

Math Rumble – Problem Set

American Mathematics Competitions

April 23–25, 2012

1. What is the difference between the sum of the first 2003 even counting numbers and the sum of the first 2003 odd counting numbers? (The counting numbers are the positive integers: 1, 2, 3, 4...)
2. A set of tiles numbered 1 through 100 is modified repeatedly by the following operation: remove all tiles numbered with a perfect square, and renumber the remaining tiles consecutively starting with 1. How many times must the operation be performed to reduce the number of tiles in the set to one?
3. What is the units digit of $19^{19} + 99^{99}$?
4. Homer began peeling a pile of 44 potatoes at the rate of 3 potatoes per minute. Four minutes later Christen joined him and peeled at the rate of 5 potatoes per minute. When they finished, how many potatoes had each person peeled?
5. The operation \otimes is defined for all nonzero numbers by $a \otimes b = \frac{a^2}{b}$. Determine $[1 \otimes (2 \otimes 3)] - [(1 \otimes 2) \otimes 3]$.
6. Let $P(n)$ and $S(n)$ denote the product and the sum, respectively, of the digits of the integer n . For example, $P(23) = 6$ and $S(23) = 5$. Suppose N is a two-digit number such that $N = P(N) + S(N)$. What is the units digit of N ?
7. How many integers between 1000 and 2000 have all three of the numbers 15, 20, and 25 as factors?
8. Miki has a dozen oranges of the same size and a dozen pears of the same size. Using her juicer, Miki can extract 8 ounces of pear juice from 3 pears and 8 ounces of orange juice from 2 oranges. She makes a pear-orange juice blend from an equal number of pears and oranges. What percent of the blend is pear juice?
9. On a twenty-question test, each correct answer is worth 5 points, each unanswered question is worth 1 point, and each incorrect answer is worth 0 points. Which scores, if any, between 90 and 100 are *NOT* possible?

10. Lenny, Moe, Nick, and Otto are good friends. Otto had no money, but the others did. Moe gave Otto one-fifth of his money, Lenny gave Otto one-fourth of his money, and Nick gave Otto one-third of his money. Each gave Otto the same amount of money. What fractional part of the group's money does Otto now have?