DIFFERENT ANGLE

Tatiana Shubin

JMM-2017, Atlanta GA

San Jose Math Circle 2014-2015

- Four Numbers Game
- Area
- Number Theory 1
- Problem Solving Practice
- BAMA – Sam Vandervelde – Marden's Marvelous Theorem
- Mastermind
- Match Wits with Ramanujan, part 1
- Match Wits with Ramanujan, part 2
- Number Theory, 2
- Russian-Style Math Circle
- BAMA – Ron Graham – (A few of) My Favorite Graph Theory Problems
- Designing Tournaments

SJMC 2014-2015 (continued)

- Number Theory, 3
- BAMA – John Stillwell – ET Math: How Different Could It Be?
- Expanding Fractions Generates Hidden Insights
- Functional Equations
- Number Theory, 4
- Monthly Contest, Problems and Solutions
- BAMA – Carl Pomerance – What We Still Don't Know About Additions and Multiplication
- Problem Solving with the USA-IMO Team Coach Po-Shen Loh
- Number Theory, 5
- The Pigeonhole Principle

SJMC 2014-2015 (continued)

- Bay Area Math Olympiad (BAMO) Prep
- BAMO Aftermath
- BAMA – Federico Ardila – Tiling in Combinatorics, Algebra, and Geometry
- Mathematical Induction
- Number Theory, 6
- Mathematics of Perspective
- Number Theory, 7
- Distances
- Three Squares and Minimal Distances
- Group Theory
- BAMA – Elwyn Berlekamp – The Game of Amazons
- Purple Comet Marathon
- Pizza and Puzzles Party

SJMC 2015-2016

- Counting the Diagonals
- Game Theory, 1
- BAMA – Ed Schaefer – The Miracle of Public Key Cryptography
- Game Theory, 2
- Shape of Space
- Perfect Squares
- Distance-Time Problems
- BAMA – Aparna Higgins – Pebbling on Graphs: Demonic Graphs and Troop Deployment
- Divisibility Rules
- Game Theory, 3
- Game Theory, 4
- BAMA – Dan Goldston – Sums and Differences of Pairs of Primes

SJMC 2015-2016 (continued)

- Mathematical Induction
- Continued Fractions and Rational Approximation
- Game Theory, 5
- A Bit of Information Theory
- Dots! And Planar Graphs
- BAMA – Richard Kubelka – Math of Walking Dogs
- Game Theory, 6
- A Gentle Introduction to Eulerian Geometry
- BAMO Prep
- BAMO Aftermath
- Game Theory, 7

SJMC 2015-2016 (continued)

- BAMA – Tadashi Tokieda – The World from a Sheet of Paper
- Around $(1+\sqrt{2})^n$
- Introduction to the Isoperimetric Problem
- Math Wrangle
- BAMA – Joe Buhler – The Power of Cooperation
- Computer Programming in the 18th Century (OK, really, Finite Differences)
- Sums and Products
- Game Theory, 8
- The Steinhaus Conjecture
- Counting Rationals Using the Calkin-Wilf Tree
- Game Theory, 9
- Pizza and Puzzles Party

Distances, 1

I. On a faraway planet *Neverland* there is a powerful nation of the *Fewer-is-better* people called *Minima*; a neighboring country, *Maxima*, is the home of the *More-is-better* people. According to the laws of Minima, when houses are built they must be placed in such a way that there are as few as possible different distances between them. But the laws of Maxima are completely opposite: there should be as many different distances as only possible. Your spacecraft ran out of fuel and you had to touch down on Neverland. You can get fuel in exchange of helping the local people to build a number of houses according to their laws. Would you be able to do it? Imagine that:

- You happen to land in Maxima, and you need to design a plan for building 3 houses. If you can design a number of *essentially different* plans you will get even more fuel!

- You land in Minima. You need to design a plan for building (a) 3 houses; (b) 4 houses; (c) 5 houses. Again, the more essentially different designs your offer the bigger is your reward.

Distances, 2

II. Could you help hungry creatures to get their food as quickly as possible?

- In a room that has the shape of a perfect cube, a fly is sitting at a corner of the ceiling. The fly spots a syrup drop at the opposite corner of the floor. What is the fastest way for the fly to get to the syrup? Is there more than one such a way?
- A cube is made out of twelve sticks glued together at the vertices. An ant sits at a vertex and wants to crawl to the opposite vertex where it noticed some sugar. What is the fastest way for the ant to get there? Is there more than one fastest way?
- A spider and a fly are sitting on the opposite vertices of a solid wooden cube. What is the shortest way for the spider to crawl to the fly? Is there more than one shortest path?

Distance-Time Problems (Raghu Subramanian)

- The circumference of the earth around the equator is 25000 miles. Let us suppose that you are a marathon runner and it takes you a year to run around the equator. Your feet cover 25000 miles, but your head describes a slightly bigger circle. How much faster does your head move compared to your feet?

- You drive from home to school at 30 miles/hour. At what speed should you return so that your average speed for both legs is 60 miles/hour?

Distance-Time Problems (R. S.)

- A boy, a girl, and a dog go for a walk. They all start at the same time from the same place. The boy walks down the road at 3 mi/h, the girl walks at 2 mi/h, and the dog trots back and forth between the boy and the girl at 5 mi/h. After one hour, what are the positions of the boy, girl, and dog?
- One morning at 6am, a monk started climbing a mountain. He traveled at widely varying speeds, sometimes walking briskly, and sometimes pausing to rest. He reached the top of the mountain at 6pm that evening. There he stayed for three full days and nights, thinking deeply about the nature of life, world, and everything. The next morning, at 6am, he started descending the mountain via the same trail. Again, he traveled at widely varying speeds, and reached the bottom at 6 pm. Prove that there exists a time of the day, T, and a point on the trail, P, such that the monk was at point P at time T on both his upward and downward journeys.

Continued Fractions and Rational Approximation (Ted Alper)

- A common typesetting error produces $x - 1$ instead of x^{-1}. For what real number or numbers are these two expressions equivalent?

- *A famous puzzle:* On a street in Louvain, Belgium, the houses are numbered consecutively, starting from 1. There are more than 50 houses on the street, but fewer than 500. Find the number of the house for which the sum of all the house numbers less than it equals the sum of the house numbers greater than it. More generally, find all solutions with no restrictions on the number of houses on the street. (*There's a story involving Ramanujan solving the general problem instantly.)*

Around $\left(1+\sqrt{2}\right)^n$ (Dmitry Fuchs)

The numbers $\left(1+\sqrt{2}\right)^n$ and $\frac{\left(1+\sqrt{2}\right)^n}{\sqrt{2}}$ (and some other similar numbers) are very close to integers. Two questions of interest are:

(1) why does it happen?

(2) to what integers are they close?

Let $a_n = \left(1+\sqrt{2}\right)^n$, and $b_n = \frac{\left(1+\sqrt{2}\right)^n}{\sqrt{2}}$

n	=	1	2	3	4	5	6	7	8
a_n	≈	2	6	14	34	82	198	478	1154
b_n	≈	2	4	10	24	58	140	338	816

Sums and Products (Lucian Sega)

- *(2005 Archimedes Contest, Romania, Grade 6)*

There exists a positive integer *n* such that

$$\frac{8000}{19}\left(\frac{1}{1+2+3+\cdots+100}+\frac{1}{1+2+3+\cdots+101}+\cdots+\frac{1}{1+2+3+\cdots+1999}\right)=n^3$$

Find *n*.

- *(1996 Turkey Math Contest)*

Let a_n be the integer closest to $\sqrt{n}$. Find *S*, where

$$S=\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\cdots+\frac{1}{a_{2070}}$$

Some of my favorite problems

- What is the number of non-facial diagonals in a Buckyball?

- What is the number of similar diagonals in a 4-dimensional cube?

- Prove that $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$ by

  (a) counting rectangles in an n-by-n square;

  (b) rearranging 1-by-1 cubes in each of m-by-m cube for $m = 1,2, \ldots, n$

More favorite problems

- *(A Moscow Math Circle, by S. Dorichenko, MCL 8)*

In a 3-dimensional space, we are given a point light source emitting light in all directions. Is it possible to choose: (a) 100 opaque balls; (b) 4 opaque balls, and place them in space in such a way that they don't intersect each other, don't touch the light source, and completely block its light, i.e., every light ray emanating from the bulb meets one of these balls?

- *(A Moscow Math Circle, by S. Dorichenko, MCL 8)*

You have a compass, a ruler, a piece of paper, a pencil, and a ball. You can draw on the surface of the ball using the compass and the pencil, and you can draw on the paper using the compass, the pencil, and the ruler. Can you draw a line segment on the paper whose length is the radius of the ball?

Sources of more good problems:

- San Jose Math Circle:

  http://www.sanjosemathcircle.org

- Math Teachers’ Circle Network:

  http://www.mathteacherscircle.org

THANK YOU!