

## Problems from the UWM Math Circle

### 1. Pirate Treasure Island

A band of pirates found a treasure map fragment. The picture is destroyed, but the following text is still legible: Stand upon the gravesite and you'll see two towering great palms above all others on the island. Count paces to the tallest of them, and turn  $90^\circ$  clockwise and count the same number of paces and mark the spot with a flag. Return to the gravesite, and count paces to the second tallest of the trees, turn  $90^\circ$  counterclockwise and count off the same number of spaces. Mark the spot with a second flag. You'll find the treasure at the midpoint of the two flags. The pirates knew which island the map referred to. Sadly, when they arrived, there was no sign of any gravesite. The captain was preparing to order his man to dig up the whole island, when one of the more geometrically inclined pirates walked around a bit, measured some things, and then pointed to a spot and began to dig. The treasure was quickly unearthed at that very spot. Can you explain the pirate's method of finding the place?

### 2. Pirate Treasure Island II

It was a dark and stormy night ... and a lonely pirate devised a scheme to bury his treasure on an island. He chose three towering palms  $T_1$ ,  $T_2$  and  $T_3$ , and two big boulders  $A$  and  $B$ . First he counted paces from  $A$  to  $T_1$ , turned  $90^\circ$  clockwise, and counted off the same number of paces. He marked the point with a flag,  $A_1$ . Next, he counted paces from  $B$  to  $T_1$ , turned  $90^\circ$  counterclockwise, and counted off the same number of steps to point  $B_1$ . He connected  $A_1$  with  $B$ ,  $B_1$  with  $A$ , and marked the intersection  $P_1$ . He did the same with the other two trees to get points  $P_2$  and  $P_3$ . To make his scheme really safe he buried the treasure at the center of the circle circumscribed about  $\triangle P_1P_2P_3$ . In a number of years the old pirate returned and to his horror he realized that there was not even a trace of the towering palms. In his anger and frustration he started to shoot around, and he shot down three birds. Having no better idea, he started the previous procedure, using the birds instead of the trees to get points  $Q_1$ ,  $Q_2$  and  $Q_3$ . He started digging at the center of the circle circumscribed about  $\triangle Q_1Q_2Q_3$ , and to his greatest surprise found the treasure. Can you explain what happened?

### 3. Truth tellers and liars

There are 100 inhabitants on the island of knights and knaves (everybody is either a knight or a knave, knaves always lie, knights always tell the truth). Each inhabitant is either a Sun worshipper, a Moon worshipper or an Earth worshipper. One time a census was conducted on the island, and everyone had to answer each of the following three questions: Are you a Sun worshipper? Are you a Moon worshipper? Are you an Earth worshipper?. 60 inhabitants answered yes for the first question, 40 said yes to the second, and 30 answered yes for the third. How many knaves and knights are there on the island?

4. The king of the octopuses has four servants, and the servants have either 6, 7 or 8 legs. Servants with 7 legs always lie, and servants with 6 or 8 legs always tell the truth. The king asks How many legs do you four have in total?, and the four octopus servants (who are standing behind a table, so you can't see their legs) answer 25, 26, 27 and 28, respectively. Who is telling the truth?
5. **Foul shooting percentages**  
Early in the season, basketball player Shanille O'Keal has a 70% foul shooting average. At the end of the season she has a 90% average. Was there necessarily a moment in between when she made exactly 80% of her foul shots?  
Early in the season, basketball player Susie Sharp has a 60% foul shooting average. At the end of the season she has an 80% average. Was there necessarily a moment in between when she made exactly 70% of her foul shots?
6. Peter and Paul play the following game. First Peter chooses a one-digit number greater than 1, then Paul multiplies this number by a one-digit number greater than 1. Next Peter multiplies the result by a one-digit number greater than 1, and they go on this way. The winner is the player who can first say a number greater than 2015.
7. There are two piles of stones, one with 10 the other with 8 stones. Two players take turns taking stones from the piles. A player can either take one stone from each pile, or just one stone from one of the piles. The winner is the one who takes the last stone(s). Does either player have a winning strategy?
9. Given a 3 in by 3 in grid made up of 1 in by 1 in squares. Can you outline the perimeter and all the grid lines if
- a) 8 pieces of yarn each 3 inches long?
  - b) 4 pieces of yarn each 6 inches long?
  - c) 6 pieces of yarn each 4 inches long?
  - d) 3 pieces of yarn each 8 inches long?

The yarn cannot be cut.

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