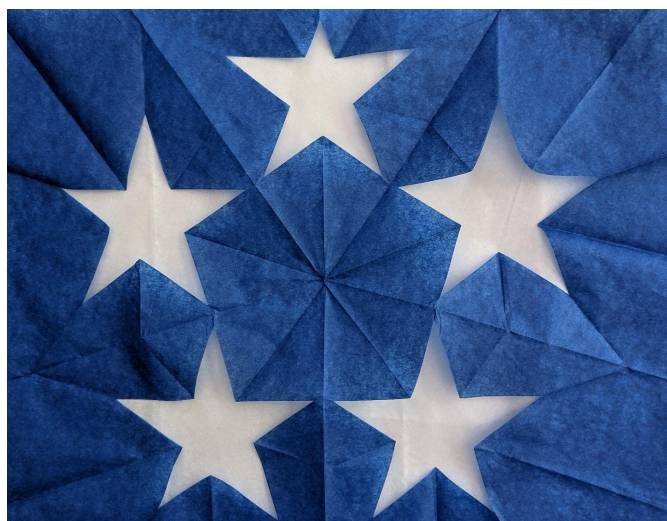


When the Sunday New York Times arrived, one of the first things I've turned to during the past several months was the crafts column in the At Home section. In fact, I was inspired to create a column for it, about a newspaper craft that is also a math puzzle, specifically for the holiday of either Flag Day or July 4th.

Just as my co-author and I were about to send it at the beginning of June, we were extremely disappointed to see that the section had been discontinued. So instead of a puzzling craft, we redesigned our article as a crafty puzzle.

Check out our JRMF Fold-and-Cut Flag video and the following article to see how to fold and cut stars with one straight cut.



Nancy Blachman
Nancy dot blachman at jrnf dot org

Skona Brittain
Skona at sbfamilieschool dot com

Fold-And-Cut Flag

You've probably done paper crafts before. This is a paper puzzle — that is, it's both a puzzle and a craft — a classic fold-and-cut puzzle with a star-spangled twist! In honor of Flag Day on June 14 or Independence Day on July 4, make your own version of a United States flag out of paper. We recommend using tissue paper, since it is thin and hence easy to fold and to cut through multiple layers, and it comes in red, white and blue.

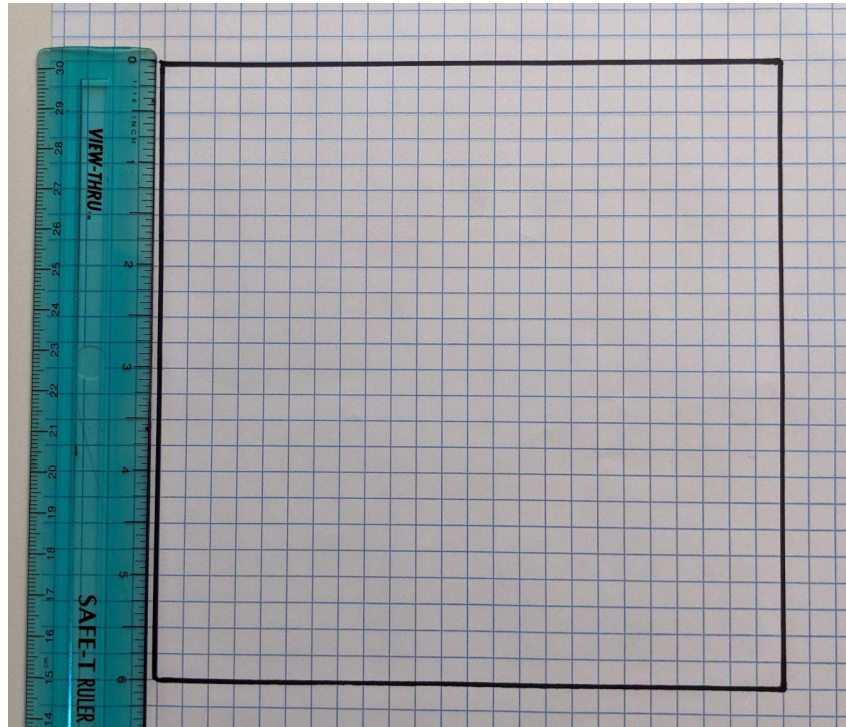
According to legend, George Washington asked Betsy Ross to sew a flag with six-pointed stars, thinking that five-pointed stars would be too difficult. But Betsy Ross was an expert fold-and-cut puzzle solver! By folding a piece of fabric, she was able to produce a symmetric five-pointed star with a single cut.

Before attempting to cut out five-pointed stars, let's try some simpler shapes.

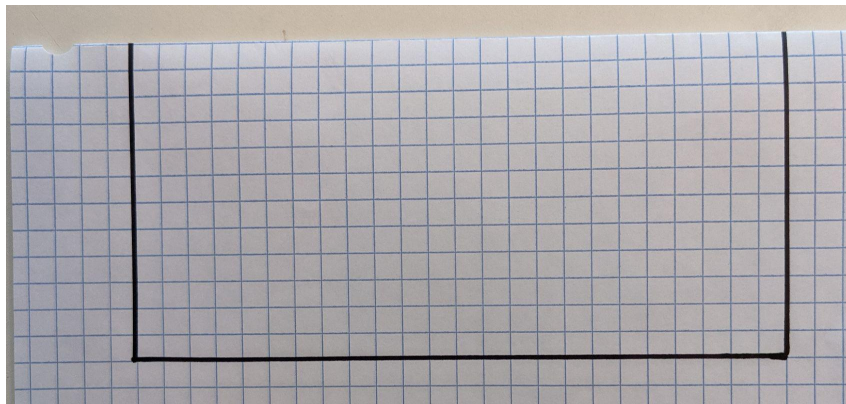
Materials

- Paper - preferably tissue paper - blue for the outline of the star(s), and red for the stripes
- A ruler or straight edge
- A protractor (only for the latter parts of this activity)
- A writing implement, preferably a thick marker
- Scissors

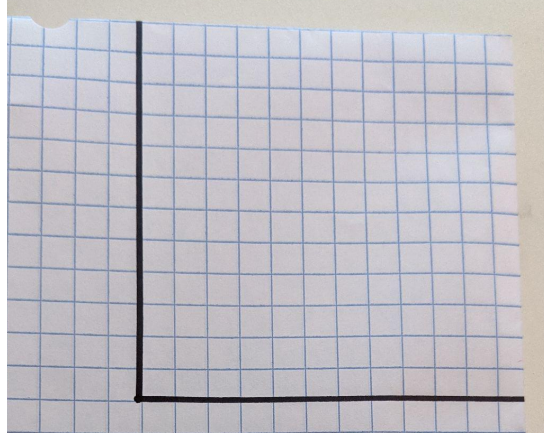
1. Draw a 6" square. We're going to cut it out with just one straight cut. I used graph paper below to make it easier to draw a square.



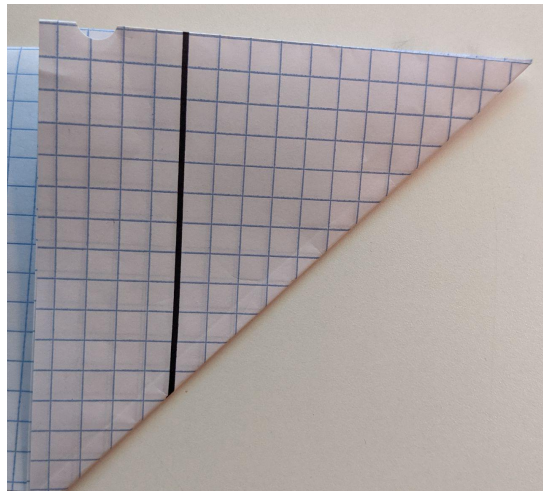
2. Fold the square in half.



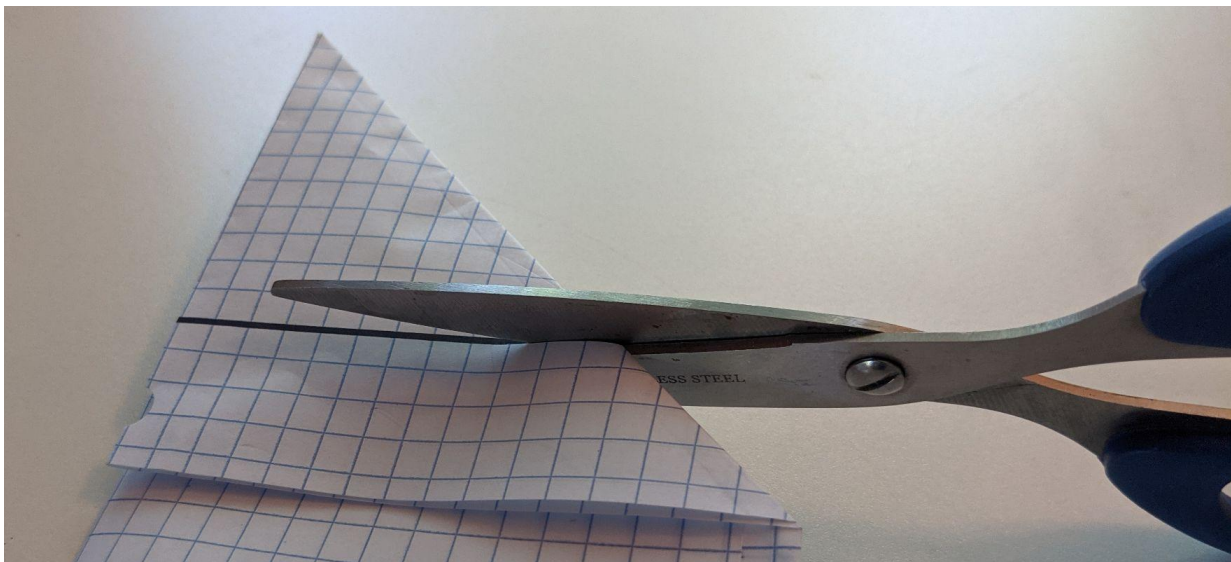
3. Then fold the visible rectangle in half in the other direction.



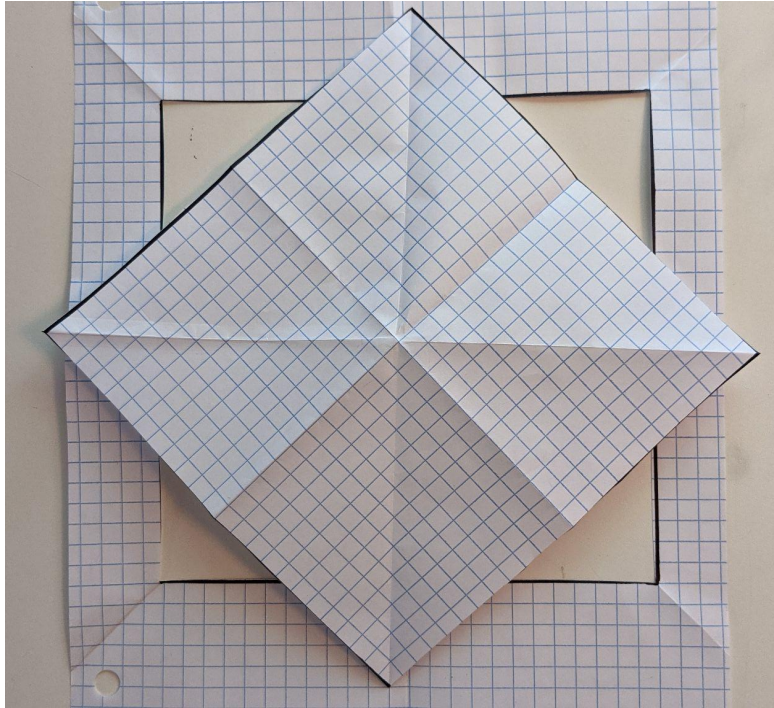
4. Next, fold the resulting square in half diagonally.



5. Cut along a visible straight line drawn.



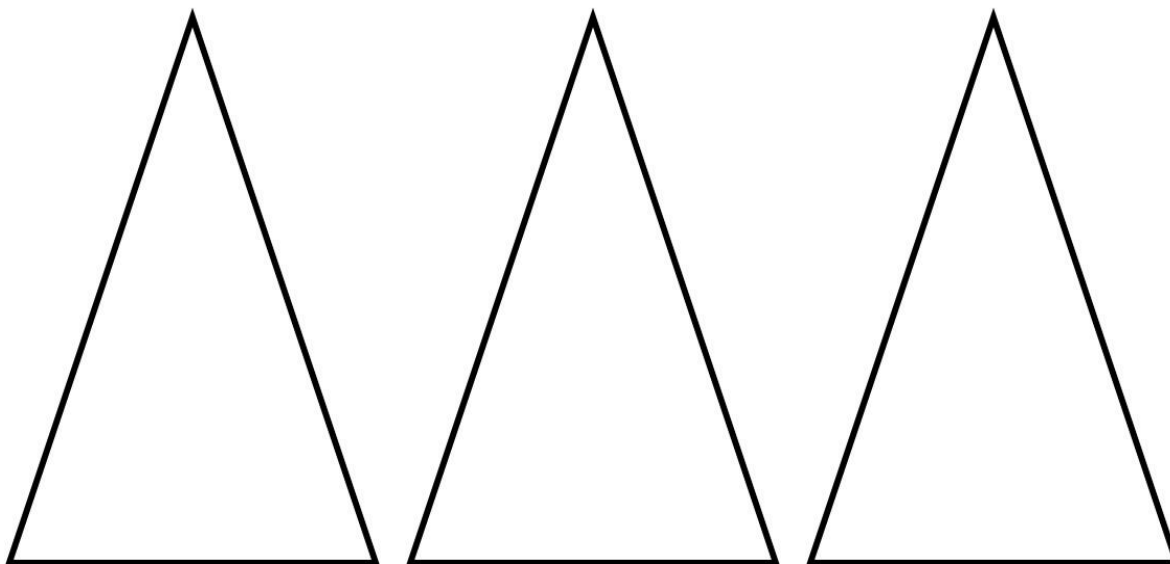
6. Unfold the page. Notice the whole 24"-perimeter square was cut with a single 3" straight cut.



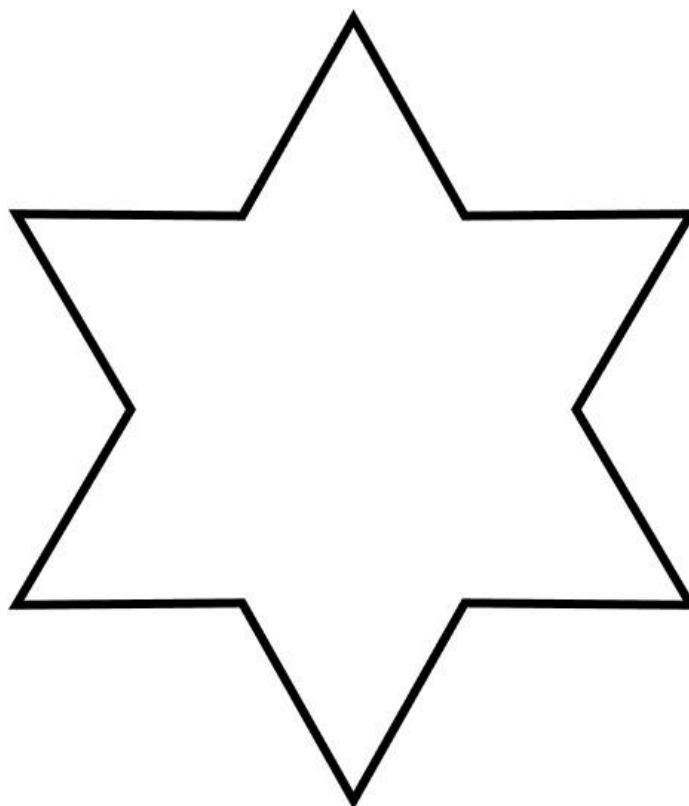
7. Can you cut out a square with fewer folds?
8. Now draw a rectangle on tissue paper and try folding it so that you can cut it out with a single straight cut. It takes the same number of folds as the square we did, but in this case part of the cut goes through 8 layers and part goes through only 4.



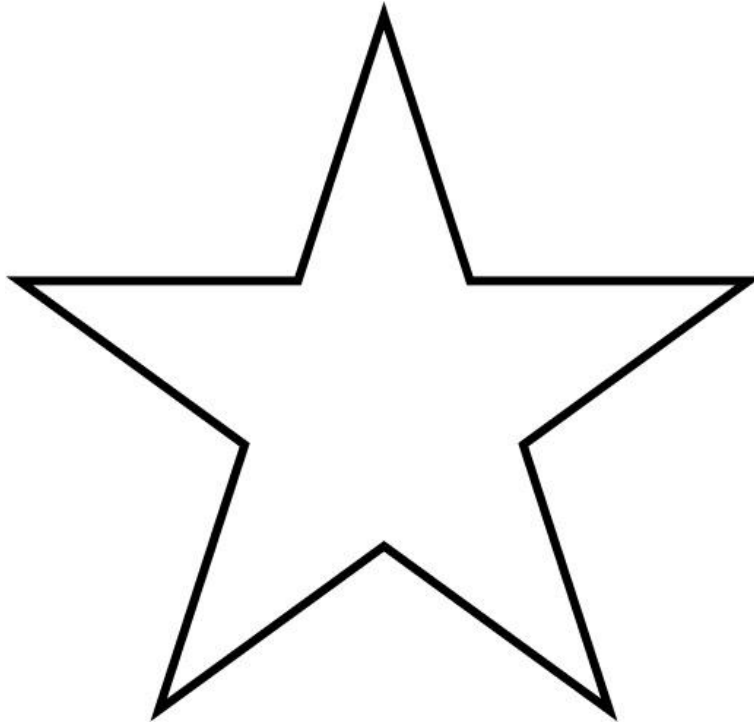
9. Next try an isosceles triangle, a triangle that has 2 sides of the same length. How many folds do you need?



10. Now on to a stellar shape with 12 sides. Can you cut this six-pointed star with a single straight cut? With how many folds?



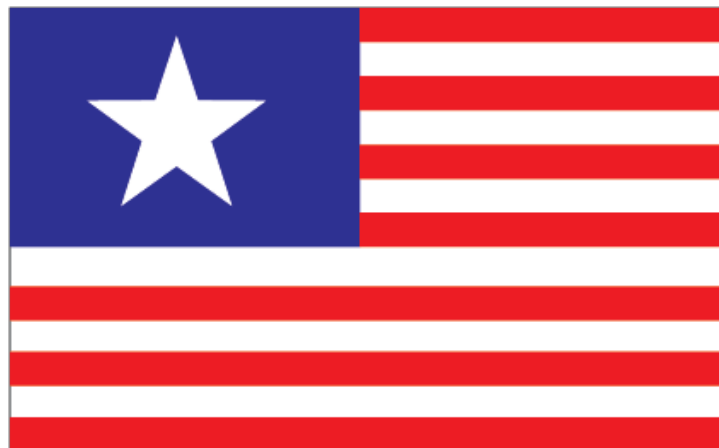
11. How do you think Betsy Ross cut out the five-pointed stars she sewed onto the United States flag?



We encourage you to experiment. Hint: The solution takes 4 folds and is included later in this article.

After you've mastered it, on blue tissue paper cut out a star and use the remaining background for the upper left corner of the United States flag.

12. Finally, fold red tissue paper and simultaneously cut 7 strips of equal width with a single cut. Use them for the 7 red stripes in the United States flag.
13. Lay out the stripes and star on a white tissue paper like this. Amaze people by telling them you made the whole thing with only 2 straight cuts!



Since your flag is not being made for the US. Federal government, legally (and luckily) its size, ratios, hues and numbers do not need to adhere to [government specification](#). But if you're feeling especially ambitious, you can make a flag like the one made by Betsy Ross. In fact, all 13 stars in the circle can even be created simultaneously with one straight cut! (Instructions based on The Colonial Flag on pages 33-34 of *Paper Capers* by Gerald M. Loe are included later in this article.)

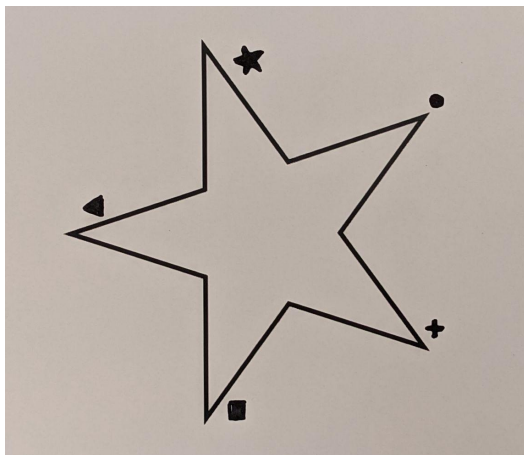


14. Send us photographs of your stars and flags or ideas for other mathematical crafts. Email us at info@jrmf.org.

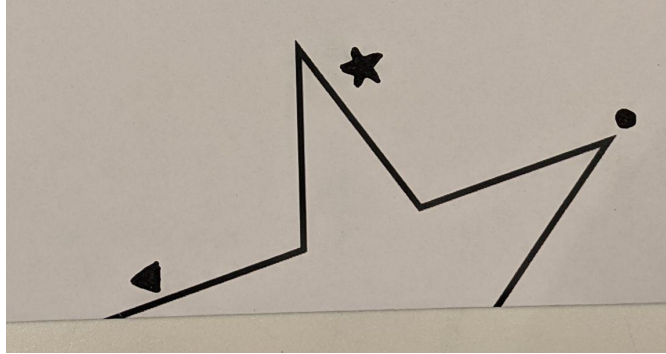
Authors: Nancy Blachman is the founder of the [Julia Robinson Mathematics Festival](#) and author of the free JRMF booklet [Fold-and-Cut Challenges](#). Skona Brittain is the mathemalinguist at JRMF and runs math circles in Santa Barbara, which occasionally do fold-and-cut challenges.

Solution: Here is one way to cut out a five-pointed star with one straight cut.

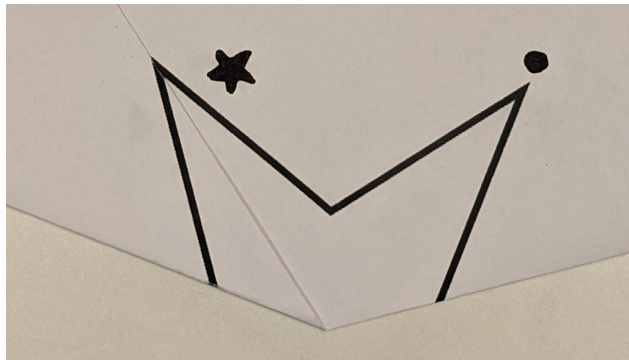
1. Draw a five-pointed star. (You can trace the one above.) Each vertex is labelled with a different symbol.



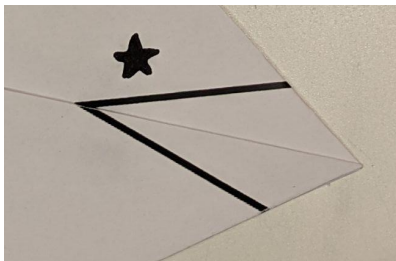
2. Fold along the center of one of the points.



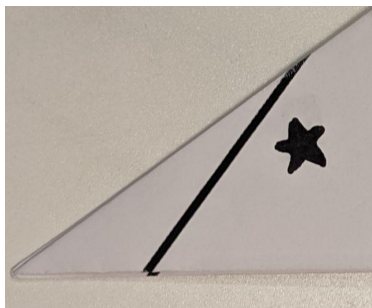
3. Fold the half-point so that it covers the top half of the visible adjacent point.



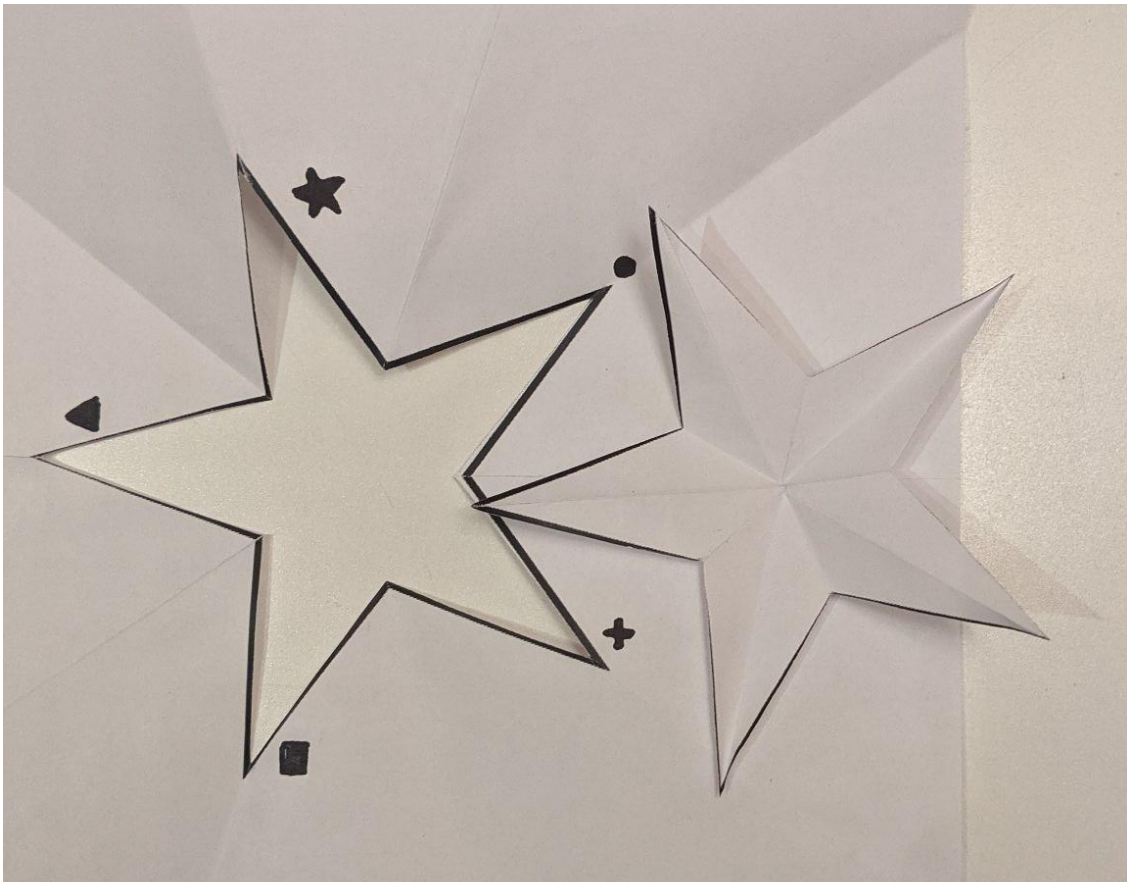
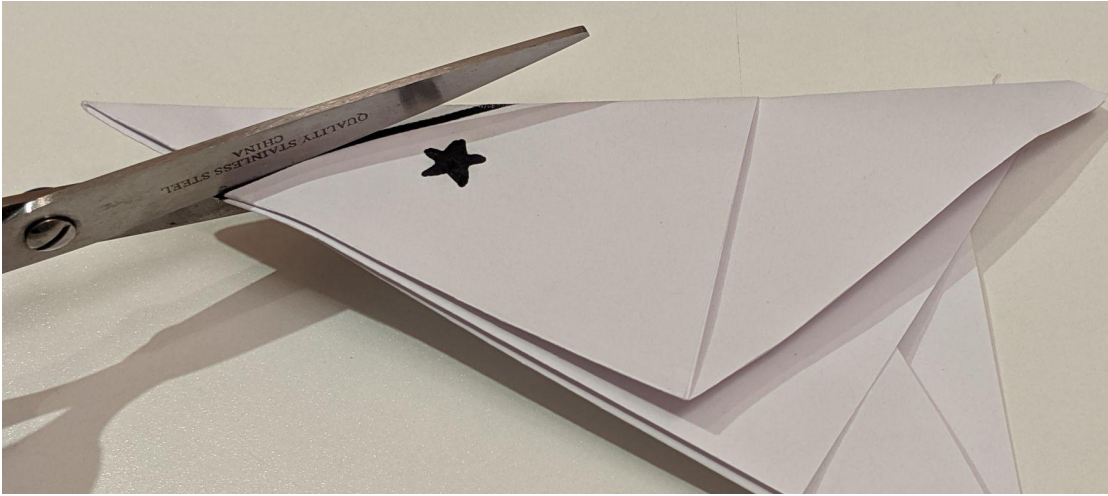
4. Then fold between the two visible points of the star so that one of them is on top of the other.



5. Fold in half so that all the outline lines are now on top of each other.



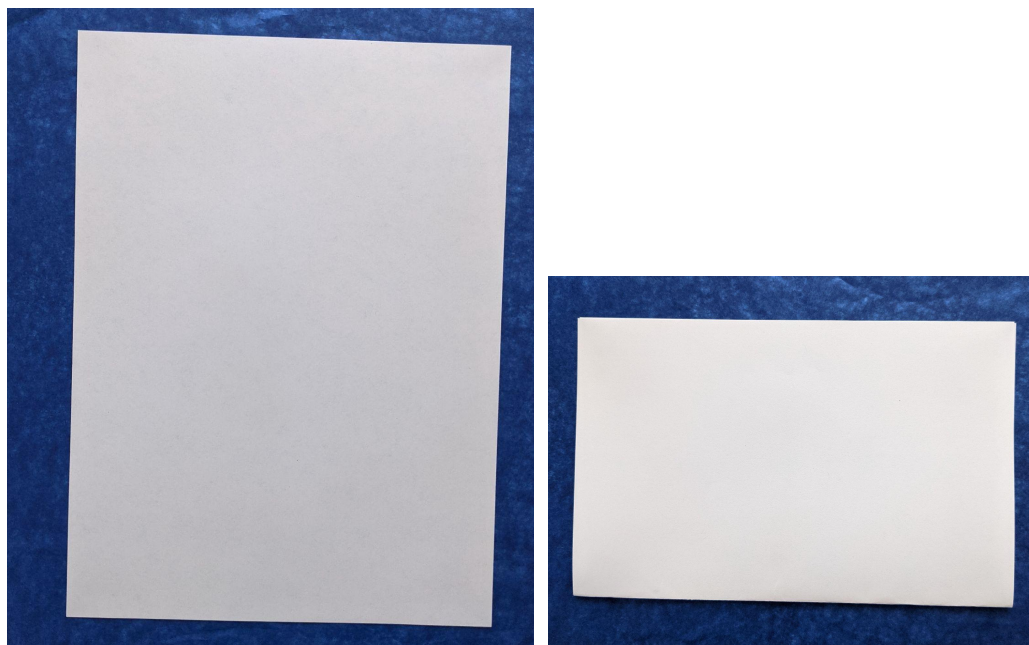
6. Cut along the outline segment of the star that is visible.



Below are instructions for folding and cutting a five-pointed star, and also 5 five-pointed stars at once, from *Paper Capers* by Gerald M. Loe. These do not require first drawing an outline of the star but they do require a protractor for measuring angles.

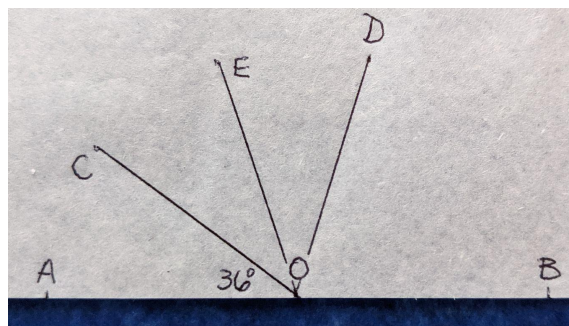
Five-pointed Star

An 8½" by 11" sheet of regular paper will work for a single star, but for the ring of 5 stars, start with a sheet of tissue paper that is at least 15" by 20". Fold it in half to create an 8½" by 5 ½" or 15" by 10" rectangle.



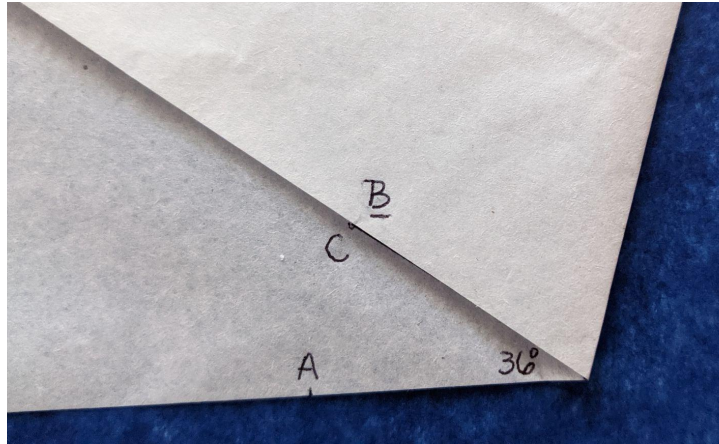
Folded edge is on the bottom

On the folded edge of the page, about halfway between the left and right edges of the page, just above the fold put a mark and label it O. Just above the fold to the far left of O, write A and on the far right side of O write B. Then measure 36 degrees (why 36 degrees?) from OA and draw a line and label the end point C. Also draw lines at 72 degrees and 108 degrees, and label them as shown below.

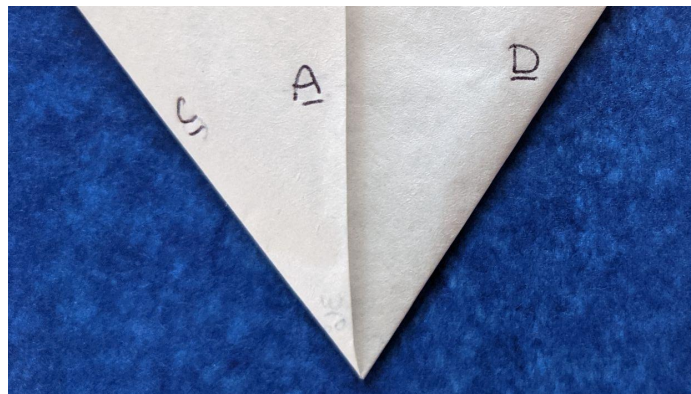


Folded edge

Fold OB up to OC and crease at OD. Since label B is now not visible, I wrote a B and underlined it to indicate that the original B is on the other side.



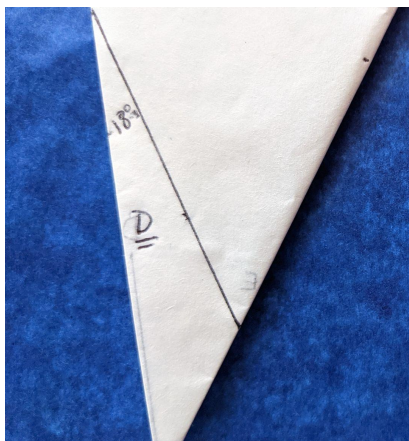
Fold OA up, creasing along OC . Labels C , A , and D are underlined to indicate that the original labels are on the other side of the folded paper.

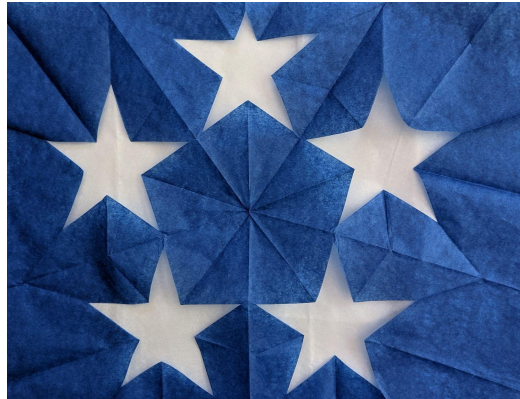


Fold edge OD to OC and crease at OA .

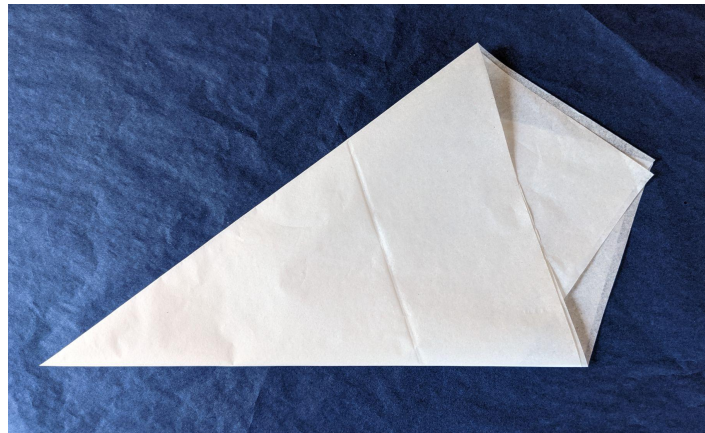


To cut out a ring of five stars, proceed to the instructions in the next section (under the heading *Ring of Five Stars*). To cut out a single five pointed star, next draw a line at 18 degrees from OC to OA and then cut along the line.

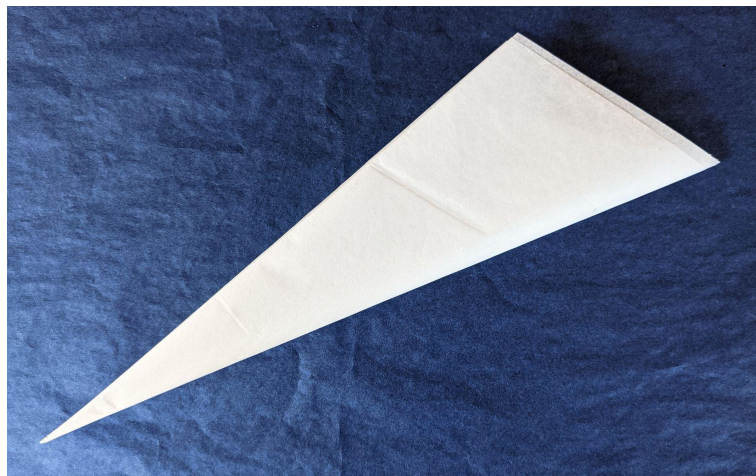


Ring of Five Stars.

Fold a 15" x 20" sheet of tissue paper into the form used for making a five-pointed star (as described above).



Crease the item through the middle of the point by folding and unfolding. Then make a fold about 4½" from the apex (the tip on the left) so that the apex lies on that crease, centered between both sides.

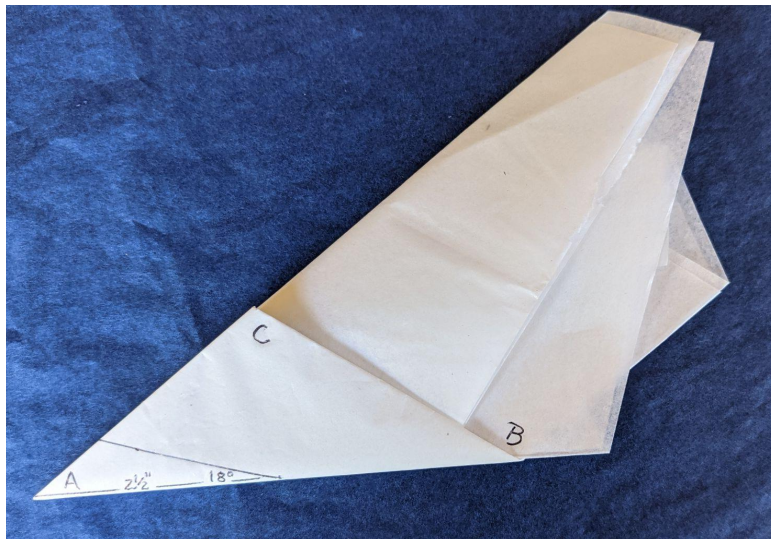




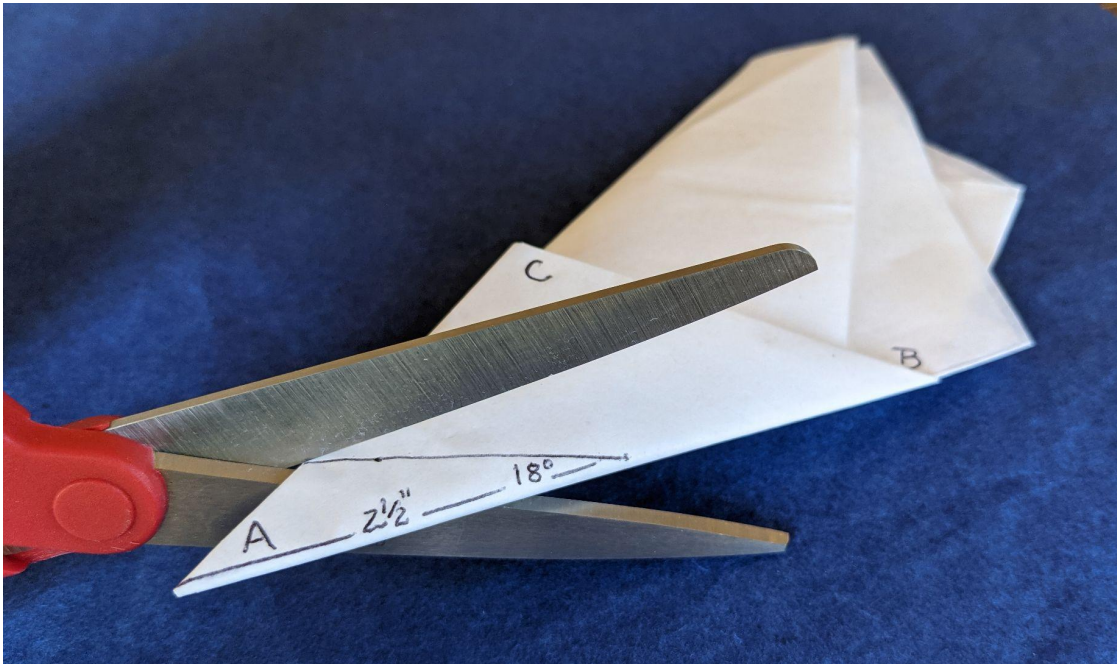
Next fold B down to the bottom edge and crease.

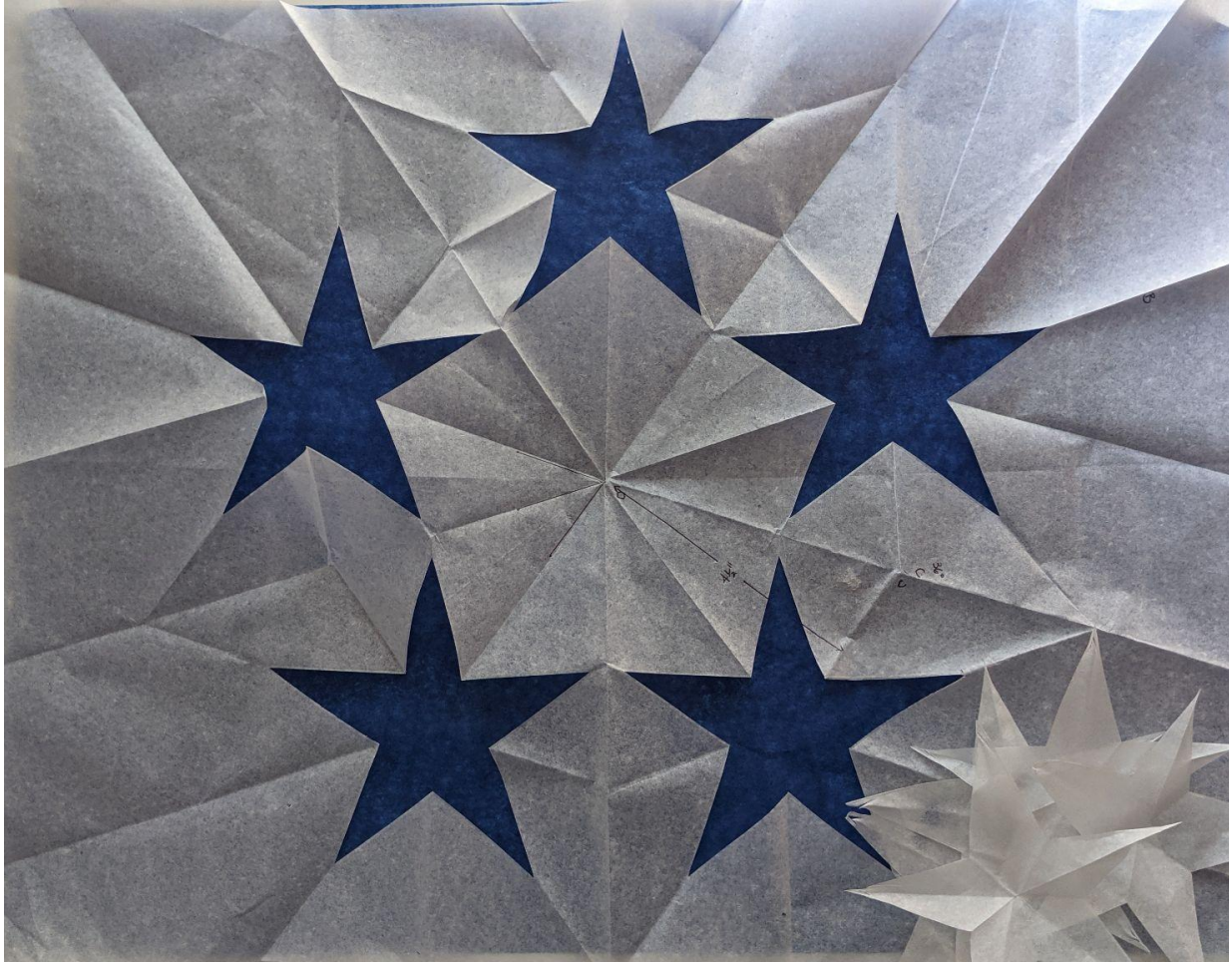


Fold C up by folding along AB. Draw a line $2\frac{1}{2}$ " from side AB of the point at 18 degrees.

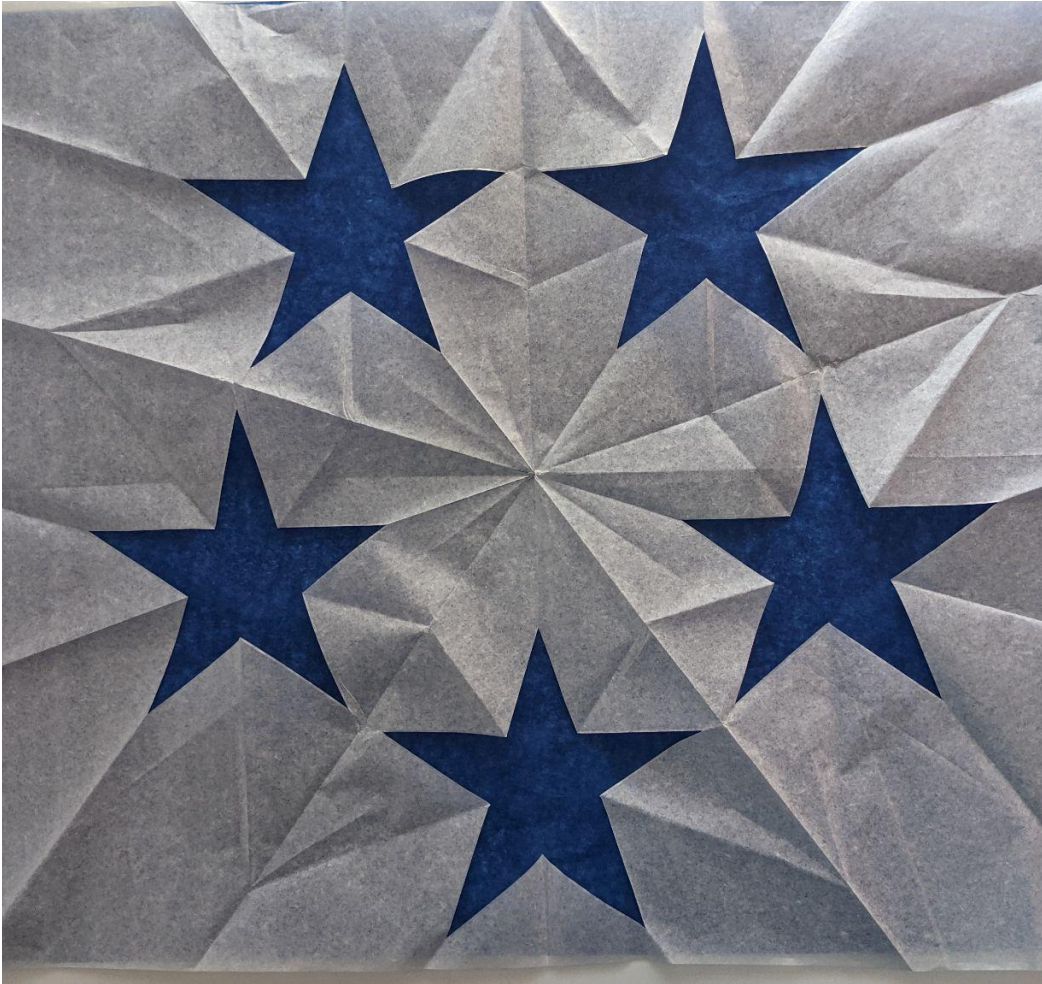
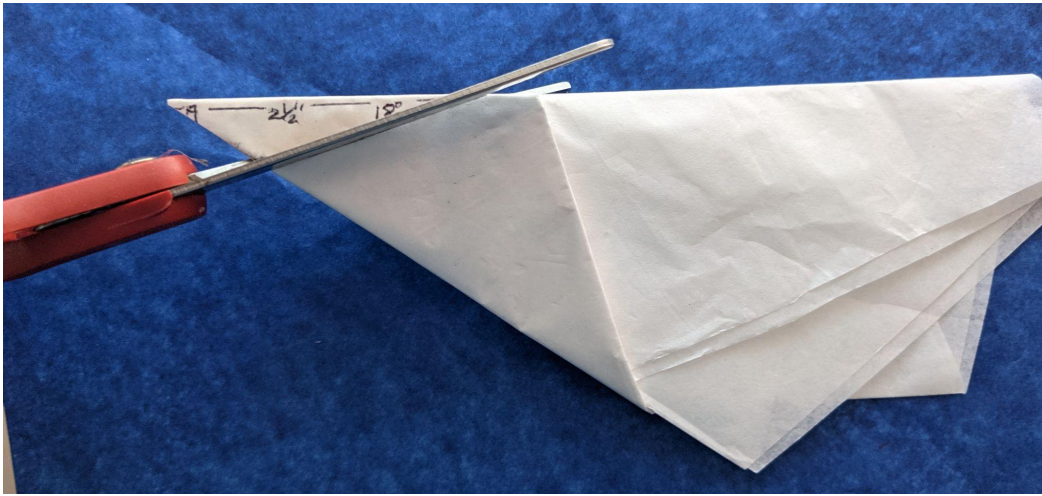


Cut along the line and carefully unfold.



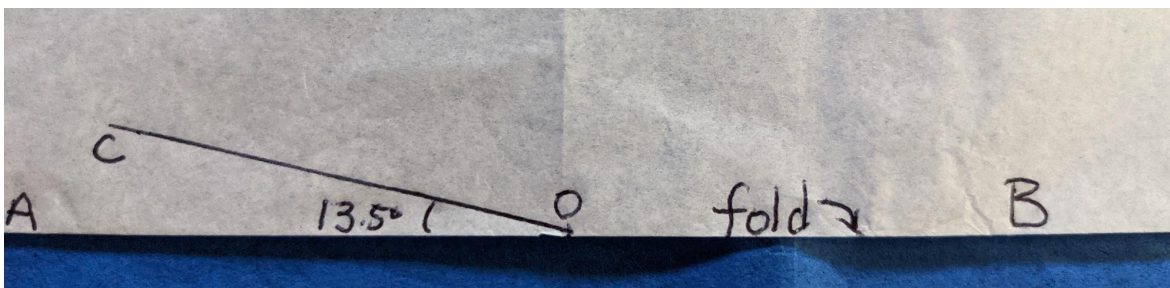


If the cut is made 18 degrees from the opposite side of the point, side AC, instead, then the stars will be rotated so that they point toward the center of the paper.

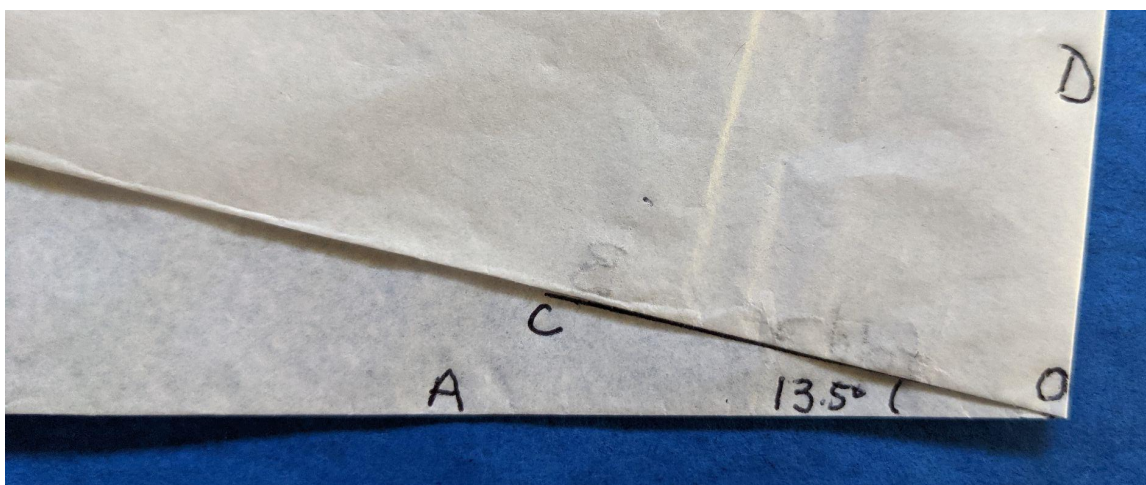


Ring of Thirteen Stars.

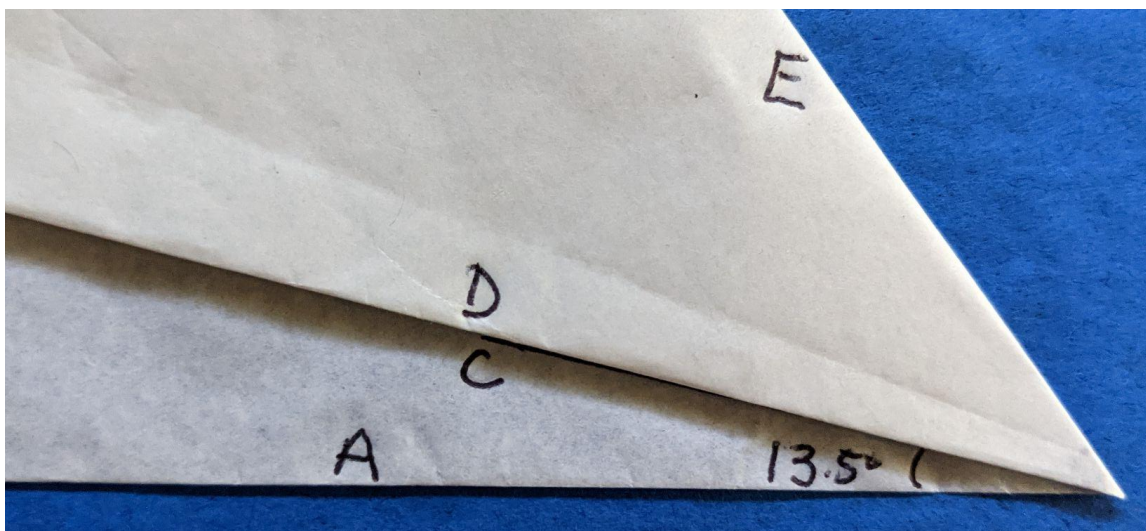
Fold a 20" x 20" sheet in half. On the folded edge of the page, halfway between the left and right edges of the page, put a mark and label it O. Just above the fold to the left of O, write A and on the right side of O write B. Then measure 13.5 degrees from OA and draw a line and label the end point C.



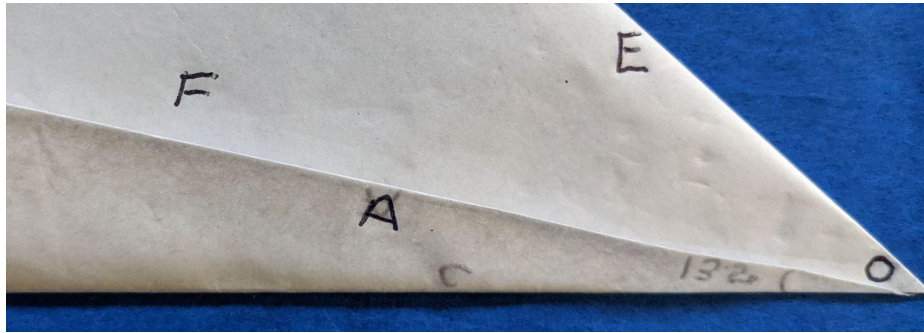
Fold OB to OC and label a point on the crease D.



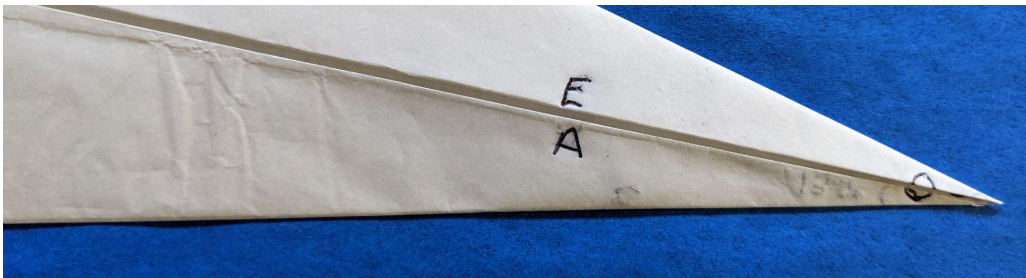
Now bring OD to OC and label a point on the crease E.



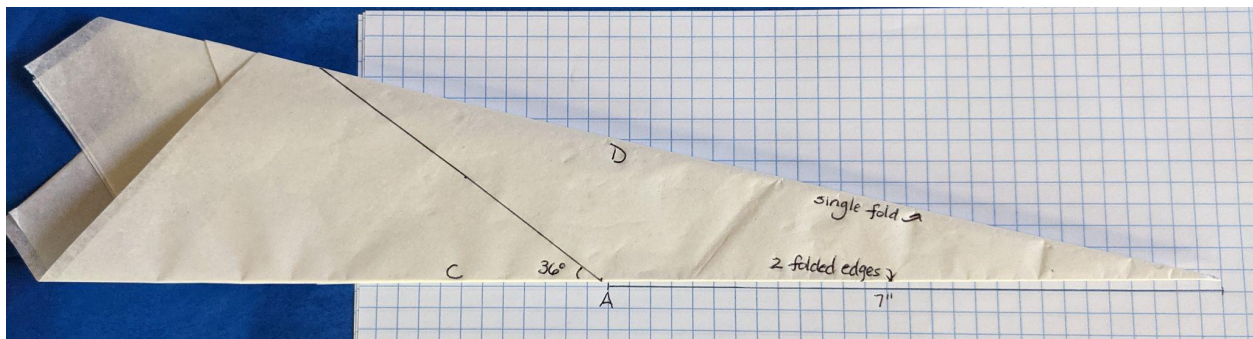
Fold OA over OC and crease at OC.



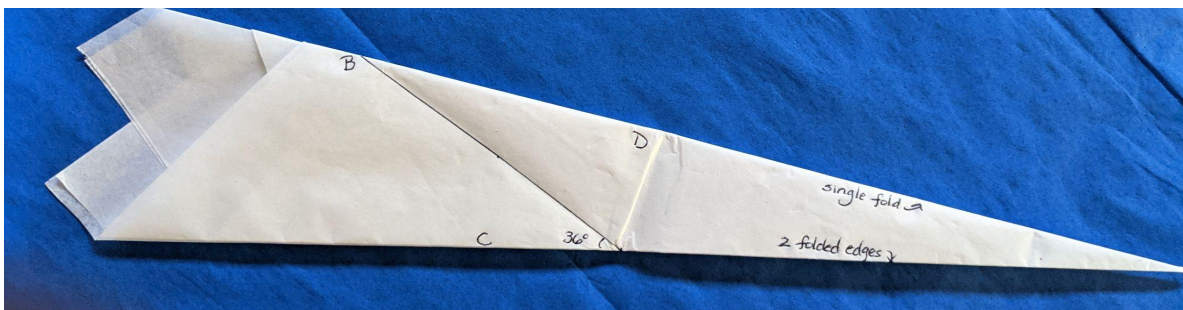
Fold edge OE to OA and label a point on the crease G.



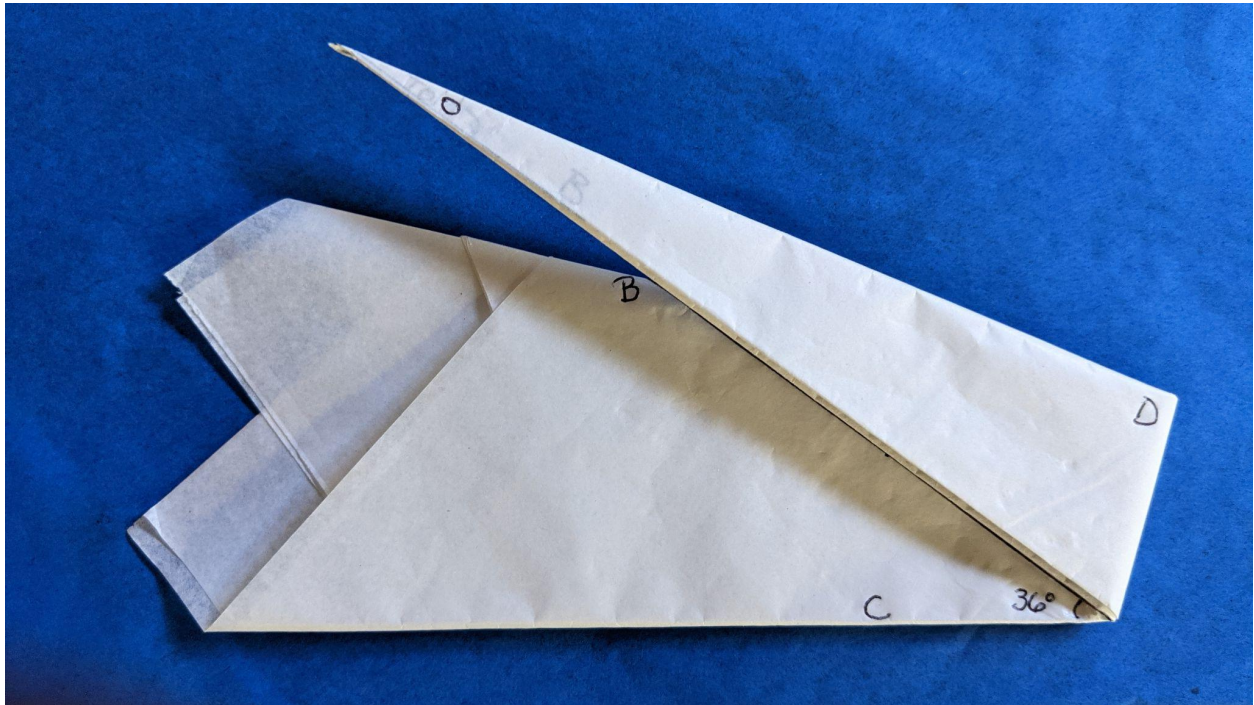
Fold edge OG over to OC, crease at OF and you will have a long wedge shaped paper. On one side of the wedge is a single fold and on the other are two folded edges. Measure 7" from the tip (labeled O) of the edge on the side with two folded edges (lower edge of the photo) and measure 36 degrees from that edge to the upper left corner.



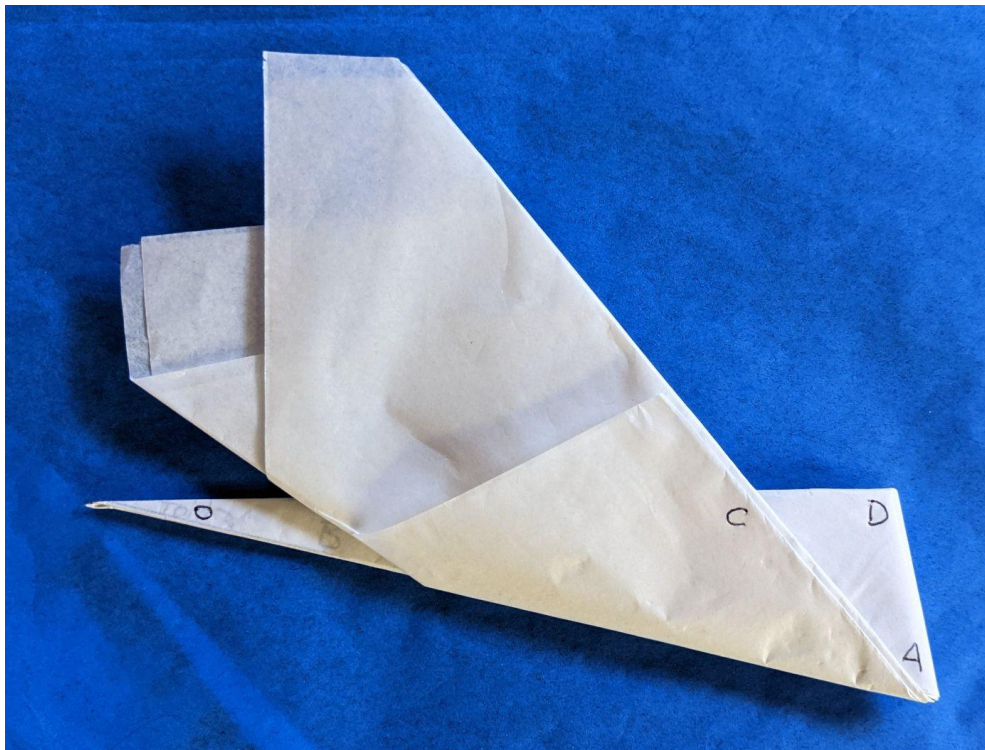
Here's what the folded paper now looks like.



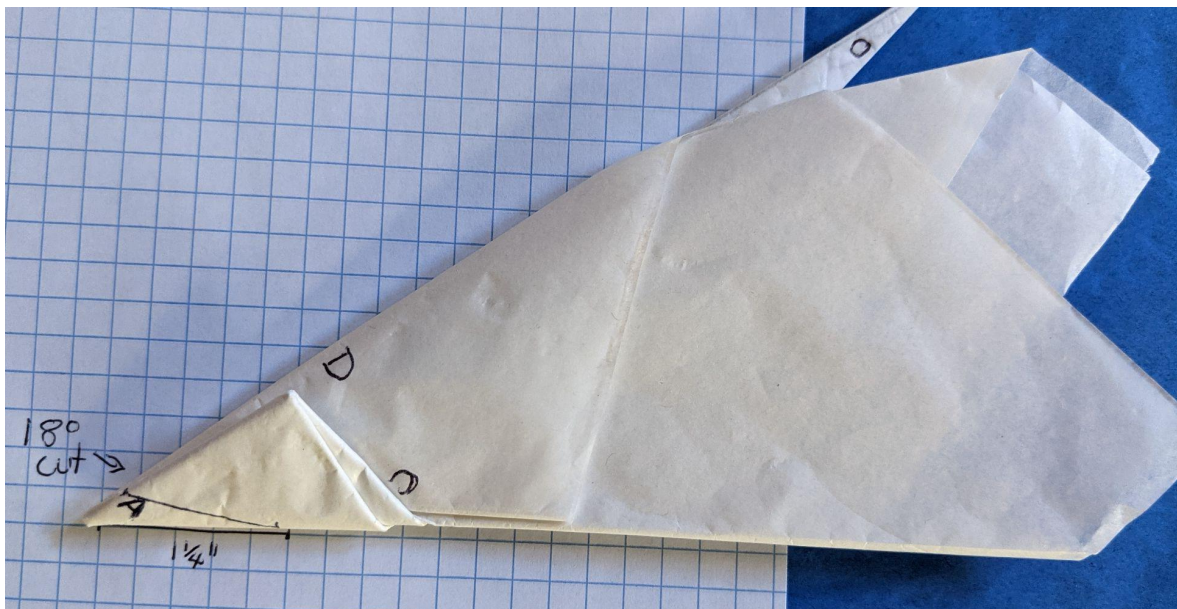
Now fold O so that the edge OA is at 36 degrees along the line that you just drew.



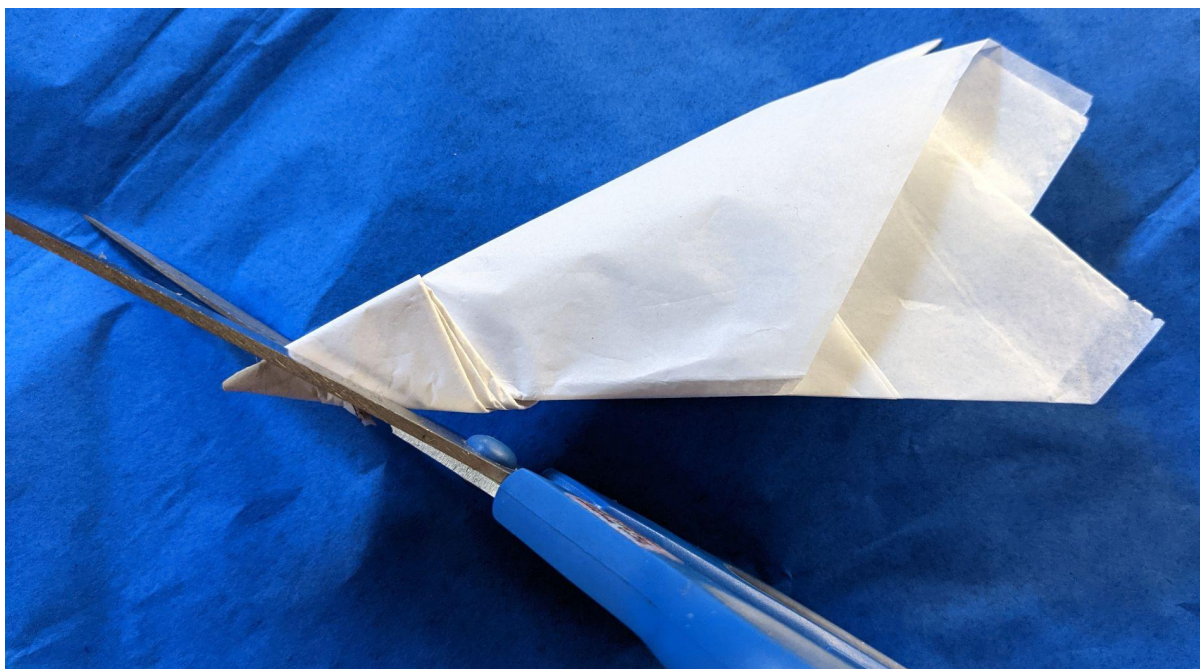
Now fold side C down along OA.



Next fold D along AC.



Measure $1 \frac{1}{4}$ " from AC and 18 degrees and cut along that line, which may not be easy since you will be cutting through 130 sheets of paper.



Here is the result.

