

Fraction Nim Game

Materials

- When playing as a whole class, you will need a whiteboard, and eraser and dry erase markers in two different colors.
- Once students begin playing against each other, they will need scrap papers and pencils or individual dry erase boards, markers, and eraser cloths.

Introducing the Game

To teach students to play the beginning version of the game, you will first need to construct the number line on the board with input from the students. Draw a number line and place the numbers 0, 1, 2, and 3 on it. Try to make the spacing as equal as possible and make the number line as long as feasible so that there is room for $\frac{1}{2}$ s, $\frac{1}{4}$ s, and $\frac{1}{8}$ s labels between the numbers. Ask the students where “one-half” should go. Ask what label should go on the number in the middle between the 1 and the 2. Place a tick mark between the 2 and the 3 and label it $2\frac{1}{2}$. Place tick marks in the middle between each pair of adjacent marks to accommodate the fourths. Ask students what numbers should go on each of the tick marks. If students seem unfamiliar with fractions, it may help to draw a “unit bar” above the number line from the 0 to the 1. Cutting the bar with the fourths marks will show them why we call these pieces fourths. Ask the students where $\frac{2}{4}$ lives on this number line, but leave it labeled as $\frac{1}{2}$. Label all of the fourths. Finally, place tick marks in the middle of each of the existing marks to accommodate the eighths. Ask students to tell you the labels for these marks.

Once the number line is drawn, explain the rules of the game. Two people take turns adding either $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$. The first person starts at 0 and makes an arc from 0 to either $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$. The next person starts where the other person landed and makes an arc that adds $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ to that number. Whoever lands exactly on the 3 at the end will win. No one is allowed to go past the 3. Explain that you have a secret strategy that allows you to win the game unless your opponent figures out the secret. Ask someone to volunteer to be your opponent even though they are probably going to lose. Ask them if they would like to go first or second. After drawing an arc, each player should say the starting number “plus” the number being added “equals” the sum. For example, the first person might say “ $0 + \frac{1}{2} = \frac{1}{2}$ ”. The second person might say “ $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ ”.

To win this game, you want to land on $\frac{3}{8}$, $\frac{3}{4}$, $1\frac{1}{8}$, $1\frac{1}{2}$, $1\frac{7}{8}$, $2\frac{1}{4}$, $2\frac{5}{8}$, and finally 3. I call these numbers the “Numbers of Doom” because your opponent is doomed if you land on them. Do not give any hints of what the strategy is to the students! Just beat them every time. Even if they accidentally land on some of the Numbers of Doom early on, they usually make a mistake eventually unless they have figured out the secret.

During the first two times, play the game as a full group first, and then allow students to try making their own number lines. After that, you may opt to play as a full group for part of the time if you wish, or students can simply play in partners the whole time. Ask students to get into pairs, and ask all students to draw their own number lines. That way, the students can play once on each of their number lines. Circulate among the students to help those who are having trouble drawing number lines or following the rules.

Encourage students to use rulers so that their marks are as equally spaced as possible. However, do not direct them to use a specific method to accomplish this – leave them free to devise their own methods. Once students catch on to drawing the number lines and labeling the fractions, you may wish to invite students to stop drawing the Nim tracks and play using pencils and unlabeled inch-marks on rulers (still

stating the fraction addition computations out loud). They can work with decimals using pencils and centimeter marks on rulers or meter sticks.

Changing the Rules

The rules of the Fraction Nim game can change once the students catch on to how to win the basic game. Here are a few variations to offer once the original game no longer offers a challenge. Each variation will have its own Numbers of Doom.

- There is a simpler version of the game where each player adds only $\frac{1}{2}$ or $\frac{1}{4}$. This makes the number line easier to draw and also makes the strategy much easier to find.
- Add $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ as before but make the number line go from 0 to 4, 5, $3\frac{1}{2}$, or any other number.
- Change the rules so that players add $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{6}$.
- Allow players to add $\frac{1}{10}$, $\frac{1}{5}$, $\frac{3}{10}$, $\frac{2}{5}$, $\frac{1}{2}$, or $\frac{3}{5}$, with a goal of $3\frac{7}{10}$.
- Change the rules so that players start at 10 and subtract $\frac{1}{4}$, $\frac{3}{4}$, or $\frac{5}{4}$ until they land at 0.
- Play the same game with decimals.
- Let them make up their own rules!