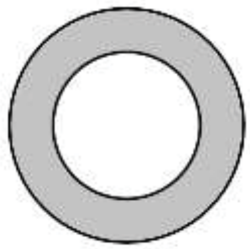
Background:
We have established what mathematician's call the Brouwer Fixed Point Theorem, at least, in dimensions one and two. (Luitzen Brouwer was a Dutch mathematician, 1881-1966.) The theorem says that any continuous map from a line segment to itself and any continuous map from a connected region in the plane (possessing no holes) to itself must each possess at least one fixed point: that is, there is sure to be at least one point that does not move under the mapping.

The theorem actually works in all dimensions. Some people like to interpret the three-dimensional version of the theorem as follows: *After stirring a cup of coffee, at least one point of liquid is sure to return to its original location.* (But there are problems with this interpretation. For example, liquid coffee is composed of discrete molecules and the theorem applies to a continuous range of points in space. Also, the spoon for stirring may separate points, causing the mixing not to be continuous.)

Exercise: Why does Brouwer's Fixed Point Theorem not apply to the this shape?



The proof of the two-dimensional Brouwer Fixed Point Theorem we presented here is due to German mathematician Emanuel Sperner (1905-1980). The lovely result about labeled triangulations and the existence of fully-labeled ABC triangles is today known as Sperners' Lemma.

The internet, of course, offers all one could possible want to learn – and more – about these results and these gentlemen.