Divide Your Cake (And Eat it, Too!)

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The circle

- Northwest Iowa Math Teachers' Circle
- Started: Fall 2014 by Tom Clark
- 10-15 participants, including pre-service and in-service teachers
- Meet once/month
- One hour of math following a meal
- My first session

The problem

First encounter: sharing snacks with my brother

Second encounter: teaching liberal arts math as a graduate student

- Two friends want to split the last piece of cake; how can they do it so that neither is envious of the piece the other receives?
- Bigger picture: how can *multiple* people – with differing values – divide an infinitely divisible heterogeneous resource fairly?
- And what does *fair* mean?

What we did

Assumptions

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- The resource is infinitely divisible (e.g., cake, but not a car, or even a dollar).
- The players' rights to the goods are symmetric; no one is more important than the others.
- The resource is heterogeneous.
- The players are cooperative, i.e., they are willing to abide by the rules of the game.
- The players act rationally, seeking to maximize their own share of the goods, and not to spite any of the other players.
- A player's values are private and not shared with any other players.

Goals

- A proportional division (each player gets at least $1/N$th of the perceived value)
- An envy-free division (no player desires any other player's share)

Then:

1. Guided exploration of the two-player division problem and its elegant solution
2. Prompts and time to think about a three-player division algorithm (with actual baked goods)
3. With some guiding, participants were able to come up with an algorithm that leads to a proportional division (moving knife)
4. Discussed an envy-free algorithm for three players (Selfridge-Conway, 1960)
5. Sealed bids for discrete division problems
6. Then we ate cake!

Response of participants

- Overall positive
- One negative comment: “this didn’t feel like math”
- That is why I love it – application of rigorous algorithmic thinking in an unfamiliar context

MP1. Make sense of problems and persevere in solving them.
What does it mean to divide something fairly?

MP3. Construct viable arguments and critique the reasoning of others.
Does your proposed algorithm work? Can I construct a counterexample? Is the division that results actually fair?

MP6. Attend to precision.
Communicate your algorithm precisely.

Possible Variations and extensions

- Play with assumptions
- Explore Sperner's lemma and its applications: Simmons-Su protocols, approximate fair division
- Rental harmony
- Fair division of discrete resources (sealed bids)

Thanks!

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