Equidistant Dots on Grids

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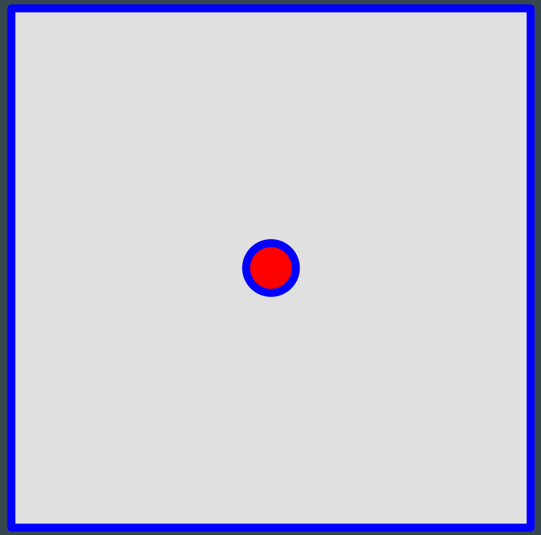
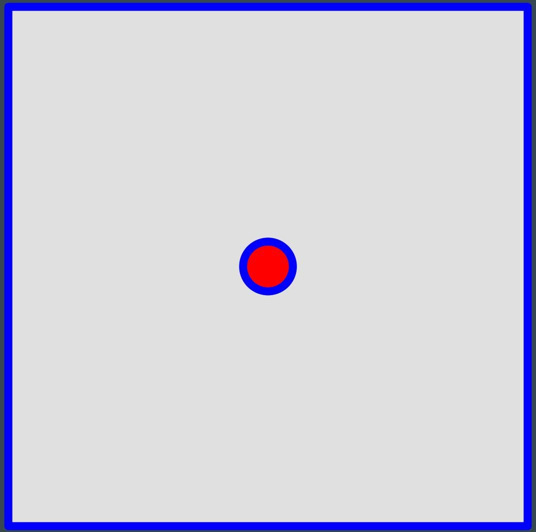
Problem

Assume that we have a grids that consist of n row and m column. We want to know what is the maximum number of dots we can place (in each cell only one point can be placed) with the condition that we do not have equidistant (list of all distances(non-negative) between placed dots may not repeat).

Example: 1-by-1 (Trivial Case)

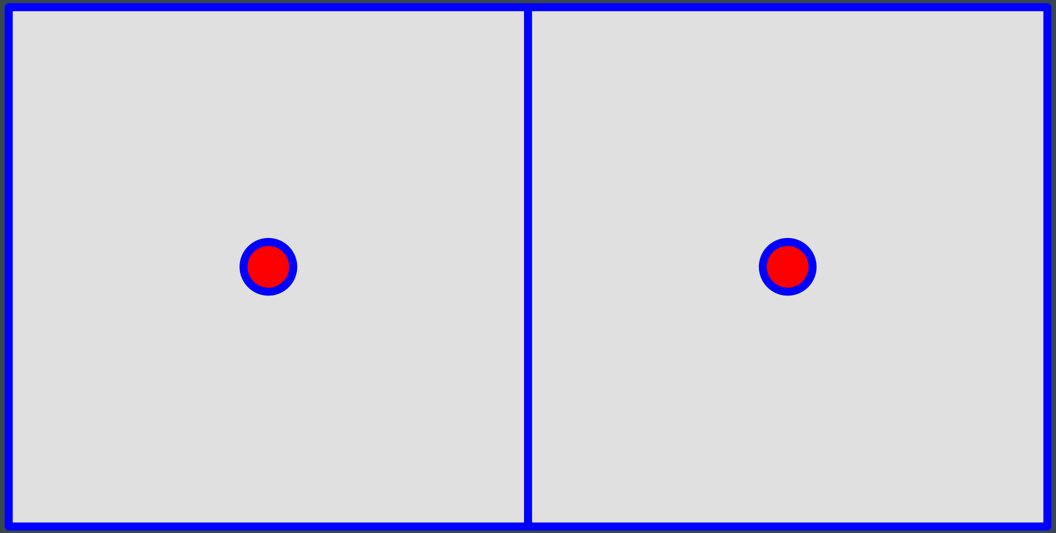


Example: 1-by-1(Trivial Case)



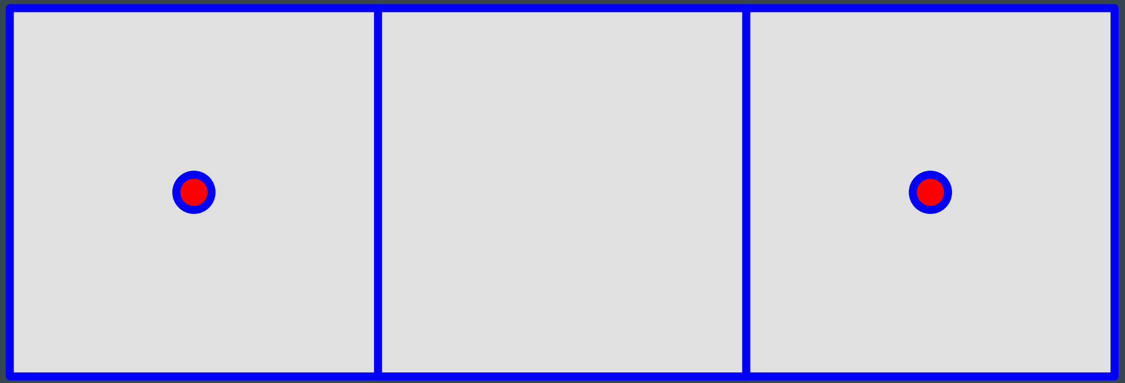
Number of Dots:1
Number of Solution:1
Possible length:0

Example: 1-by-2



Number of Dots:2
Number of Solution:1
Possible length:1

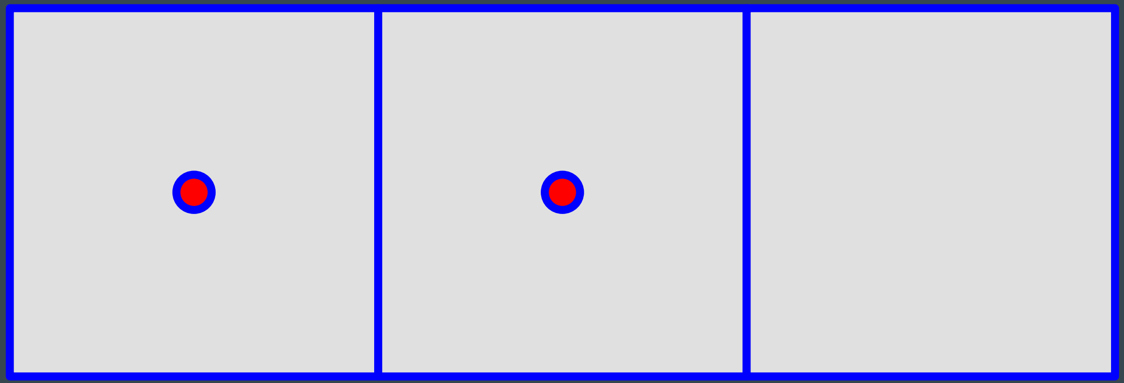
Example: 1-by-3



First Solution

Number of Dots:2
Number of Solution:1
Possible length:1,2

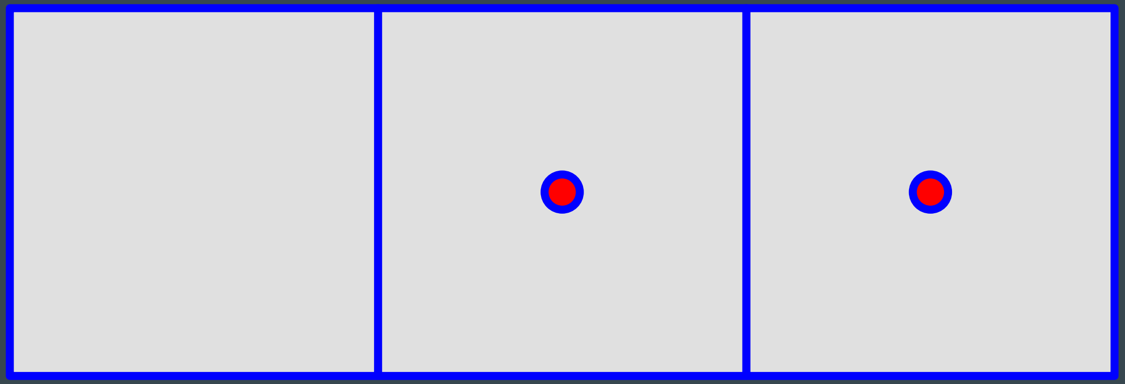
Example: 1-by-3



Second Solution

Number of Dots:1
Number of Solution:1
Possible length:1,2

Example: 1-by-3



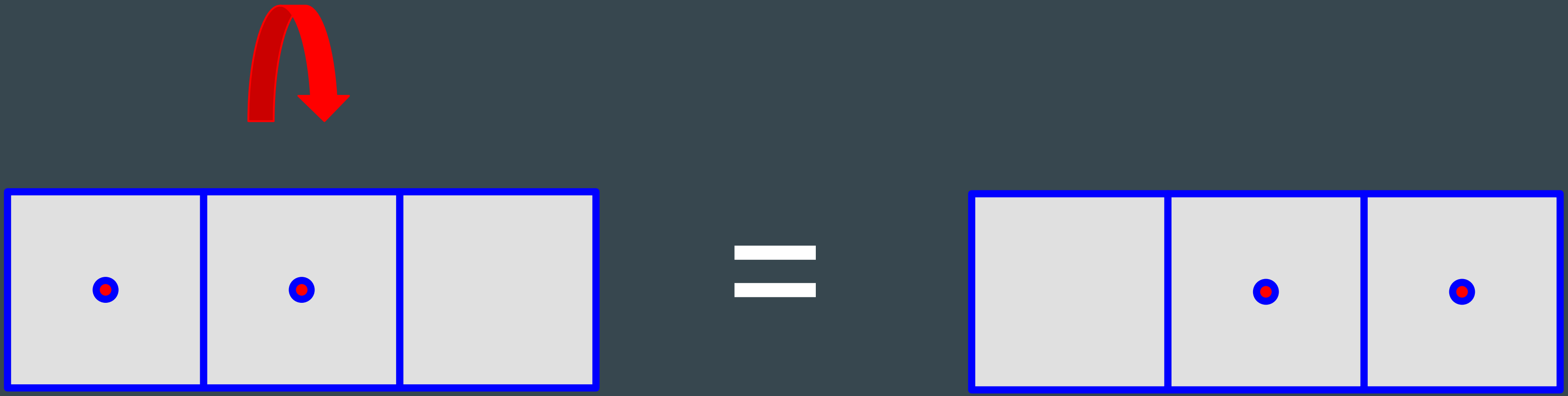
Third Solution

Number of Dots:1
Number of Solution:1
Possible length:1,2

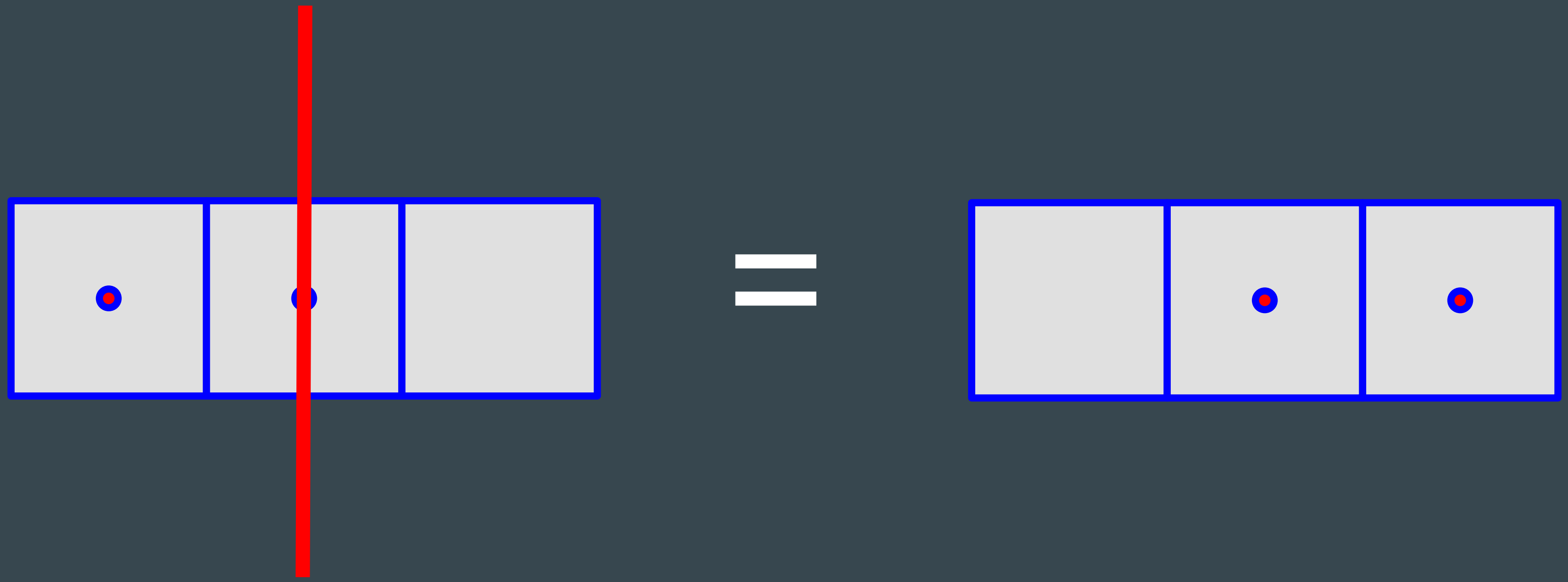
Isomorphic(Similar,Equivalent) Solution 1-by-3

Question: How we can Go from First Solution to the second one without moving the dots?

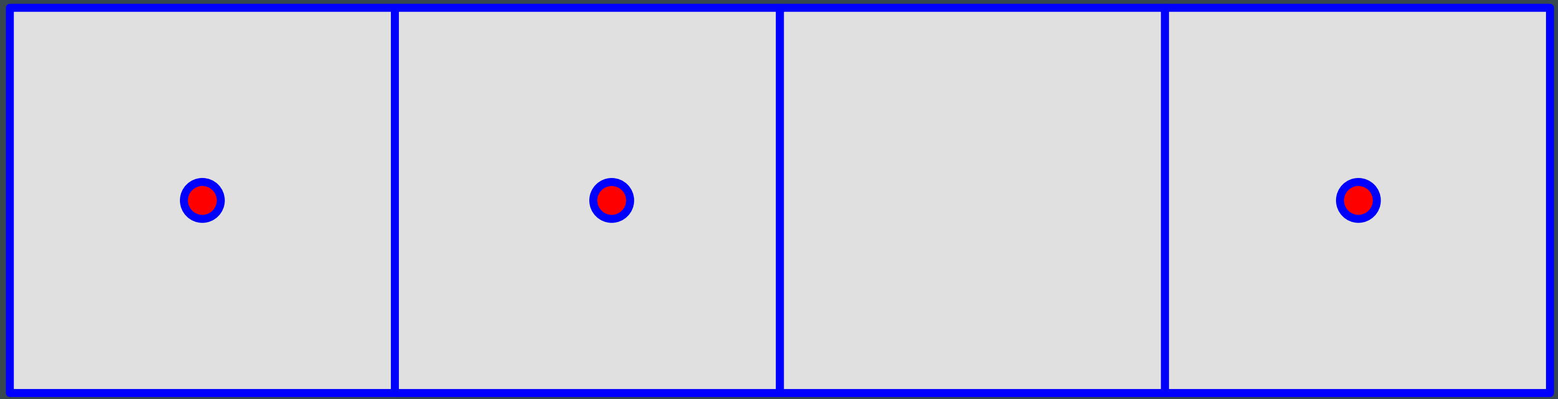
Rotation



Reflection(Mirror Line)



1-by-4



Number of Dots:3
Number of Solution up to isomorphism:1
Possible length:1,2,3

Conjecture 1(Easy)(Upper Bound???)

For 1-by-n grid. Number of dots always less than or equal to n.

Conjecture 1(Easy)

For 1-by-n grids. Number of dots always less than or equal to 1+n.

Conjecture 2(intermediate)

For 1-by-n grids. Number of dots always greater than or equal to floor(n/2).

Conjecture 3(induction???)

If number of dots in 1-by-n is k then number of dots in 1-by-(n+1) is either k or k+1.

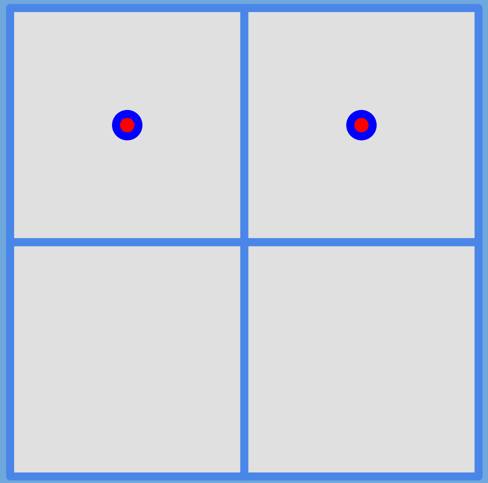
Conjecture 3

If number of dots in 1-by-n is k then number of dots in 1-by-(n+1) is either k or k+1.

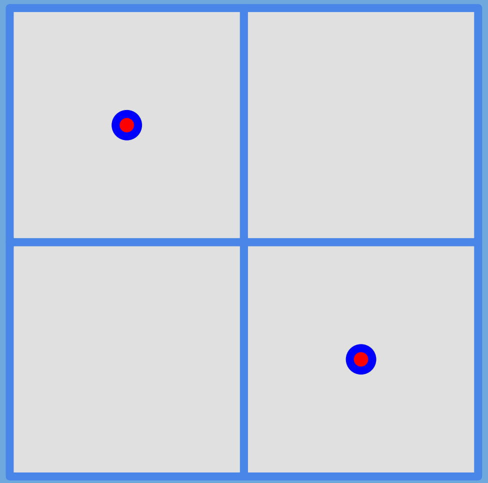
Conjecture 4

If number of dots in 1-by-n is k then number of dots in 1-by-(n+1) is equal to k if n is even and k+1 if n is odd.

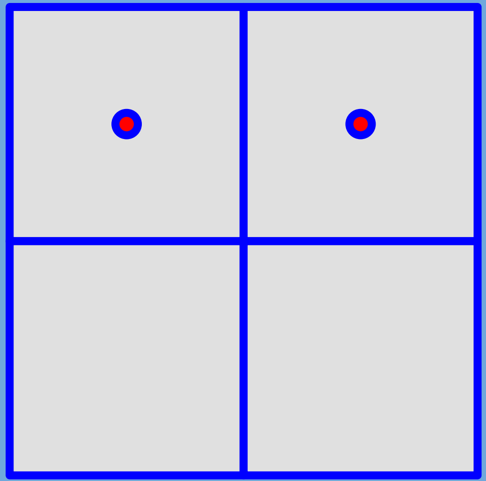
Two Dimensional Example (2-by-2)



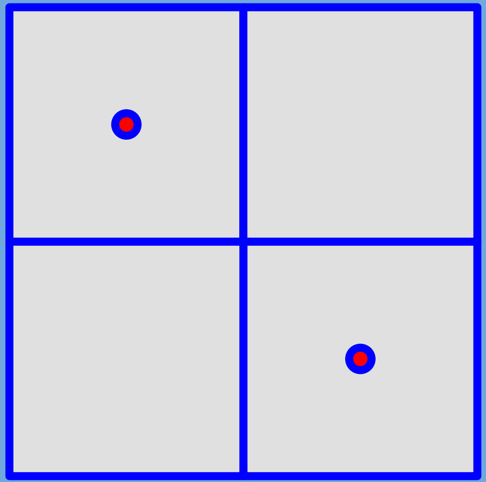
First Solution



Second Solution



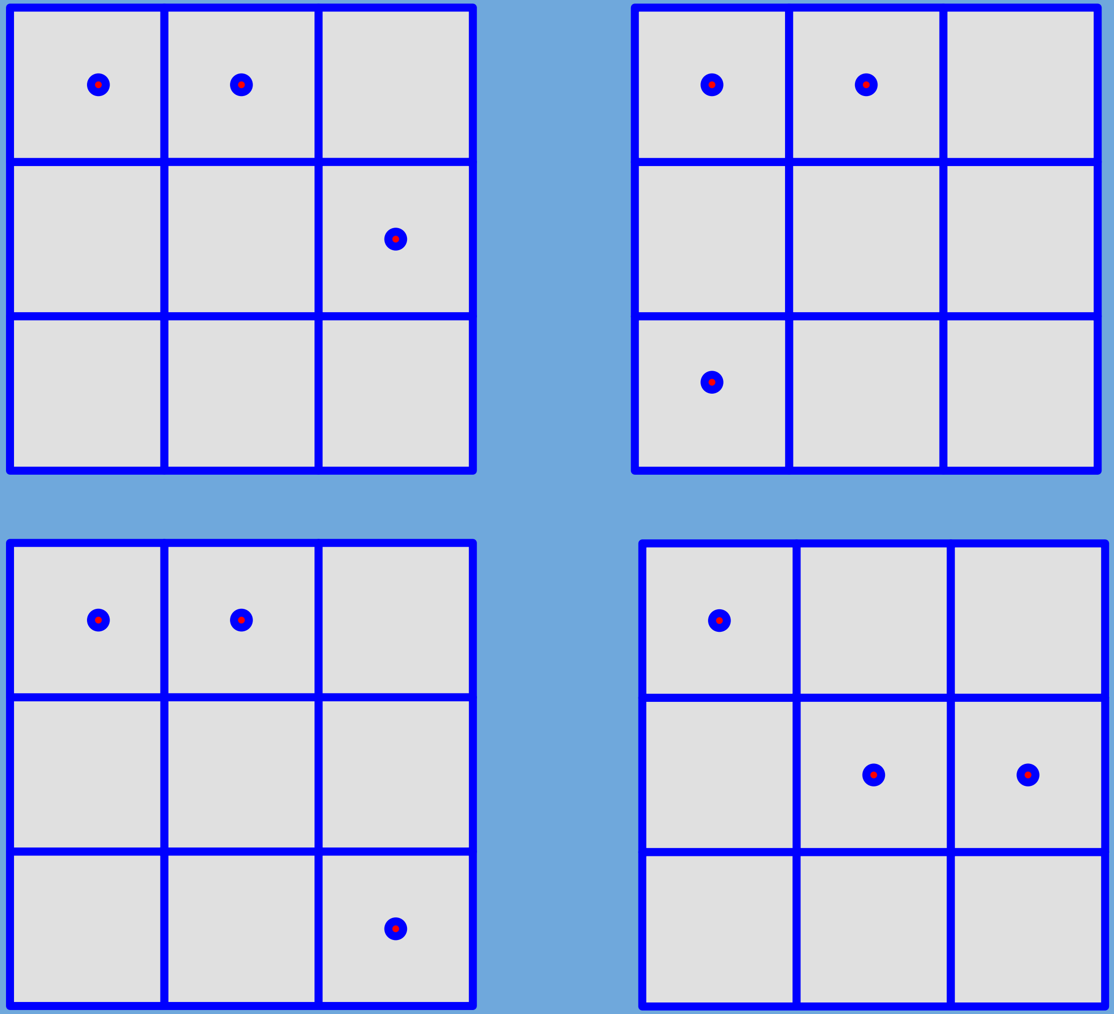
First Solution



Second Solution

Number of Dots:2
Number of Solution up to isomorphism:2
Possible length:1,√2

3-by-3



Observation:What Do you See in Solutions for 3-by-3?

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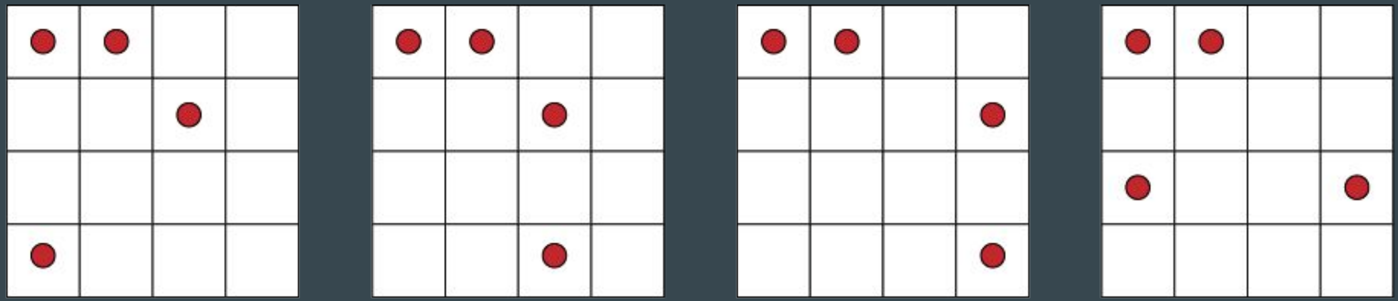
Hint: -What are invariant in all these solution?

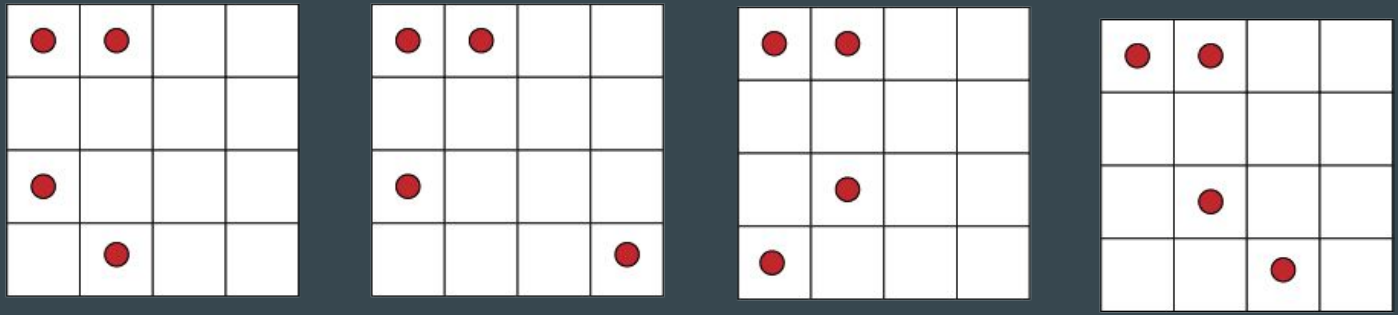
Possible Answer:

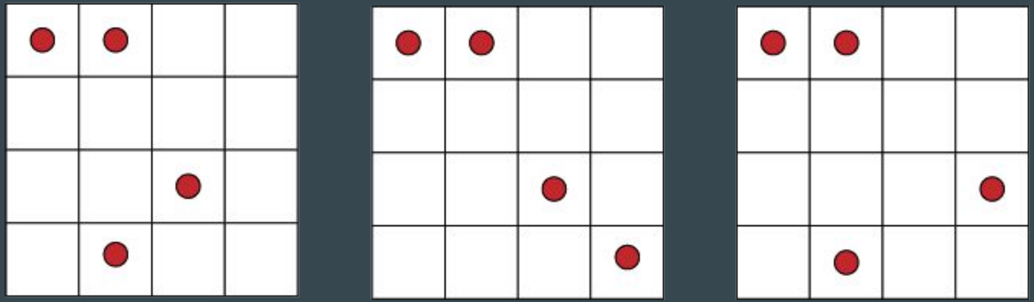
-One Row has no dot on it.

-At least one corner has dot on it

-If these dots are vertices of triangle what kind of triangle will they create?







4-by-4

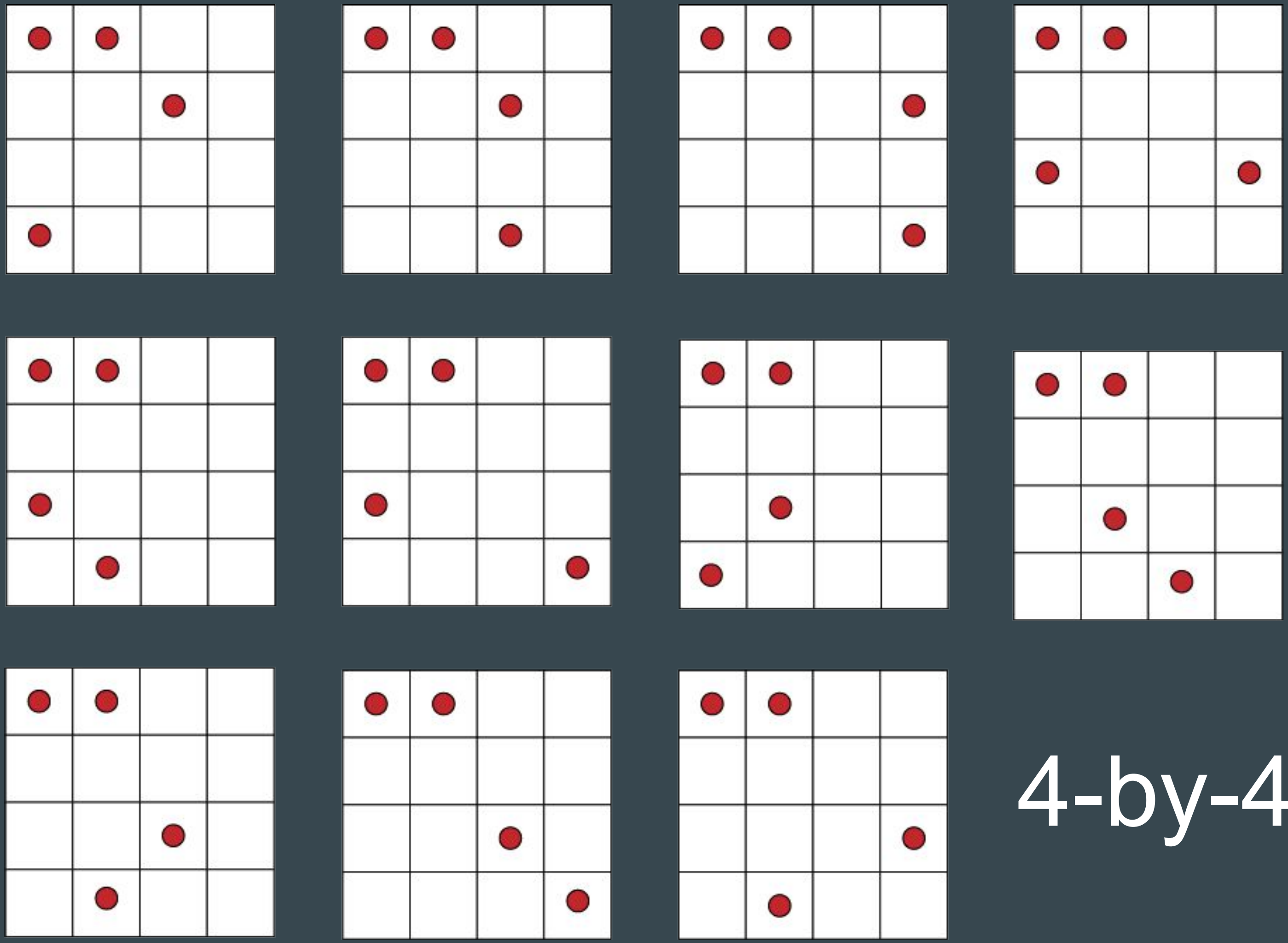
Conjecture

For n-by-n grid you can place n dots without distance between them being repeated.

Possible Question: When you go from 1-by-2 to 2-by-2 number of dots does not change. Can you find another example similar to this? That When you go from 1-by-n to 2-by-n number of dots will be same (in all or in some of the solution).

Extension 1

Ask the same question but this time in Taxicab geometry

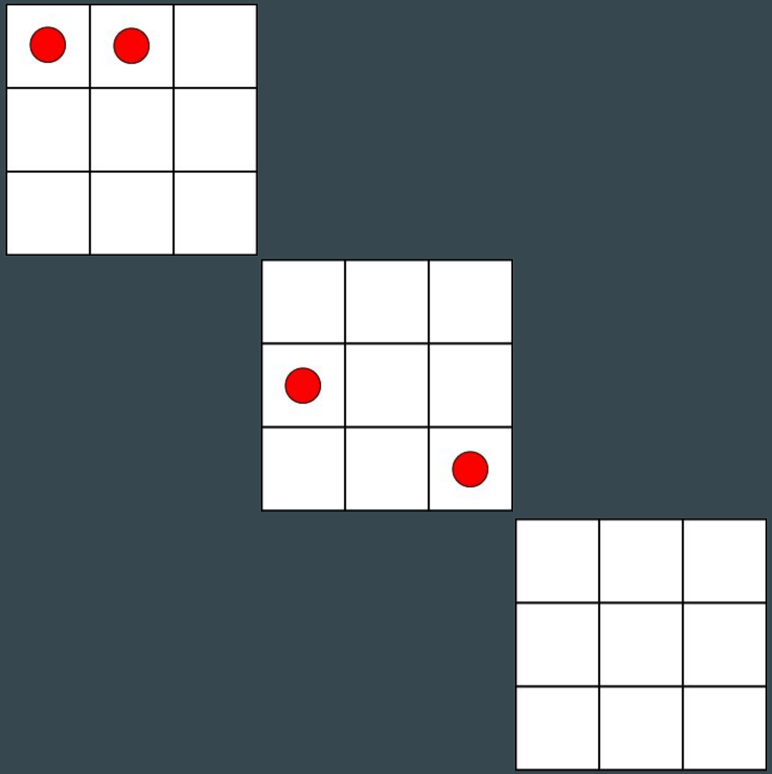


4-by-4

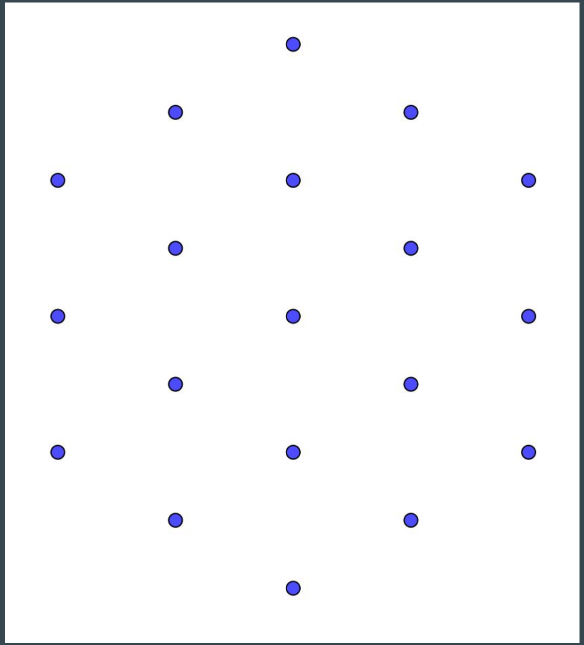
Extension 2

Ask the same question but for 3D. That is you can place one dot (or ball) inside each unit cube.

Example: 3x3x3

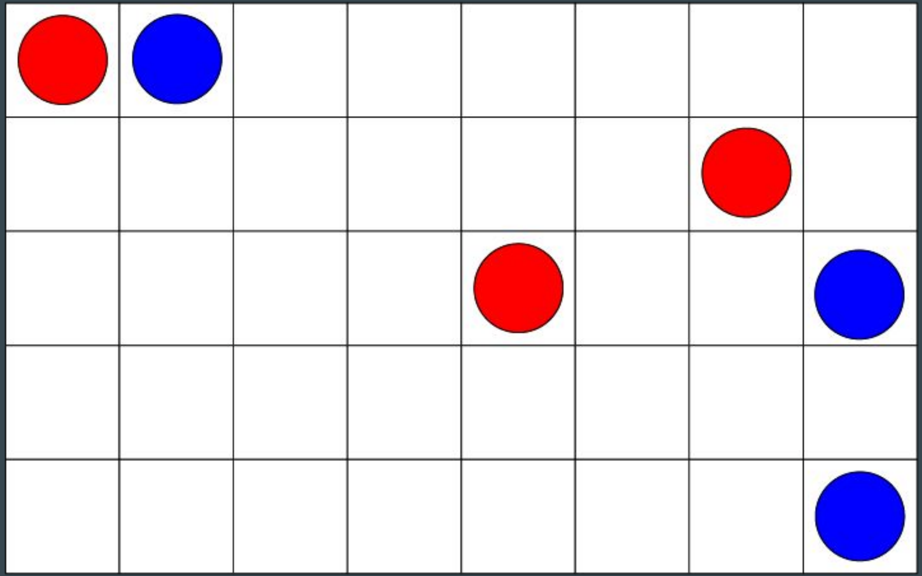


Extension 3: Isometric Grid

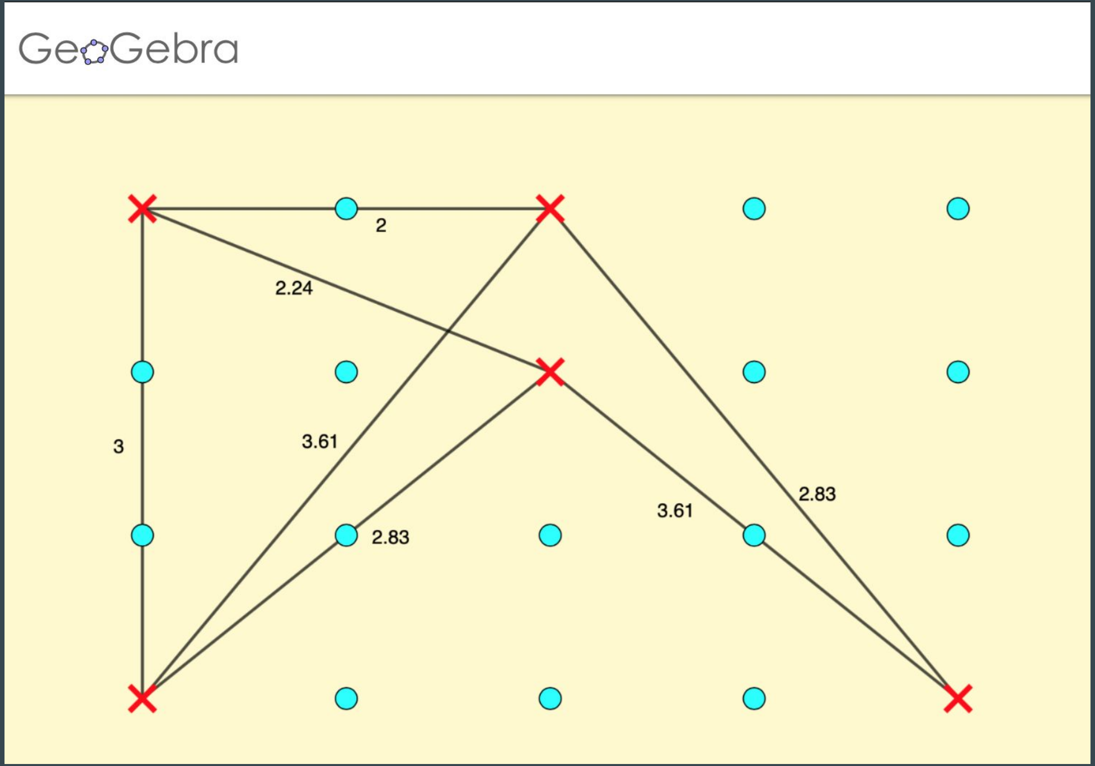


Extension 4: As a two player game

The last person who place dots on the board will win!



Using Geogebra



Thank you

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