Math Rumble – Problem Set – Solutions

American Mathematics Competitions

April 23–25, 2012

1. Each even counting number, beginning with 2, is one more than the preceeding odd counting number. Therefore the difference is 2003.
(Source: 2003 AMC 10A)

2. Consider the following table, which shows that the process takes 18 iterations of the operation to complete.

Iteration	Tiles Removed	Tiles Remaining
1	10	90
2	9	81
3	9	72
4	8	64
5	8	56
6	7	49
7	7	42
8	6	36
9	6	30
10	5	25
11	5	20
12	4	16
13	4	12
14	3	9
15	3	6
16	2	4
17	2	2
18	1	1

(Source: 2002 AMC 10)

3. The units digit of 19^{19} is the units digit of 9^{19}. Note that $9^1 = 9$, $9^2 = 81$, $9^3 = 729$, and, in general, the units digit of odd powers of 9 is 9, whereas the units digit of even powers of 9 is 1. Since both exponents are odd, the sum of their units digits is $9 + 9 = 18$, the units digit of which is 8.
(Source: 2000 AMC 8)

4. After 4 minutes Homer had peeled 12 potatoes. When Christen joined him, the combined rate of peeling was 8 potatoes per minute, so the remaining 32 potatoes required 4 minutes to peel. In these 4 minutes,

Christen peeled 20 potatoes and Homer peeled an additional 12 potatoes, bringing Homer's total to 24 potatoes.
(Source: 2001 AMC 8)

5. We have

$$[1 \bigotimes (2 \bigotimes 3)] = 1 \bigotimes \frac{2^2}{3} = 1 \bigotimes \frac{4}{3} = \frac{1^2}{(\frac{4}{3})} = \frac{3}{4},$$

and

$$[(1 \bigotimes 2) \bigotimes 3] = \frac{1^2}{2} \bigotimes 3 = \frac{1}{2} \bigotimes 3 = \frac{(\frac{1}{2})^2}{3} = \frac{\frac{1}{4}}{3} = \frac{1}{12}.$$

Therefore,

$$[1 \bigotimes (2 \bigotimes 3)] - [(1 \bigotimes 2) \bigotimes 3] = \frac{3}{4} - \frac{1}{12} = \frac{9}{12} - \frac{1}{12} = \frac{8}{12} = \frac{2}{3}.$$

(Source: 2000 AMC 8)

6. Suppose $N = 10a+b$. Then $10a+b = ab+(a+b)$. It follows that $9a = ab$, which implies that $b = 9$, since $a \neq 0$.
(Source: 2001 AMC 10)

7. A number with 15, 20, and 25 as factors must be divisible by their least common multiple (LCM). Because $15 = 3 \times 5$, $20 = 2^2 \times 5$, and $25 = 5^2$, the LCM of 15, 20, and 25 is $2^2 \times 3 \times 5^2 = 300$. There are three multiples of 300 between 1000 and 2000: 1200, 1500, and 1800.
(Source: 2003 AMC 8)

8. Suppose we use 6 pears to make 16 ounces of juice and 6 oranges to make 24 ounces of juice for a total of 40 oz of juice blend. The percent of pear juice is $\frac{16}{40} = \frac{4}{10} = 40\%$.
(Source: 2002 AMC 8)

9. To get a score in the 90s, a student must get 18 or 19 correct answers. If the number is 18, then the other two questions are worth 0+0, 0+1, 1+0, or 1+1, producing total scores of 90, 91, or 92. If the number correct is 19, then the total is 95+0 or 95+1. Therefore, the only possible scores in the 90s are 90, 91, 92, 95, and 96. The remaining scores are impossible: 93, 94, 97, 98, 99.
(Source: 2001 AMC 8)

10. Only the fraction of each friend's money is important, so we can assume any convenient amount is given to Otto. Suppose that each friend gave Otto \$1. If this is so, then Moe had \$5 originally, Lenny had \$4, and Nick had \$3. The original amount of money the four friends had was thus $\$5 + \$4 + \$3 = \$12$, of which Otto now has \$3. So Otto has $\frac{3}{12} = \frac{1}{4}$ of the group's money. This same reasoning applies to any amount of money.
(Source: 2002 AMC 8)