Folding Fractions

Tom Clark & Laura Janssen

MathFest 2018: Math Teachers' Circle Demo

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1 Left Folds

Take a strip of paper. Suppose the strip is one unit long, with the left edge representing 0 and the right edge representing 1. Fold the strip in half and crease it at the midpoint. Label this crease $\frac{1}{2}$. Now make a *left* fold: Fold the left edge to your most recent crease. Label this new crease. Continue making left folds and labeling each crease.

1. Write the first seven fractions in the sequence of crease marks.

2. The sequence of crease marks seems to converge (get closer and closer) to what value? Why does this make sense?

3. Can you come up with a multiplication problem that would represent making three left folds? Five left folds? Twelve left folds?

2 Right Folds

Take a new strip of paper. Again, fold the strip in half and label the crease $\frac{1}{2}$. But this time make a *right* fold: Fold the right edge of the strip to your most recent crease. Label this new crease. Continue making right folds and labeling each new crease.

1. Write the first seven fractions in the sequence of crease marks.

2. The sequence of crease marks seems to converge (get closer and closer) to what value? Why does this make sense?

3. Can you come up with an addition problem that would represent making two right folds? three right folds? Seven right folds?

4. How do your answers to problem 2 and 3 relate?

3 More Patterns

Now, we've done a sequence of left folds and we can call this pattern LLLL.... We've also done a sequence of right folds, RRRR.... What if we were to do another pattern? Let's try the pattern LRLR...: Begin by folding your strip in half and labeling the crease "Fold 1." Now do left fold to that crease and label it "Fold 2." Do a right fold to fold 2 and label it "Fold 3." Continue making left and right folds (always to the most recent crease). What do you notice?

What if we started with a fold other than $\frac{1}{2}$? Take a new strip and make a first crease anywhere on it. Label this "Fold 1." Then begin the LRLR... pattern as above. What do you notice?

What do you predict would be the outcome of the pattern RLRL...?

Make a conjecture about the RLRL... and LRLR... patterns.

4 Writing an Equation

Take a strip and make a first crease anywhere. Label this crease x. Make a left fold to x and label the crease a. Next, make a right fold to x and label the crease b.

1. What is the distance from a to b?

2. What does a equal in terms of x?

3. What does b equal in terms of x?

We can use these equations and a spreadsheet to test our conjecture about RLRL... and LRLR... folding patterns.

5 Even More Patterns

Let's try the pattern LLRLLR.... What do you predict will happen? Begin by folding your strip in half and labeling the crease "Fold 1." Continue to label each crease as you go. What do you notice?

What would you predict about the RRLRRL... pattern? Try it.

6 Resources

We are happy to share the handout electronically with anyone interested. E-mail us at: `Tom.Clark@dordt.edu` and `Laura.Janssen@dordt.edu`.

Here is the GeoGebra file:

`https://www.geogebra.org/classic/zpba4jsb`

Tom's Version of the activity as done at the Northwest Iowa MTC. `https://www.dordt.edu/sites/default/files/Tom%20Clark/foldingnumbers.pdf`

Earlier version that inspired ours and includes "bucket pouring" variations by Steve Dunbar and Anne Schmidt at Lincoln, NE MTC. `https://www.mathteacherscircle.org/assets/session-materials/DunbarSchmidtFoldingPouring.pdf`

Even earlier version by James Tanton in Chapter 9, *Mathematics Galore!*, 2012, MAA, Washington DC.