroad gave minicourses in music and mathematics at the 2005 JMM and the 2008 JMM. At the 2008 minicourse he had us dancing the symmetries of the square.

What is Contra Dancing?

- Each contra dance begins with two lines, everyone facing a partner. Sets of four dancers (hands-four) execute 8 figures for 64 beats of music.

- Typically, the band plays either jigs or reels, consisting of musical phrases which are eight beats long.

- The footwork in contra dancing consists of walking in time with the music: one step for each beat..

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A Few Basic Moves

- Circle – All in foursome hold hands and circle counterclockwise or clockwise by 90, 180, 270, or 360 degrees. https://www.youtube.com/watch?v=DBvhyVata9I
- Star – All in foursome put left hands (or right hands) in center and circle by 90, 180, 270, or 360 degrees.
- Allemande- Partners face each other and hold a hand with elbows bent and thumbs up and circle. Can be with left or right hands and can be half turn or full turn or even 1 ½ turns.

Note: There are many contra dance figures (https://www.cdss.org/elibrary/dart/appendix_b.htm) like box the knat, gypsy, promenade, slide, twirl.

A Very Simple Dance

- Circle 180 counterclockwise (in video they say left or right, ¼, ½, ¾, or full turn)

  (What kind of geometric transformation does this mimic? What move would bring us back to original positions? What mathematical operation are we mimicking?)

A Very Simple Dance

https://www.youtube.com/watch?v=64CbgbUL1K0

- Circle 180 counterclockwise
- Star right 360

A Very Simple Dance

- Circle 180
- Star right 360
- Allemande 1 ½ with partner (How can we describe this as a geometric transformation?)

A Very Simple Dance

https://www.youtube.com/watch?v=64CbgbUL1K0

- Circle 180

  (What kind of geometric transformation does this mimic? What move would bring us back to original positions? What mathematical operation are we mimicking?)
- Star right 360
- Allemande 1 ½ across y=0.
- Allemande 1 ½ across x = 0.
- Allemande across y = x.
- Allemande across y = -x.

  (Are we back at original positions? Can we use one of these moves to get back to original positions? Is this always true no matter which combination of moves we execute?)

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A Very Simple Dance

https://www.youtube.com/watch?v=64CbgbUL1K0

- Circle 180
- Star right 360
- Allemande 1 ½ across y=0.
- Allemande 1 ½ across x = 0.
- Allemande across y = x.
- Allemande across y = -x.
- Star left 360
- Circle 180

Setting

Math circles for middle grades math teachers

- Immersion workshop Feb 21 – 23 2011

  (one morning (supported by teacher quality partnership grant)

- Summer week-long workshop in 2012 focusing on geometry (supported by teacher quality grant)

Common Core Standards

MCC8.G.1 Verify experimentally the properties of rotations, reflections, and translations,

MCC8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them; and

MCC8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Initial Questions

1. Identify a dance move or series of dance moves that mimic translations, rotations, reflections, dilations.
2. Plot each dancer, in a set of four, at the vertex of a square centered at the origin. Describe how each dancer moves to new coordinates for each possible circle and each possible allemande.
3. What move results in a flip across the line

$y = x$? $y = -x$? $y = 0$? $x = 0$?

More Questions

4. What if you do a circle and then an allemande one after the other? Would such a combination be the same as a single circle or allemande?
5. Can you return to the initial configuration with one circle or allemande?
6. What if you look at each move as a permutation of 1,2,3,4? What permutations are possible with the moves we've defined?
7. What is the radius of the circumscribed circle?
8. Simulate the dance on Geometer's Sketchpad?

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R90			**Fx**	
(1,1) (-1,1) (-1,-1) (1,-1)			(1,1) (-1,1) (-1,-1) (1,-1)	
R180			**Fy**	
(1,1) (-1,1) (-1,-1) (1,-1)			(1,1) (-1,1) (-1,-1) (1,-1)	
R270			**Fy=x**	
(1,1) (-1,1) (-1,-1) (1,-1)			(1,1) (-1,1) (-1,-1) (1,-1)	
R360			**Fy=-x**	
(1,1) (-1,1) (-1,-1) (1,-1)	(1,1) (-1,1) (-1,-1) (1,-1)		(1,1) (-1,1) (-1,-1) (1,-1)	

$\otimes$	R_{90}	R_{180}	R_{270}	R_{360}	$F_{h1,4}$	$F_{v1,2}$	$F_{d1,3}$	$F_{d2,4}$
R_{90}	3412 R_{180}	2341 R_{270}	1234 R_{360}	4123 R_{90}	3214 $F_{d1,3}$	1432 $F_{d2,4}$	2143 $F_{v1,2}$	4321 $F_{h1,4}$
R_{180}	2341 R_{270}	1234 R_{360}	4123 R_{90}	3412 R_{180}	2143 $F_{v1,2}$	4321 $F_{h1,4}$	1432 $F_{d2,4}$	3214 $F_{d1,3}$
R_{270}	1234 R_{360}	4123 R_{90}	3412 R_{180}	2341 R_{270}	1432 $F_{d2,4}$	3214 $F_{d1,3}$	4321 $F_{h1,4}$	2143 $F_{v1,2}$
R_{360}	4123 R_{90}	3412 R_{180}	2341 R_{270}	1234 R_{360}	4321 $F_{h1,4}$	2143 $F_{v1,2}$	3214 $F_{d1,3}$	1432 $F_{d2,4}$
$F_{h1,4}$	1432 $F_{d2,4}$	2143 $F_{v1,2}$	3214 $F_{d1,3}$	4321 $F_{h1,4}$	1234 R_{360}	3412 R_{180}	2341 R_{270}	4123 R_{90}
$F_{v1,2}$	3214 $F_{d1,3}$	4321 $F_{h1,4}$	1432 $F_{d2,4}$	2143 $F_{v1,2}$	3412 R_{180}	1234 R_{360}	4123 R_{90}	2341 R_{270}
$F_{d1,3}$	4321 $F_{h1,4}$	1432 $F_{d2,4}$	2143 $F_{v1,2}$	3214 $F_{d1,3}$	4123 R_{90}	2341 R_{270}	1234 R_{360}	3412 R_{180}
$F_{d2,4}$	2143 $F_{v1,2}$	3214 $F_{d1,3}$	4321 $F_{h1,4}$	1432 $F_{d2,4}$	2341 R_{270}	4123 R_{90}	3412 R_{180}	1234 R_{360}

	R_{90}	R_{180}	R_{270}	R_{360}	$F_{h1,4}$	$F_{d1,3}$	$F_{v1,2}$	$F_{d2,4}$
R_{90}	R_{180}	R_{270}	R_{360}	R_{90}	$F_{d2,4}$	$F_{h1,4}$	$F_{d1,3}$	$F_{v1,2}$
R_{180}	R_{270}	R_{360}	R_{90}	R_{180}	$F_{v1,2}$	$F_{d2,4}$	$F_{h1,4}$	$F_{d1,3}$
R_{270}	R_{360}	R_{90}	R_{180}	R_{270}	$F_{d1,3}$	$F_{v1,2}$	$F_{d2,4}$	$F_{h1,4}$
R_{360}	R_{90}	R_{180}	R_{270}	R_{360}	$F_{h1,4}$	$F_{d1,3}$	$F_{v1,2}$	$F_{d2,4}$
$F_{h1,4}$	$F_{d2,4}$	$F_{v1,2}$	$F_{d1,3}$	$F_{h1,4}$	R_{360}	R_{270}	R_{180}	R_{90}
$F_{d1,3}$	$F_{h1,4}$	$F_{d2,4}$	$F_{v1,2}$	$F_{d1,3}$	R_{270}	R_{360}	R_{90}	R_{180}
$F_{v1,2}$	$F_{d1,3}$	$F_{h1,4}$	$F_{d2,4}$	$F_{v1,2}$	R_{180}	R_{90}	R_{360}	R_{270}
$F_{d2,4}$	$F_{v1,2}$	$F_{d1,3}$	$F_{h1,4}$	$F_{d2,4}$	R_{90}	R_{180}	R_{270}	R_{360}

Permutations of 4 dancers:

R_{90}: $\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 4 & 1\end{pmatrix}$ R_{180}: $\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 1 & 2\end{pmatrix}$

R_{270}: $\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 1 & 2 & 3\end{pmatrix}$ R_{360}: $\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 2 & 3 & 4\end{pmatrix}$

$F_{h1,4}$: $\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix}$ $F_{v1,2}$: $\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix}$

$F_{d1,3}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 2 & 1 & 4\end{pmatrix}$ $F_{d2,4}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 4 & 3 & 2\end{pmatrix}$

How many other permutations are possible? 16 additional permutations are possible.
What are they?

$\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 2 & 4 & 3\end{pmatrix}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 3 & 4\end{pmatrix}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 1 & 2 & 4\end{pmatrix}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 1 & 3 & 2\end{pmatrix}$

$\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 3 & 2 & 4\end{pmatrix}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 1 & 4\end{pmatrix}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 1 & 4 & 2\end{pmatrix}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 2 & 1 & 3\end{pmatrix}$

$\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 3 & 4 & 2\end{pmatrix}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 1 & 3\end{pmatrix}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 2 & 4 & 1\end{pmatrix}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 2 & 3 & 1\end{pmatrix}$

$\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 4 & 2 & 3\end{pmatrix}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 3 & 1\end{pmatrix}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 2 & 1\end{pmatrix}$ $\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix}$

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In $F_{d1,3}$, the formation $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ changes to $\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$.

What matrix multiplied times $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ produces $\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$?

(Recall multiplication of 2 x 2 matrices: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} aA+bC & aB+bD \\ cA+dC & cB+dD \end{pmatrix}$.)

Express each of the original 8 symmetries of the square in matrix form.

Possibilities

Transformations

Compositions

Identities, inverses, commutative property

Fractions

Coordinate system, lines

Group theory

Permutations

Matrices

Contradancing

What is it?
http://www.sbcds.org/contradance/whatis/
http://www.contradancelinks.com/resources.html

History of Contra dance
http://www.heinerfischle.de/history/c-history.htm

Contra dancing and mathematics
http://www.edmath.org/copes/contra/
http://www.danceofmathematics.com/
http://www.edmath.org/copes/contra/MAA.html

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